Mini-symposium FNMCPMF
Fast numerical methods for complex porous media flows

Résumé

The numerical computation of multiphase flows in porous media is important for the prediction of water discharges and quality in aquifers, as well as for predicting how to produce and store oil and gases in reservoirs. Years of investigation have resulted in highly-complex mathematical models, with numerous physical parameters to be calibrated (or at least discussed). Parameter variations are however a computationally-demanding task. In practice, it is therefore useful to consider adequate acceleration methods, which allow one to explore fast the parametric dependency of the numerical models. In the minisymposium, problems related to the numerical simulation of complex parameter-dependent models for porous media flows will be presented. Real-life examples will be discussed. Some recent techniques to address the computational challenge will also be described, like the Reduced-Basis method for instance.

Organisateur(s)

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Liste des orateurs

1. **Donald Brown**, Institut für Numerische Simulation
   Titre : Multiscale modeling of pore-scale processes.
2. **Ondrej Budáč**, ANMC, MATHICSE, EPFL
   Titre : A reduced basis finite element heterogeneous multiscale method for Stokes flow in porous media.
3. **Riad Sanchez**, IFPEn
   Titre : The certified reduced-basis method for Darcy flows in porous media.
4. **Felix Schindler**, Applied Mathematics Münster, WWU
   Titre : The online adaptive localized reduced basis multi-scale method for flow in porous media.

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Introduction

The aim of this session is to give an overview of recent numerical techniques developped in order to obtain more accurate results when simulating porous media flows within reasonable simulation times.

In this field, the difficulty is twofold due to the multiscale nature and/or the parametric dependence of the flows.

The complex microstuctures of the porous media often have a significant influence on the flows which are usually simulated at an upper scale. The challenge then consists in developping numerical methods which are able to combine in a efficient manner simulations at the pore and Darcy scales or simulations with two grid resolutions, the space variations of the parameters being only known or modelled at the finest one.

Multiscale finite element methods represent a promising approach to tackle this issue. Based on a divide-and-conquer strategy, multiscale methods first split up the resolution of the flow problem into several local cell problems. The local problems allow one to compute basis functions whose shapes take the small scale variations of the flows into account. These basis functions are used in a following stage to solve the initial flow problem at a coarser scale. But, even if the cell problems can be solved in parallel, their solution may be nonetheless costly. Model reduction based on the reduced basis methods can be used to decrease this cost.

Many other physical parameters than the geometry also enter in the models and need being varied (for calibration, uncertainty quantification...). One can also resort to model reduction based on the reduced basis method in cases where the flow problem has to be solved for several values of the physical parameters of the model.

This session is therefore intended to show how recent numerical techniques can be implemented to reduce the complexity inherent in the simulation of these flow problems and how to combine them.

1 Multiscale modeling of pore-scale processes

Many porous media applications exhibit complex microstructure at the pore level and are intrinsically multiscale in nature. The possible applications include heat conduction in metallic foams, lithium ion batteries, and subsurface modeling. To efficiently simulate large scale problems in these application domains, new computational tools that can handle the multiscale nature of the problem need to be developed. Broadly speaking, we seek computational techniques that reduce order or complexity.
In this talk, we develop a multiscale method to solve problems in complicated porous microstructures. By using a coarse-grid quasi-interpolation operator to define a fine detail space and local orthogonal decomposition, we construct multiscale corrections to coarse-grid basis functions with microstructure. By truncating the corrector functions we are able to make a computationally efficient scheme. We give a few numerical examples and detail the technical subtleties related to the Poincare Constants in perforated domains.

