



Université  
de Toulouse

# Couplage des équations de St-Venant 1D-2D et assimilation variationnelle. Application aux plaines d'inondations

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*INSA & Institut de Mathématiques de Toulouse (IMT)*

## Plan

- **Partie 1.** De l'assimilation variationnelle de données appliquée à une plaine d'inondation

*En collaboration avec C. Puech (Cemagref Montpellier) et X. Lai (Niglas, Nanjing, Ac. Sc. Chine)*

- **Partie 2.** Un algorithme de couplage 1D-2D avec assimilation simultanée

*En collaboration avec J. Marin (Ing. Inria Grenoble) et I. Gejadze (Univ. Strathclyde)*

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- Pour terminer, quelques expérimentations num. en cours

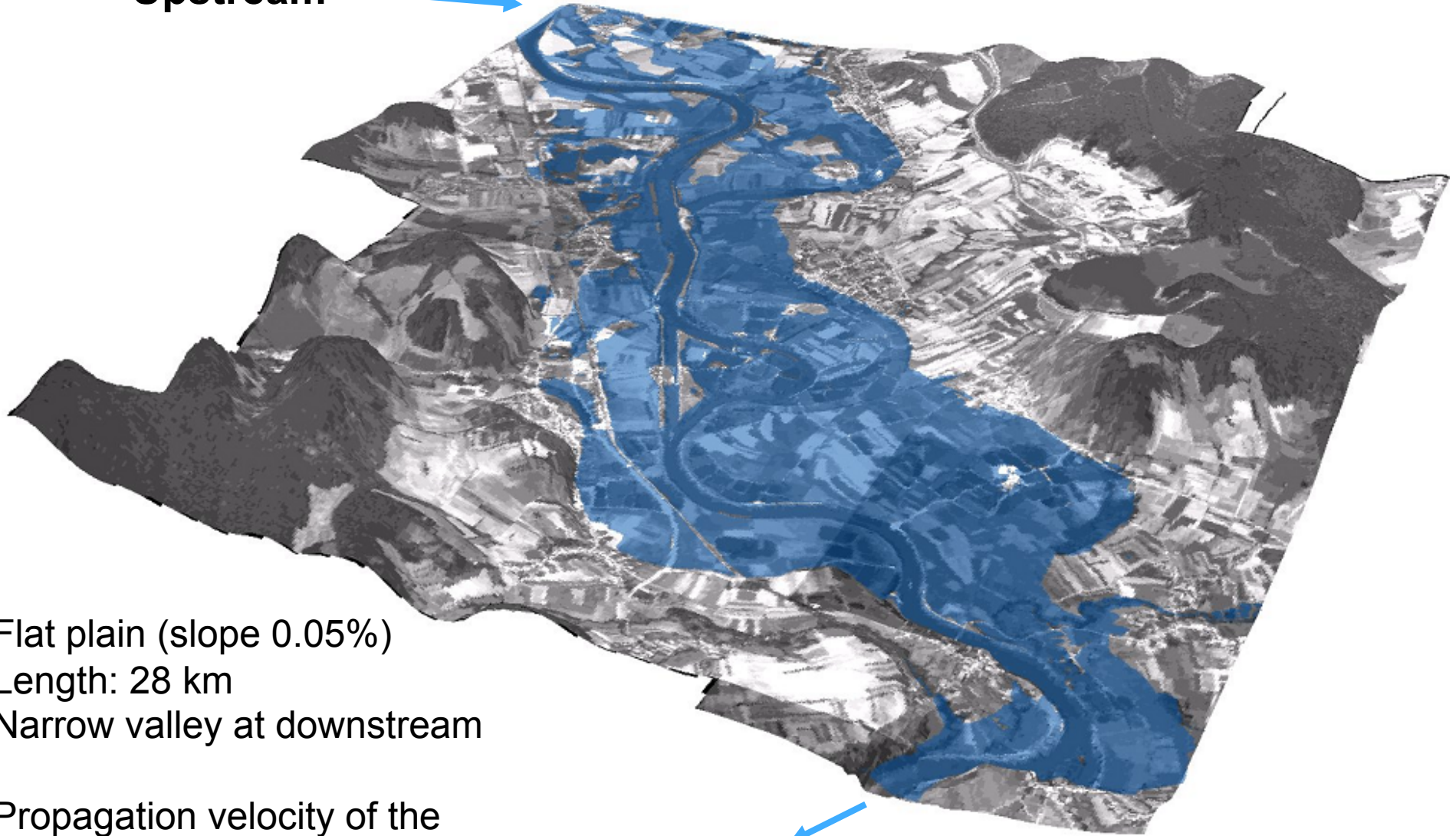


*En collaboration avec F. Couderc (IR Cnrs), R. Madec (IR Anr Amac) et JP Vila (Insa) de l'IMT*

Part 1. Mosel river (at border France-Germany).  
Flow configuration



**Upstream**



Flat plain (slope 0.05%)  
Length: 28 km  
Narrow valley at downstream

Propagation velocity of the  
flood peak: 2 km/h  
Peak discharge: 1450 m<sup>3</sup>/s

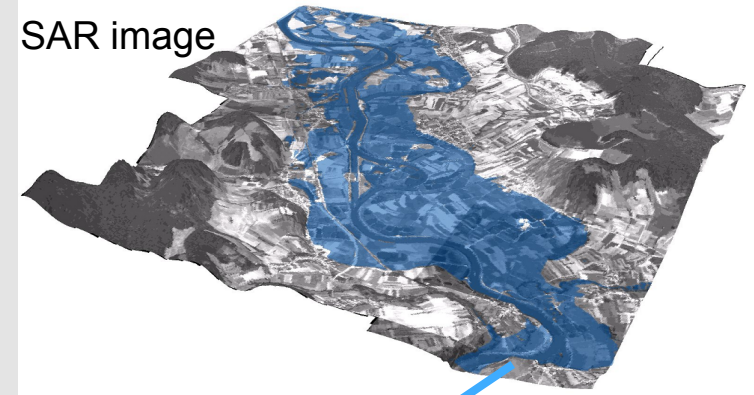


**Downstream**

## Image analysis



SAR image



downstream

Step 1: Extracting waters: a fuzzy mapping...

Step 2: Transforming 2D in 3D

by merging image and DTM

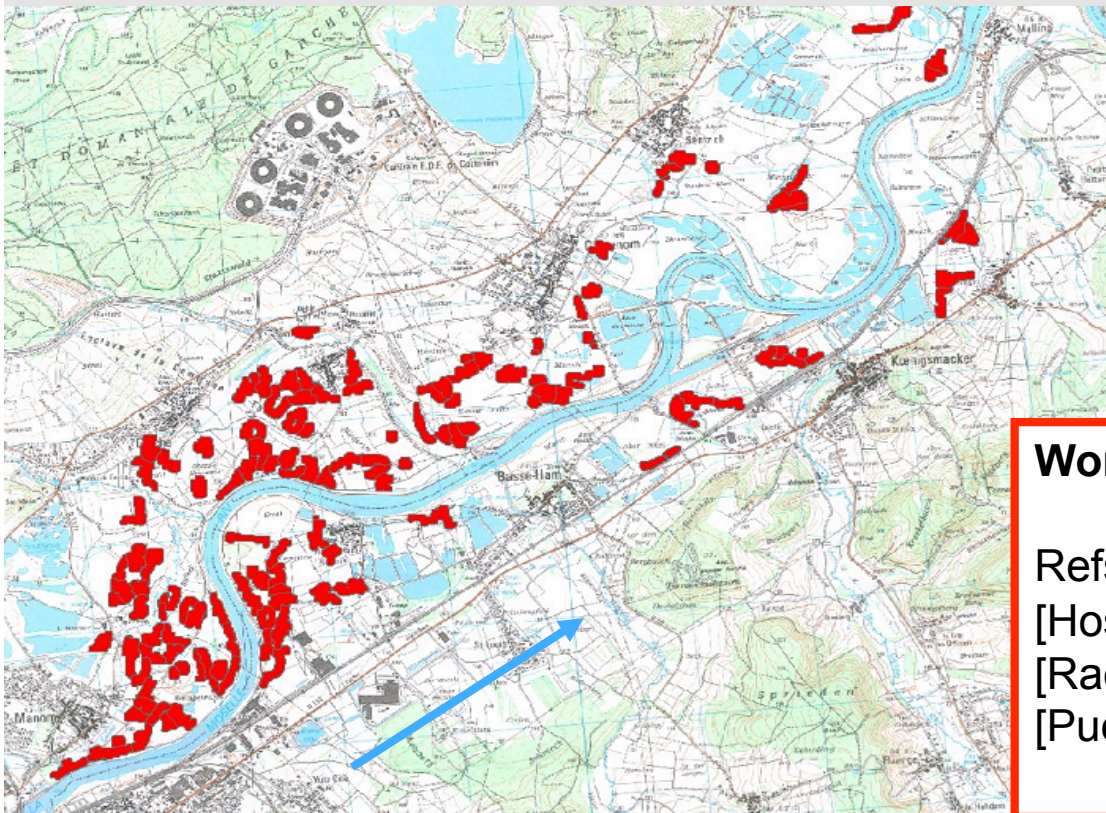
Step 3: Localization of informative points

