Mini-symposium EQCIN
Equations cinétiques

Mini-symposium porté en partie par le projet ANR Calibration (2011 BS01 010 01)

Résumé

Même si l'équation de Boltzmann, et plus généralement les équations cinétiques associées, sont étudiées depuis de nombreuses années, l'activité de recherche sur ces sujets est toujours intense et de nombreux résultats intéressants ont été obtenus récemment.Ce minisymposium s'attache à mettre en évidence des résultats récents portant sur la dérivation rigoureuse de modèles cinétiques et l'usage d'équations cinétiques dans des champs d'application variés.Le premier exposé, de Claude Bardos, traite de la relation de Maxwell-Boltzmann, donnant la densité électronique en fonction du potentiel. Elle est souvent utilisée en physique des plasmas. Claude Bardos la justifiera d'un point de vue mathématique.Le second exposé, de Maxime Herda, traite de la dérivation rigoureuse d'un système cinétique-fluide utilisé dans les plasmas magnétisés. Le dernier exposé, de Stéphane Junca, traite du système de classement des joueurs d'échecs ELO, qui donne lieu à une équation cinétique. Le comportement en temps grand des solutions de cette équation permet d'évaluer le système.

Organisateur(s)

1. **Anne Nouri**, Institut de Mathématiques de Marseille, Université d'Aix-Marseille.

Liste des orateurs

1. **Claude Bardos**, Laboratoire Jacques-Louis Lions, Paris
   Titre : About the Maxwell-Boltzmann relation for fluid and plasmas.
2. **Maxime Herda**, Institut Camille Jordan, Lyon
   Titre : Derivation of a kinetic-fluid model for a multispecies magnetized plasma with massless electrons.
3. **Stéphane Junca**, Laboratoire Alexandre Dieudonné, Nice
   Titre : A kinetic PDE for the ELO rating system.

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1 About the Maxwell-Boltzmann relation for fluid and plasmas (C. Bardos)

This is a preliminary report on a joint work with Francois Golse, Claudia Negulescu and Remi Sentis.
The standard description of the evolution of a plasma at the kinetic level involves the coupling of two Vlasov equations one for electron and one for ions.
The Maxwell-Boltzmann relation is used to replace the electrons equation by a formula for the for the potential : Φ of the following type :

$$-\Delta\Phi + e^{\lambda\Phi} = \int f_{\text{ions}}(x,v,t)dv\,.$$

The way to justify such formula consist in assuming that the electrons have reached some type of thermodynamical equilibrium.
A formal proof can be obtained by taking in account the relaxation to a Maxwellian due the collision term

$$\sigma_{\text{electrons}} C(f_{\text{electrons}})$$

(Boltzmann or Fokker Planck kernel) and looking for times large enough with respect to σ .
Then at this formal level things are easier to formulate and to understand than for the Landau damping or for the metastable solutions the $2d$ Euler equation. On the other hand these non linear effects makes the obtention of complete proofs more difficult and they may be only available for some small fluctuations .

2 Derivation of a kinetic-fluid model for a multispecies magnetized plasma with massless electrons (M. Herda)

We consider a three-dimensional kinetic model for a two species plasma consisting of electrons and ions confined by an external nonconstant magnetic field. Then we derive a kinetic-fluid model when the mass ratio m_e/m_i tends to zero.
Each species initially follows a Vlasov-type equation and the electrostatic coupling follows from a Poisson equation. In our modelling, ions are assumed non-collisional while a Fokker-Planck collision operator is added in the electron equation. As the mass ratio tends to zero we show convergence to a new system where the macroscopic electron density satisfies an anisotropic drift-diffusion equation. To achieve this task, we overcome some specific technical issues of our model such as the strong effect of the magnetic field on the electrons and the lack of regularity at the limit. With methods usually adapted to diffusion limit of collisional kinetic equations and including renormalized solutions, relative entropy decrease and velocity averages, we establish the rigorous derivation of the limit model.

3 A kinetic PDE for the ELO rating system (S. JUNCA)

The ELO rating system was first introduced by Arpad Elo, an American physicist and a chess player, for chess competitions. This system and several variants are now used to rank individual players or teams in many sports, or online games. The Elo system bases its ratings on encounters between players or teams, resulting on a win for one of the player. An encounter is typically the outcome of a game but nothing prevents the system from being applied to even more general settings ; the result of an encounter could be decided by voting for instance. After either each encounter, or a series of them, the system updates the ratings of each player depending on their wins or losses and on the ratings of the players against whom they competed.
A natural question is the validity of the Elo system : Is the ranking after many interactions between players fair and in which sense ? Our goal is to investigate the validity of the system in the specific case with a large number of players, which is typical for online games for example. We investigate the validity of the rating system by looking at their large time behavior.

Références

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