Mini-symposium BoND
Dispersive equations : modeling, boundary conditions and numerics

Mini-symposium porté par l'ANR BoND

Résumé

This mini symposium is dedicated to evolution problems in which dispersion is predominant compared to dissipative or dissipative mechanisms. The model dispersive equations that are being considered include Korteweg-de Vries, Nonlinear Schrodinger equations but also the more complicated systems of Euler-Korteweg (for capillary fluids) and of Green-Naghdi (for water waves).

In particular, the minisymposium will focus on problems in which boundaries play an important role, be they "physical", fixed boundaries such as walls or pipe extremities in fluid flows, artificial boundaries introduced for numerical purposes, moving boundaries like shocks in compressible fluids or free surface. Problems related to dispersive patterns (periodic waves, dispersive shock waves, solitary waves) and control of water waves will also be discussed.

Organisateur(s)

1. **Pascal Noble**, INSA Toulouse, Institut de Mathématiques de Toulouse.
2. **Frédéric Lagoutière**, Université Paris Sud, Département de Mathématiques d'Orsay.

Liste des orateurs

1. **Thomas Alazard**, ENS Paris, DMA
   Titre : Control for water waves.
2. **Ingrid Lacroix-Violet**, Université Lille 1, Laboratoire Paul Painlevé
   Titre : Conditions limites artificielles discrètes pour l'équation KdV.
3. **Nicolas Pavloff**, Université Paris Sud, Laboratoire Physique Théorique et Modèles Statistiques
   Titre : Superfluidity and non-linear phenomena in Bose-Einstein condensates.
4. **Qinling Tang**, Université de Lorraine, Institut Éile Cartan de Lorraine
   Titre : An efficient numerical method for simulating dynamics of rotating Bose-Einstein condensate.

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Introduction

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In particular, the minisymposium will focus on problems in which boundaries play an important role, be they "physical", fixed boundaries such as walls or pipe extremities in fluid flows, artificial boundaries introduced for numerical purposes, moving boundaries like shocks in compressible fluids or free surface (speakers : Ingrid Lacroix-Violet, Qinling Tang). Problems related to dispersive patterns like periodic waves, dispersive shock waves, solitary waves (speaker : Nicolas Pavloff) and control of water waves (speaker : Thomas Alazard) will also be discussed.

1 Boundaries and Dispersives equations

Ingrid Lacroix-Violet : Conditions limites artificielles discrètes pour l'équation KdV

Dans cet exposé nous nous intéresserons à l'équation de Koteweg-de-Vries linéarisée en une dimension d'espace. Après avoir brièvement expliqué comment obtenir pour cette équation des conditions limites transparentes pour le cas continu, nous présenterons la construction de conditions limites artificielles totalement discrètes adaptées à deux schémas numériques différents, tous deux basés sur la méthode des trapèzes en temps

Qinling Tang : An efficient numerical method for simulating dynamics of rotating Bose-Einstein condensate

We will present an efficient numerical methods for simulating rotating Bose-Einstein condensate (BEC) that are well modeled by Gross-Pitaevskii equation (GPE) with a rotating term in the mean field limit. The method consists two main merits : (i) we first introduce the rotating Lagrangian coordinate system and reformulate the original GPE into a new GPE without rotating term in the new coordinate. (ii) then, we truncate the new GPE into a bounded domain problem which are solved via a PML approach. Some interesting numerical results will also be presented.

2 Dispersive Patterns

Nicolas Pavloff : Superfluidity and non-linear phenomena in Bose-Einstein condensates

The realization of Bose-Einstein condensation of ultracold atomic vapors has paved the way of detailled studies of the phenomenon of superfluidity in theses sytems. In particular, a mechanism of breakdown of superfluidity predicted by Landau in 1941 has been experimentally checked by several groups. The actual disappearance of superfluidity is monitored by the ejection of nonlinear waves (solitons, vortices and dispersive shocks). I will present the theoretical and experimental situation in this field.

3 Control of dispersive equations

Thomas Alazard : Control for water waves

Water waves are disturbances of the free surface of a liquid. They are, in general, produced by the immersion of a solid body or by impulsive pressures applied on the free surface. The question we discuss in this talk is the following : which waves can be generated by blowing on a localized portion of the free surface. Our main result asserts that one can generate any small amplitude, periodic in x, two-dimensional, gravity-capillary water waves. This is a result from control theory. More precisely, we prove the local exact controllability of the incompressible Euler equation with free surface. This is a joint work with Pietro Baldi and Daniel Han-Kwan [**?**]