for local  $h$  values (ie. no trees, no urban, no steep slopes)

Step 4: "hydraulic coherence" imposed

(min-max elevation following the steepest descent)

**Final Result: elevation  $h$  at "image blocks" with uncertainty estimates ( $\pm 15$  cm in average)**



downstream

**Work done by C. Puech et al.  
Cemagref Montpellier, France**

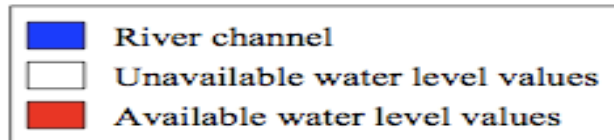
Refs.

[Hostache-Puech et al] Revue teledetection'06

[Raclot] Int. J. remote-sensing'06

[Puech-Raclot] Hydro. processes'02

## Summary of data available



### Data 1) SAR image

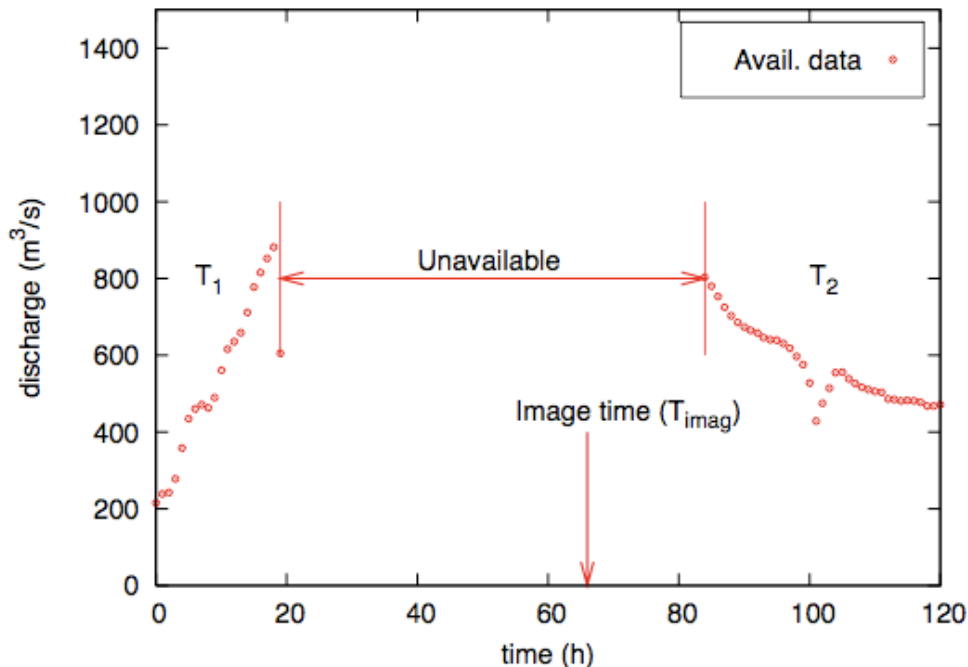
After analysis → local H-values at image time  
with quantified uncertainties ( $\pm 15\text{cm}$ )