2 A reduced basis finite element heterogeneous multiscale method for Stokes flow in porous media

In this talk we present a numerical multiscale method for Stokes flow in porous media. We consider a two-scale model that is based on the Stokes homogenization results [6, 7, 8]. The presented method is named the *reduced basis Darcy-Stokes finite element heterogeneous multiscale method* (RB-DS-FE-HMM) [5], which is based on the DS-FE-HMM [3].
The (RB)-DS-FE-HMM methods use finite elements to solve a Darcy equation on a macroscopic mesh, where the missing permeability data are extracted from the Stokes micro problems at each macroscopic quadrature point. The computational cost of the DS-FE-HMM is heavily dominated by solving a large amount of micro problems. The RB-DS-FE-HMM is addressing this bottleneck by applying the reduced basis (RB) methodology [4] to the micro problems. We map the Stokes micro problems to a reference

domain, where a single (fine) microscopic mesh is considered and the coefficients of the mapped problems are parametrized by macroscopic coordinate.
The RB framework is divided into two stages : an offline stage and an online stage. In the offline stage (performed only once) we use a greedy algorithm to choose a small set of parameters at which we numerically compute the Stokes micro problems (the reduced basis). Micro problems can be then projected into the low-dimensional solution space spanned by the reduced basis, which yields a (small but dense) reduced system. The online stage can be performed repeatedly and it provides a fast assembling and solution of the reduced system and a subsequent evaluation of the output of interest (effective permeability) for any parameter value (macroscopic coordinate). It can be shown that the presented method satisfies approximation and algebraic stability properties, while keeping a favorable computational cost that is independent of the microscopic mesh size.
A priori and a posteriori estimates of the RB-DS-FE-HMM can be derived and a residual-based adaptivity approach can be applied. Two and three dimensional numerical experiments confirm the accuracy of the RB-DS-FE-HMM and demonstrate the cost reduction compared to the DS-FE-HMM.

3 The certified reduced-basis method for Darcy flows in porous media

The reduced basis (RB) method [10] is a model reduction technique yielding substantial savings of computational time when a solution to a parametric equation has to be computed for many values of the parameter. A reduced basis approach aims at reducing the use of discretization schemes (based on finite elements or finite volumes) whose repeated usages may be costly. This approach builds a low-order approximation of the true set of the discrete solutions which is based on two stages : in a first offline step, the parameters values are sampled and the corresponding numerical solutions are computed ; then, in a second online step, new solutions for different parameter values are quickly estimated by calculating the Galerkin projection of the solution on the subset spanned by the precomputed snapshots. A certification of the error approximation may also be achieved when an a-posteriori error estimate is known. This error estimation is also crucial for the design of the sampling procedure used during the construction of the reduced basis. Under appropriate assumptions, this error bound and projection steps are computed with an algorithm whose complexity does not depend of the size of the discrete problem.
In this work we extend this method to solve problems of two-phase flows in porous media discretized with a cell-centered finite volume scheme. Our first step in this study is to develop an efficient reduced basis scheme for the pressure equation. Considering as a first test oil-water flows in a heterogeneous porous medium, the water viscosity is assumed to be variable due to operational conditions. In the oil industry, a certain concentration of polymer may be added to water to reduce the contrast of mobility with the oil phase and obtain a more efficient sweeping. Here one possible benchmark consists in approximating all the pressure fields obtained at the final time step according to the values of the water viscosity using a RB approach. An a posteriori error estimate is built giving an upper bound on the residual obtained when replacing the discrete solution by the RB approximation. This error estimate is used in the offline stage by the so-called greedy algorithm and the empirical operator interpolation method is used to obtain a fast approximation in the online stage. As an illustration, numerical experiments performed on the SPE10 benchmark [9] are presented.

4 The online adaptive localized reduced basis multi-scale method for flow in porous media

We present recent advances in the context of the localized reduced basis multi-scale method (LRBMS) for parametric elliptic multi-scale problems with possibly heterogeneous diffusion coefficient (see [2, 1]). The numerical treatment of such parametric multi-scale problems is characterized by a high computational complexity, arising from the multi-scale character of the underlying differential equation and the additional parameter dependence. The LRBMS method can be seen as a combination of numerical multi-scale methods and model reduction using reduced basis methods to efficiently reduce the computational complexity with respect to the multi-scale as well as the parametric aspect of the problem, simultaneously. We present new error estimates that are based on conservative flux reconstruction and provide an efficient and rigorous bound on the full error with respect to the weak solution. We will also present an online adaptive basis enrichment algorithm, based on this localized error estimate, and present experiments to demonstrate the applicability of the resulting algorithm to single phase flow in heterogeneous media.

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