Mini-symposium FUCO
Problèmes numériques de la fusion contrôlée

Mini-symposium soutenu par l'IPL Inria Fr@res

Résumé

Controlled fusion is one of the energy source that can possibly replace the use of fossil fuel whose adverse environmental consequences are extremely serious. The studies on controlled fusion and the design of the experiments make use in a decisive manner of large scale numerical simulations. These simulations are extremely costly because of the large range of space (3 to 5 orders of magnitude) and time (5 to 7 orders of magnitude) scales met in these problems. They are also extremely complex and require well-designed numerical methods due to the strong anisotropy of the dynamics, fast along field lines and slow transverse to them.This symposium will consider numerical challenges in this field both for MHD fluid and kinetic models.

Organisateur(s)

1. **Hervé Guillard**, Inria Sophia Antipolis Méditerranée and Université de Nice Sophia Antipolis, LJAD, UMR 7351, 06100 Nice.

Liste des orateurs

1. **Yanick Sarazin**, CEA, IRFM, Saint-Paul-Lez-Durance,
   Titre : Physical challenges and numerical issues in controlled fusion plasmas.
2. **Emmanuel Franck**, Inria Nancy-Grand-Est
   Titre : Hierarchie de modèles fluides et solveurs en temps pour les plasmas de Tokamak.
3. **Charles Prouveur**, Institut Camille Jordan, Université Claude Bernard Lyon 1
   Titre : High order time discretization for backward semi-Lagrangian methods.
4. **Nicolas Crouseille**, Inria Rennes Bretagne Atlantique
   Titre : Méthode de splitting pour les équations de Vlasov-Maxwell.
5. **José Costa**, Inria Sophia Antipolis Méditerranée et Université de Nice Sophia Antipolis, LJAD, UMR 7351
   Titre : Full-MHD implementation for X-point geometry in Tokamaks.

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1 Yanick Sarazin, ”Physical challenges and numerical issues in controlled fusion plasmas”

In the route towards harnessing controlled fusion on Earth, the international ITER (”the way” in Latin) project represents a major step forward. ITER, currently under construction at Cadarache, France, is a tokamak, a device in which a hot plasma at about 150.10^6 Kelvin is confined by strong magnetic fields of a few Tesla within nested toroidal magnetic surfaces. Critical physical issues arise from this specific configuration. First of all, the magnetic configuration, which partly results from the current flowing into the plasma, should be stable with respect to large scale magneto- hydro-dynamical (MHD) instabilities. Secondly, the insulating property of the magnetic topology should be optimum as it governs the overall fusion performance, i.e. the ratio of fusion power over injected power. It turns out that the energy confinement is mostly governed by micro-scale turbulence in tokamak plasmas when MHD activity is quiescent. So far, the main physical parameters (size, shape, etc.) of new tokamaks, including ITER, have been chosen on the basis of empirical scaling laws, which predict how the energy confinement time τ_E varies with critical dimensionless parameters. Both the inherent dispersion of the multi-machine data points, of about 15%, and the fact that this τ_E curve is extrapolated out of the operational regime of current devices, ask for grounding these scaling laws within well understood theoretical bases. Importantly, controlled fusion will operate with small margins in terms of performance, so that optimization is required. Reliable understanding and control are therefore mandatory. Hence the need for first principle modeling of both MHD instabilities and turbulence dynamics, in view of understanding, predicting - including outside the current experimental domain - and possibly controlling the plasma confinement. Two main models are currently being used to address these issues, namely i) dimensional fluid models for MHD stability and transport studies at the plasma periphery, and ii) dimensional so- called gyrokinetic models for core turbulence. First, these intrinsically nonlinear models reveal extremely challenging from the numerical point of view, partly because of the broad range of involved spatial (3 to 5 orders of magnitudes) and temporal (5 to 7 orders of magnitude) scales, and also because of the strong anisotropy of the dynamics, fast along field lines and slow transverse to them. As a matter of fact, each of the associated codes already uses several tens of millions CPU hours per year on the largest supercomputers worldwide, and would require even more for ITER relevant parameters. Second, these models however suffer critical limitations, which would need being overcome in the near future. Two kinds of limitations can be distinguished : either intrinsic, within each model, or extrinsic, i.e. requiring the treatment of all the various physical issues within the same description to look for possible competing and/or synergistic effects. The talk will review the main physical challenges in tokamak plasmas and the current status of the first principle models, from a theoretical and a numerical point of view. The GYSELA, JOREK and TOKAM3X codes, which are developed and/or used at IRFM, will serve as examples : their development involves multi-disciplinary teams and spans several years, in between 5 and 15 years. The last part of the talk will address their limitations, will discuss tentative ways to overcome them and possible numerical bottlenecks.

