

*Mini-symposium DMRI*  
*Analysis and simulation in diffusion magnetic resonance  
imaging (dMRI)*

**Résumé**

This mini-symposium brings together mathematicians, physicists and experimentalists who work in mathematical analysis, numerical simulation and experimental aspects of diffusion magnetic resonance imaging (dMRI). The first two talks discuss the application to porous media, including an overview of the methodology and challenges as well as more recent results. The next three talks focus on biological applications, with the third talk explaining the formulation of a theoretical model of the time-dependent diffusion coefficient, the fourth talk on numerically simulating the dMRI signal of the neuronal network of the Aplysia, and the last talk on the efficient processing and interpretation of the dMRI signal from the brain white matter fibers.

**Organisateur(s)**

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**Liste des orateurs**

1. **Stephane Rodts**, Laboratoire Navier - Université Paris Est  
*Titre* : Mesure RMN de diffusion restreinte en milieux poreux : pratique et interprétations.
2. **Denis Grebenkov**, Laboratoire de Physique de la Matière Condensée, CNRS, Ecole Polytechnique  
*Titre* : IRM de diffusion dans des milieux poreux : défis théoriques.
3. **Simona Schiavi**, INRIA-Saclay, Equipe DEFI  
*Titre* : A macroscopic model for the dMRI signal accounting for time-dependent diffusivity.
4. **Khieu Van Nguyen**, Institut d'imagerie biomédicale (I2BM), NeuroSpin, CEA  
*Titre* : Numerical simulation of dMRI in Aplysia neuronal network.
5. **Rutger Fick**, INRIA-Sophia Antipolis, Equipe Athena  
*Titre* : Efficient processing of dMRI data of the brain white matter.

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## Introduction

This mini-symposium brings together mathematicians, physicists and experimentalists who work in mathematical analysis, numerical simulation and experimental aspects of diffusion magnetic resonance imaging. The first two talks discuss the application to porous media, including an overview of the methodology and challenges as well as more recent results. The next three talks focus on biological applications, with the third talk explaining the formulation of a theoretical model of the time-dependent diffusion coefficient, the fourth talk on numerically simulating the dMRI signal of the neuronal network of the Aplysia, and the last talk on the efficient processing and interpretation of the dMRI signal from the brain white matter fibers.

## 1 Mesure RMN de diffusion restreinte en milieux poreux : Pratique et interprétations

Outre ses applications spectaculaires dans le domaine bio-médical, la mesure RMN de diffusion par gradient de champ pulsé (RMN-GCP, aussi appelée NMR-PGSE en langue anglaise) compte au nombre des outils standards de caractérisation des fluides contenus dans les milieux poreux. A cause des effets de restriction géométrique, le propagateur de diffusion mesuré n'est en général pas Gaussien, notamment lorsque les échelles de longueur sondées par la mesure (généralement de 10 à 100 microns pour les mesures sur l'eau) coïncident avec les échelles d'organisation caractéristiques du réseau de pores. Un courant de pensée s'est ainsi développé entre les années 1990–2000 visant à identifier les liens entre la topologie du milieu et les caractéristiques non-Gaussiennes mesurées, autant pour comprendre les propriétés de transport des fluides à travers des réseaux poreux complexes que pour faire de la RMN-GCP un outil pratique de caractérisation micro-géométrique des matériaux [1][2][3].

L'exposé proposé abordera les problèmes expérimentaux — et imparfaitement résolus — se posant lors des mesures de diffusion en milieux fortement hétérogènes, et leurs conséquences sur l'interprétation quantitative des résultats. On évoquera les inhomogénéités de champ magnétique induites par les contrastes de susceptibilité, la relaxivité de surface, les écarts à l'approximation des impulsions courtes, ainsi que les séquences RMN développées pour contrer certains de ces effets.

L'exposé montrera ensuite les principaux cadres et voies d'interprétation, connus et moins connus, permettant de comprendre et d'exploiter les aspects non Gaussien de la diffusion, la plupart du temps directement dans l'espace réciproque (q-space). Nous proposerons une cartographie type du q-space permettant de synthétiser ces résultats et de faire apparaître leurs domaines respectifs de validité [4].

Enfin, si le temps le permet, nous évoquerons les méthodes de calcul de modes propres de diffusion-relaxation par techniques de différences finies, qui permettent d'étudier par une programmation informatique simple et abordable les phénomènes concernés. Nous discuterons quelques perspectives dans ce domaine pour améliorer la précision des schémas [5].

## 2 Diffusion MRI in porous media : theoretical challenges

Diffusion MRI is a non-invasive experimental technique which is often used to probe the microstructure of biological tissues (brain, lungs) and mineral porous media (rocks, cements) [6, 7, 8, 9]. In this talk, we review some theoretical and numerical studies of the intricate relation between the microstructure and the macroscopic signal [10]. Starting from the classical Gaussian phase approximation, we discuss limitations of apparent diffusion coefficients and notable deviations from Gaussian behavior, potential pitfalls of phenomenological models, and more recent developments such as matrix formalism and two-dimensional correlation experiments [11, 12].