By [Puech et al. '06]

### Data 2) In-situ measurements

#### q at gauge station (EDF)

Plot: q at the beginning & the end of the  
flood event

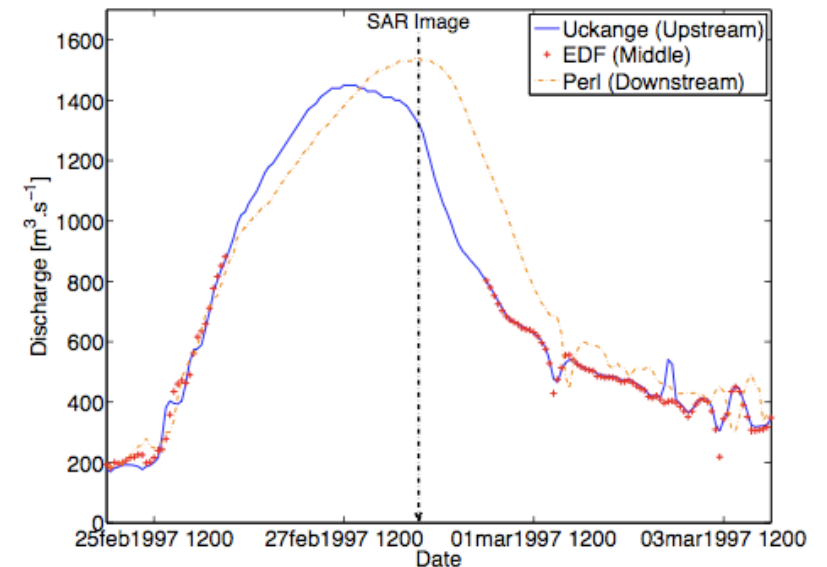


### Boundary conditions known:

q at upstream & h at downstream

Plot:

discharges at downstream & upstream



**Forward model:** 2D S.W.E. inviscid, in var. (h, qx, qy).

**Cost function.** The control variable k = Manning coef. or inflow discharge + I.C.

$$j(k) = \frac{1}{2} \|h - h^{obs}\|_{Image}^2 + \frac{\alpha_1}{2} \|u^{simul}(h - h^{obs})\|_{Image}^2 + \text{regularization terms}$$

$$+ \frac{\gamma_1}{2\sigma_Q^2} \left[ \int_0^{T_1} (Q(t) - Q^{obs}(t))^2 dt \right]$$

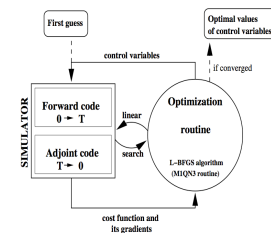
Obs. at gauge station

Image term: net mass flux

## Local sensitivity analysis:

One (1) run of the forward + adjoint models gives the gradient value  $\nabla j(k)$   
hence a local sensitivity information