2 Emmanuel Franck, ”Hierarchie de modèles fluides et solveurs en temps pour les plasmas de Tokamak”

Ce travail se place dans le contexte de la simulation des instabilités de bords pour les plasma dans un tokamak de type ITER. Pour simuler ces instabilités on résout des modèles de type MHD dans des géométrie toroïdal complexe. Le code JOREK initialement développé au CEA est un des codes plus important en Europe pour la simulation de ces instabilités. Pour cela on résout le modèle à l’aide d’un schéma implicite en temps, d’une méthode d’élément finis dans le plan poloidal et de méthode de Fourier dans la direction toroïdale. Actuellement on utilise un solveur GMRES préconditionné par une matrice diagonale par bloc (de fourier) ou chaque bloc est résolu exactement. Ce type de méthode est peu efficace en phase non linéaire et très coûteux en mémoire. Dans un premier temps on donnera une hiérarchie de modèles fluide (systèmes hyperboliques avec termes de diffusion) qui permettent de modéliser les plasmas. Pour chacun de ces modèles on discutera le problème de conservation de l’énergie et de stabilité. La seconde partie de l’exposé portera sur un préconditionneur dit physique qui est basé sur une approximation et une réécriture des équations. L’idée de base est d’approcher la Jacobienne du système hyperbolique par un opérateur d’onde du second ordre. Ce système d’onde est un opérateur plus facile à inverser avec des méthodes type multi-grilles. La résolution de l’opérateur d’onde implicite correspond à la phase de

préconditionnement. Cette méthode est compatible avec les méthodes "jacobian-free" utile pour réduire le coût mémoire.
Ce type de préconditionnement sera testé d'abord sur un problème d'onde raides avec termes sources utilisé en transfert radiatif (photonique) puis sur un modèle de MHD réduite. Pour finir on donnera des pistes pour optimiser l'algorithme et l'étendre à des problèmes plus complexes.

3 Charles Prouveur, "High order time discretization for backward semi-Lagrangian methods"

We introduce different high order time discretization schemes for backward semi-Lagrangian methods. These schemes are based on multi-step schemes like Adams-Moulton and Adams-Bashforth schemes combined with backward finite difference schemes. We apply these methods to transport equations for plasma physics applications and for the numerical simulation of instabilities in fluid mechanics. In the context of backward semi-Lagrangian methods, this time discretization strategy is particularly efficient and accurate when the spatial error discretization becomes negligeable and allows to use large time steps.

4 Nicolas Crouseille, "Méthode de splitting pour les équations de Vlasov-Maxwell"

Dans cet exposé, nous proposons des discrétisations temporelles nouvelles pour les équations cinétiques de Vlasov-Poisson et de Vlasov-Maxwell. L'approche repose sur une méthode de splitting d'opérateurs, motivée par une décomposition du Hamiltonien correspondant. Cela permet de construire des schémas d'ordre arbitrairement élevé par composition de chaque étape.
Dans le cas de Vlasov-Poisson, la décomposition s'effectue entre l'énergie cinétique et l'énergie électrique, ce qui correspond au splitting directionnel classique. Les relations d'ordre permettant de calculer les coefficients de splitting peuvent être simplifiées par rapport au cas général, et des méthodes dédiées d'ordre élevé (jusqu'à l'ordre 6), mettant en jeu un nombre réduit d'étapes sont alors construites.
Dans le cas de Vlasov-Maxwell, la décomposition s'effectue entre l'énergie cinétique, l'énergie électrique et l'énergie magnétique. Le schéma obtenu est alors nouveau et par composition, il peut être d'ordre arbitraire ((Strang : ordre 2, triple-jump : ordre 4, ?). De plus, ce schéma préserve la charge : il permet de garantir la contrainte de Gauss automatiquement, sans la résoudre explicitement.
Ces travaux ont été effectués en collaboration avec F. Casas (Université de Valence), L. Einkemmer (Université de Innsbrück), E. Faou (Inria-Rennes) et M. Mehrenberger (Université de Strasbourg).

5 José Costa, "Full-MHD implementation for X-point geometry in Tokamaks"

In the context of "controlled fusion" by magnetic confinement of the plasma in tokamaks devices, reduced MagnetoHydroDynamic (MHD) models can be derived, with the following approximations
– The toroidal magnetic field amplitude dominates over the poloidal field strength
– The toroidal magnetic field strength is constant in time.
The assumption of stationarity of the toroidal component of the magnetic field eliminates the fast compressional magneto-acoustic waves. Moreover, it avoids the compression of the toroidal magnetic field which is consistent with observations in existing devices (JET, ...). The frequencies of fast acoustic waves are higher than most phenomena of interest at the resistive time scale. These assumptions are then reasonable for high aspect ratio devices, when the the plasma flow regime is relatively close to the Grad-Shafranov equilibrium. In this case the suppression of fast compressional magneto-acoustic waves have reduced impact on the global dynamics leading to numerical efficiency in terms of computational cost, as the CFL condition is less restrictive. This model has been used to calculate various instabilities and has the great advantage of being easy to manipulate and less non-linear than the Full MHD model.
For highly nonlinear MHD instabilities associated to ITER device, however, the flow regime will certainly go out of these assumptions. Therefore, full MHD modeling is an important step towards the design of numerical strategies that will be potentially able to produce reliable and predictive results for critical

situations In ITER. For that, the divertor configuration - with X-point - must be simulated while paying special attention to the boundary conditions concerning the interactions of the plasma with a material wall. These conditions, named Bohm boundary conditions, are well established for simplified configurations where the magnetic field is almost orthogonal to the boundary. In this case, a simple analysis set out a lower bound of the parallel velocity to the sound speed. This result in a particular case is very often used for general context, with some marginal adjustments. For the full MHD model, we have to make these Bohm conditions consistent with the hyperbolic nature of the system under consideration where boundary conditions can be fixed only for quantities carried by incoming characteristics. This becomes very challenging by the effect of fast acoustic waves.