Mini-symposium SoP
Statistics of Processes

Résumé

This mini symposium is aimed to describe some recents advances in statistical studies of processes : the first session is about asymptotic properties of the discretized trajectories of a jump process. The study of the jumps is particularly important in finance modeling. The second session deals with limit theorems for realized co-volatility vectors, and this subject is also very popular in financial economics. The third session is about autoregressive processes, more precisely correlation testing. Finally the last session deals with a generalization of the Bessel clock.

Organisateur(s)

1. **Marguerite Zani**, Mapmo, Université d'Orléans.

Liste des orateurs

1. **Emmanuelle Clément**, Lama, Université Paris–Est Marne-la-Vallée
   Titre : Local Asymptotic Mixed normality property for discretely observed stochastic differential equations driven by stable lévy processes.
2. **Hacène Djellout**, Laboratoire de Mathématiques, Université Blaise Pascal
   Titre : Large Deviations of the realized (co)–volatility.
3. **Frédéric Proia**, LAREMA, Université d'Angers
   Titre : Correlation testing procedures in the autoregressive process.
4. **Marguerite Zani**, Mapmo, Université d'Orléans
   Titre : Large deviations for clocks of semi-stable processes.

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Introduction

The first part of this mini symposium deals with statistical study of stochastic processes with jumps, from high frequency data. This has been the subject of many recent works. A major issue is to determine if the jump part is relevant to model the observed phenomenon. Especially, for modeling of asset prices, the assessment of the part due to the jumps in the price is an important question.

Another theme presented in this symposium is the study of co-volatility vectors. Realized statistics based on high frequency returns have become very popular in financial economics. In recent years, different non-parametric estimators of the variation of a log-price process have appeared. Among them are the realized quadratic (co-)variation which is perhaps the most well known example, providing a consistent estimator of the integrated (co-)volatility when the logarithmic price process is continuous.

The third part is devoted to the study of autoregressive processes, and the correlation testing.

Last, there will be a presentation of large deviation results for clocks of semi-stable Lévy processes. These objects generalize the well known Bessel clock which appears in the Asian option studies.

1 Local Asymptotic Mixed normality property for discretely observed stochastic differential equations driven by stable lévy processes

In [1] the authors prove prove the Local Asymptotic Mixed Normality property from high frequency observations, of a continuous time process solution of a stochastic differential equation driven by a pure jump Lévy process. The process is observed on the fixed time interval $[0, 1]$ and the parameter appears in the drift coefficient only. They compute the asymptotic Fisher information and find that the rate in the LAMN property depends on the behavior of the Lévy measure near zero. The proof of this result contains a sharp study of the asymptotic behavior, in small time, of the transition probability density of the process and of its logarithm derivative.

2 Large deviations of the realized (co)–volatility

In [2] the authors propose to study the large deviation properties of realized (co-)volatility. More specifically, they consider a bivariate model with synchronous observation schemes and correlated Brownian motions of the following form : $dX_{\ell,t} = \sigma_{\ell,t} dB_{\ell,t} + b_\ell(t,\omega)dt$ for $\ell = 1, 2$, where X_ℓ denotes the log-price, they are concerned with the large deviation estimation of the vector $V_t^n(X) = \left(Q_{1,t}^n(X), Q_{2,t}^n(X), C_t^n(X)\right)$ where $Q_{\ell,t}^n(X)$ and $C_t^n(X)$ represente the estimator of the quadratic variational processes $Q_{\ell,t} = \int_0^t \sigma_{\ell,s}^2 ds$ and the integrated covariance $C_t = \int_0^t \sigma_{1,s}\sigma_{2,s}\rho_s ds$ respectively, with $\rho_t = cov(B_{1,t}, B_{2,t})$. The main motivation is to improve upon the existing limit theorems such as the weak law of large numbers or the central limit theorem which have been proved in different contexts. As an application they provide the large deviation for the standard dependence measures between the two assets returns such as the realized regression coefficients up to time t, or the realized correlation. They also prove the same results for the threshold estimator in the presence of the jump component. This study should contribute to the recent trend of research on the (co-)variance estimation problems, which are quite often discussed in high-frequency financial data analysis.

3 Correlation testing procedures in the autoregressive process

In [3] the author is interested in the implications of a linearly autocorrelated driven noise on the usual estimates in a stable autoregressive process. He suggests a sharp analysis on the asymptotic behavior of the least squares estimators of the autoregressive and the serial correlation parameters. Then, he derives a statistical procedure enabling to test for correlation of any order in the residuals of an autoregressive modelling. He concludes discussing on some well-known testing procedures that appear to be particular cases of this study.

4 Large deviations for clocks of self-similar processes

The Lamperti correspondence gives a prominent role to two random time changes : the exponential functional of a Lévy process drifting to ∞ and its inverse, the clock of the corresponding positive self-similar process. In [4] the authors describe asymptotical properties of these clocks in large time, extending the results of Yor and Zani.

Conclusions

Statistical studies of processes are particularly important in the process of modelisation for disciplines as various as finance, econometrics, etc... This mini symposium is a review of recent progresses in this field.

Références

[1] E.Clément and A.Gloter., *Local Asymptotic Mixed Normality property for discretely observed stochastic differential equations driven by stable Lévy processes.*, Stochastic Processes and their Applications, 125, p2316-2352, 2015.

[2] H. Djellout, A. Guillin and Y. Samoura, *Large deviations of the realized (co)–volatility vector*, arXiv :1411.5159 .

[3] F. Proia, *Testing for residual correlation of any order in the autoregressive process* , arXiv :1312.2240.

[4] N. Demni, A. Rouault and M. Zani, *Large deviations for clocks of self-similar processes* , arXiv :1411.5543 , to appear in the special Volume of Springer LNM in honour of M. Yor.