### 3 A macroscopic model for the diffusion MRI signal accounting for time-dependent diffusivity

Diffusion Magnetic Resonance Imaging (dMRI) encodes the displacement of water due to diffusion and is a powerful tool to obtain information on the tissue micro-structure. An important quantity measured in dMRI at the voxel level is the apparent diffusion coefficient (ADC) and it is well-established from imaging experiments that, in the brain, in-vivo, the ADC depends significantly on the diffusion time. To aid in the understanding and interpretation of the ADC, we derived a new asymptotic model for the dMRI signal from the Bloch-Torrey equation that governs the water proton magnetization under the influence of diffusion-encoding magnetic gradient pulses. Our new model was obtained using a particular choice of scalings for the biological cell membrane permeability, the diffusion time, the diffusion-encoding magnetic field gradient strength and a periodicity length of the cellular geometry. The ADC of the resulting model is time-dependent. We numerically validated this model for a wide range of diffusion times for two dimensional geometrical configurations.

### 4 Numerical simulation of diffusion MRI in Aplysia neuronal network

The nerve cells (up to hundreds of micrometers) of the Aplysia are much larger than mammalian neurons. The neuronal network of the Aplysia can be modelled by clusters of spheres connected by long thin cylinders, the shapes of which can be discerned at the resolution of MRI using T2 contrast (not diffusion contrast). From a segmented geometry, the dMRI signal can be numerically simulated by solving the Bloch-Torrey equation using a method that couples finite volume discretization in space to Runge Kutta Chebyshev time stepping. We present preliminary results of the numerical simulations at different diffusion times and in different diffusion-encoding directions for several values of biological membrane permeability. We then discuss the calibration of the unknown parameter of the Bloch-Torrey equation, the membrane permeability, using the experimental dMRI data.

### 5 Efficient processing of diffusion MRI data of the brain white matter

Diffusion MRI (dMRI) is a unique Magnetic Resonance Imaging modality, which is able to quantify in-vivo and non-invasively the average random thermal movement (diffusion) of water molecules in biological tissues such as brain white matter. Using the water diffusion as a probe, dMRI makes it possible to reconstruct white matter fiber pathways and segment major fiber bundles that reflect the structures in the brain which are not visible to other non-invasive imaging modalities. This modern imaging modality, of great interest to neuroscientists and clinicians, has opened a number of challenging problems. In this talk, the important problems of efficiently processing complex dMRI data will be introduced and recently developed solutions and advances will be presented.

Applications to computational brain imaging will also be presented and discussed with a particular emphasis on the recovery of microstructure parameters of the tissue, as well as the tracking, the reconstruction and the clustering of the bundles of white matter fibers.

Higher order models will also be presented that go beyond the classical Diffusion Tensor Model, which is well known to be inadequate in crossing tissue structures. Both non-parametric (or model-free) methods and tissue compartment based methods will be presented. These new approaches open the possibility of inferring and recovering a more detailed geometric description of the anatomical connectivity between brain areas and their underlying tissue structure. The presentation of some open problems currently being investigated by the dMRI community will conclude the talk.

## Références

- [1] P.P. MITRA, *Diffusion in porous materials as probed by pulsed gradient NMR measurements*, Physica A, 241 : 122-127, 1997.

- [2] S. RODTS AND P. E. LEVITZ, *Time domain analysis : an alternative way to interpret PGSE experiment*, Magnetic Resonance Imaging, 19 : 465-467, 2001.
- [3] S. RODTS AND P. E. LEVITZ, *Probing confining geometries with molecular diffusion : a revisited analysis of NMR-PGSE experiment*, Dynamics in Small Confining Systems V., J.M. Drake, P.E. Levitz, R. Overney, M. Urbakh (Eds.), Material Research Society, T3.6, 2001.
- [4] P. E. LEVITZ AND S. RODTS, *Brownian motions in confining geometry : a multiscale analysis*, Ann. Chim. Sci. Mat. 30(4) : 345-352, 2004.
- [5] D. BYTCHENKOFF AND S. RODTS, *Structure of the two-dimensional relaxation spectra seen within the eigenmode perturbation theory and the two-site exchange model.*, Journal of Magnetic Resonance, 208 : 4-19, 2011.
- [6] P. T. CALLAGHAN, *Principles of Nuclear Magnetic Resonance, Microscopy*, Clarendon Press, Oxford, 1991.
- [7] W. S. PRICE, *NMR Studies of translational Motion : Principles and Applications*, Cambridge University Press, 2009.
- [8] D. S. TUCH, T. G. REESE, M. R. WIEGELL, AND V. J. WEDEEN, *Diffusion MRI of Complex Neural Architecture*, Neuron **40**, 885-895 (2003).
- [9] D. LE BIHAN AND H. JOHANSEN-BERG, *Diffusion MRI at 25 : Exploring brain tissue structure and function*, NeuroImage **61**, 324-341 (2012).
- [10] D. S. GREBENKOV, *NMR survey of reflected Brownian motion*, Rev. Mod. Phys. **79**, 1077-1137 (2007).
- [11] D. S. GREBENKOV, *Use, Misuse and Abuse of Apparent Diffusion Coefficients*, Conc. Magn. Reson. A36, 24-35 (2010).
- [12] D. S. GREBENKOV, *Exploring diffusion across permeable barriers at high gradients. II. Localization regime*, J. Magn. Reson. 248, 164-176 (2014).