Mini-symposium ONDES
Sur les ondes en milieux aléatoires

Résumé

L'objectif du mini-symposium est de faire le point sur les récents développements - théoriques ou appliqués - du domaine des ondes en milieux aléatoires, ondes classiques ou quantiques. On pense notamment aux théories d'existence d'EDP non-linéaires à coefficients aléatoires, à leurs limites asymptotiques (e.g. homogénéisation, limite semi-classique), ou à l'imagerie en milieux complexes (imagerie médicale, seismologie).

Organisateur(s)

1. **Olivier Pinaud**, Colorado State University.
2. **Christophe Gomez**, Aix Marseille Université.

Liste des orateurs

1. **Romain Duboscq**, Institut de Mathématiques de Toulouse
   Titre : Equations de Schrödinger avec opérateurs bruités.
2. **Christophe Gomez**, Aix Marseille Université
   Titre : Asymptotic Behaviors for the Random Schrödinger Equation with Long-Range Correlations.
3. **Olivier Pinaud**, Colorado State University
   Titre : Stochastic Schrödinger equations with fractional multiplicative noise.
4. **Chrysoula Tsogka**, University of Crete
   Titre : Signal to Noise Ratio analysis in virtual source array imaging.

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Résumé du premier exposé

Dans cet exposé, nous aborderons la construction de la solution fondamentale de l'équation de Schrödinger stochastique avec opérateur hamiltonien quadratique. Ce résultat s'inspire des travaux de D. Fujiwara et passe par l'étude des trajectoires de particules classiques soumises à l'hamiltonien associé à l'équation. L'intérêt principal de cette construction est d'obtenir une formulation quasiment explicite de la solution. Nous nous intéresserons ensuite aux effets qui découlent de l'introduction d'opérateurs bruités dans l'équation de Schrödinger.

Résumé du second exposé

Wave propagation in random media with long-range correlations was recently stimulated by data collections showing that the underlying media of propagation can exhibit long-range correlation effects. Under the forward scattering approximation, the time-harmonic wave equation can be reduced to a random Schrödinger equation. In this talk, we will study the Schrödinger equation with a slowly decorrelating random potential, and show that the energy density of the wave function exhibits some anomalous diffusion phenomena, which can be captured numerically.

Résumé du troisième exposé

This talk is devoted to non-linear stochastic Schrödinger equations with multiplicative fractional noise, where the stochastic integral is defined following the Riemann-Stieljes approach of Zähle. Under the assumptions that the initial condition is in the Sobolev space $H^q(\mathbb{R}^n)$ for a dimension n less than three and q an integer greater than or equal to zero, that the noise is a $Q-$fractional Brownian motion with Hurst index $H \in (\frac{1}{2}, 1)$ and spatial regularity $H^{q+4}(\mathbb{R}^n)$, as well as appropriate hypotheses on the non-linearity, we obtain the local existence of a unique pathwise solution in $\mathcal{C}^0(0, T, H^q(\mathbb{R}^n)) \cap \mathcal{C}^{0,\gamma}(0, T, H^{q-2}(\mathbb{R}^n))$, for any $\gamma \in [0, H)$. Contrary to the parabolic case, standard fixed point techniques based on the mild formulation of the SPDE cannot be directly used because of the weak smoothing in time properties of the Schrödinger semigroup. We follow here a different route and our proof relies on a change of phase that removes the noise and leads to a Schrödinger equation with a magnetic potential that is not differentiable in time. We will also discuss the derivation of such Schrödinger equations from asymptotics of the wave equation.

Résumé du quatrième exposé

We consider the problem of virtual source array imaging. Motivated by geophysical applications, we assume that the illuminating array is at the surface of the earth while the reflector to be imaged is located in a homogeneous slab at some depth. We also assume that the medium between the reflector and the illuminating array is complex and strongly scattering. In this setup traditional migration imaging fails since the echoes from the reflector are lost in the noisy backscattered echoes from the ambient medium. In virtual array imaging, noisy traces are recorded on an auxiliary receiver array that is located in the homogeneous slab above the reflector and below the strongly scattering medium. Imaging is performed by migrating the cross correlations of the recorded field. We will illustrate with numerical results the robustness of virtual array imaging and present an analysis of the signal to noise ratio of the obtained image.