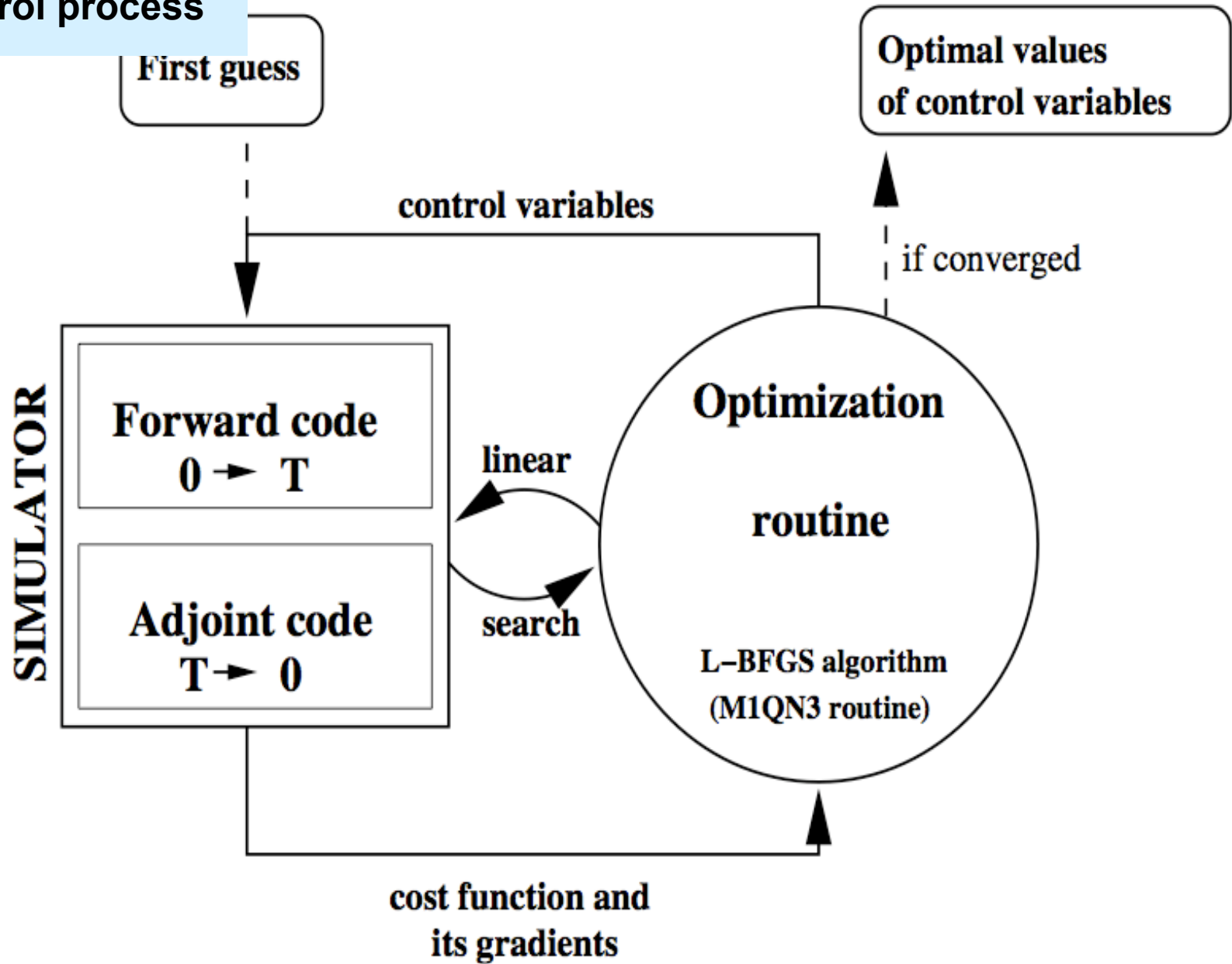
**Calibration:**  $\min_K j(k) \rightarrow \text{optimal control loop}$



## Our software DassFlow (see webpage)

- o Forward models: SWEs. Also: transport, sedimentation (FV), Stokes ALE non-newtonian(FE)
- o FV schemes: explicit HLLC or implicit Van Leer.
- o Adjoint code: automatic differentiation (Tapenade software, Inria)
- o From libraries: minimization (BFGS, Inria), linear algebra (Mumps, U. Toulouse)
- o MPI Fortran codes

# Optimal control process



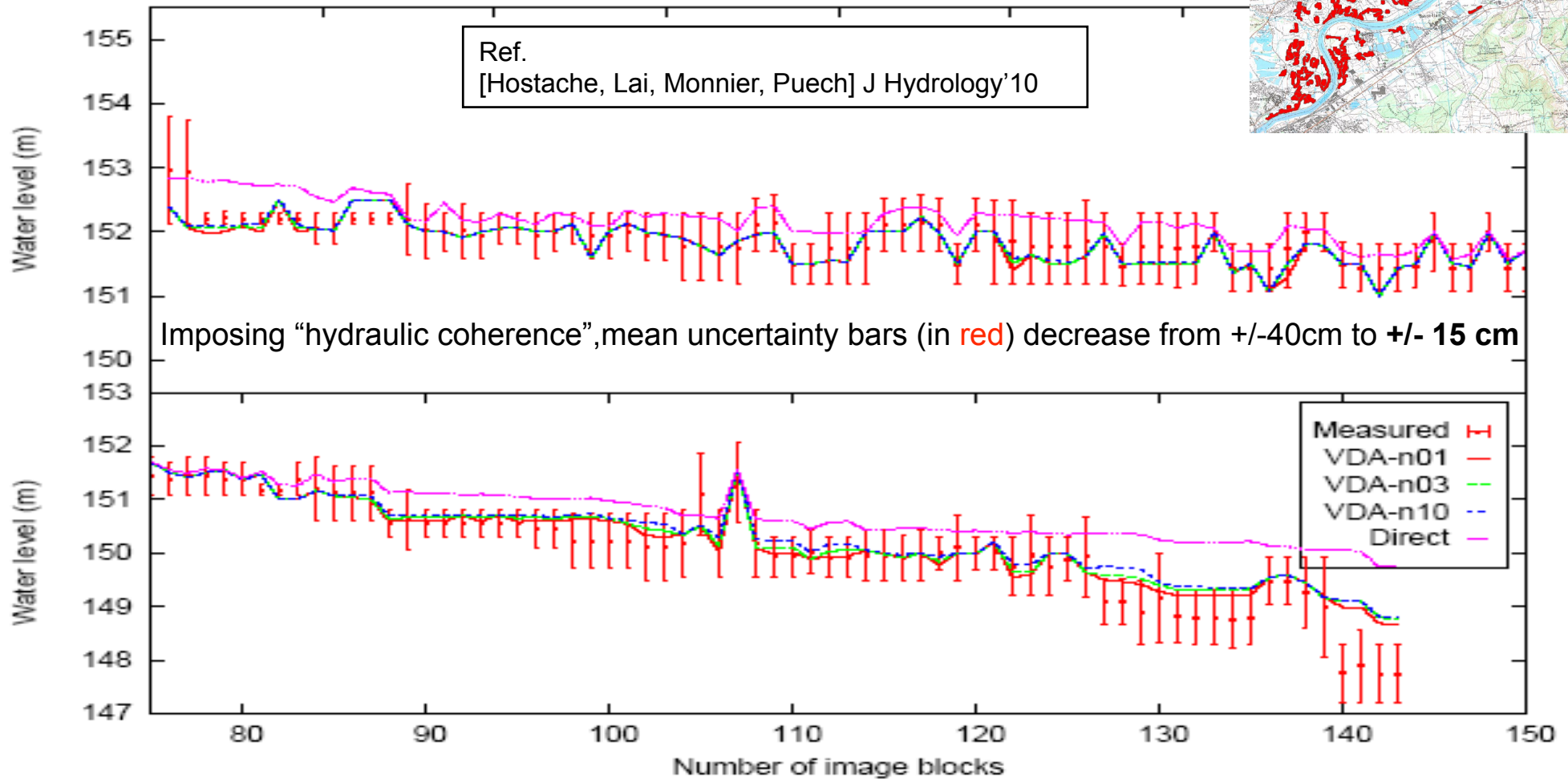
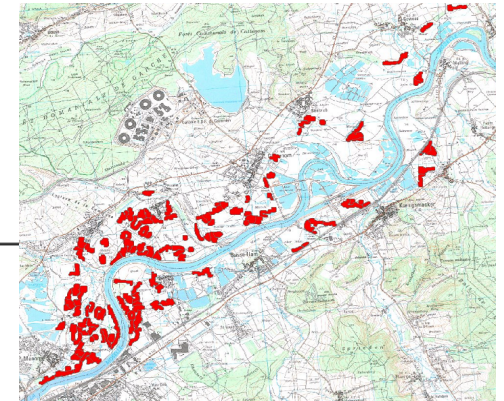
**DassFlow: Data Assimilation for Free-Surface Flows.  
Computational platform**

# Mosel river flood-plain flow: calibration of Manning-Strickler coefficients using 10 land-use (with main channel = constant Manning coef.)

Water elevation at image bocks (in red):

calibration by hand (forward run, pink) vs VDA (4D-var, blue)

vertical bars = observed h from image with computed uncertainty

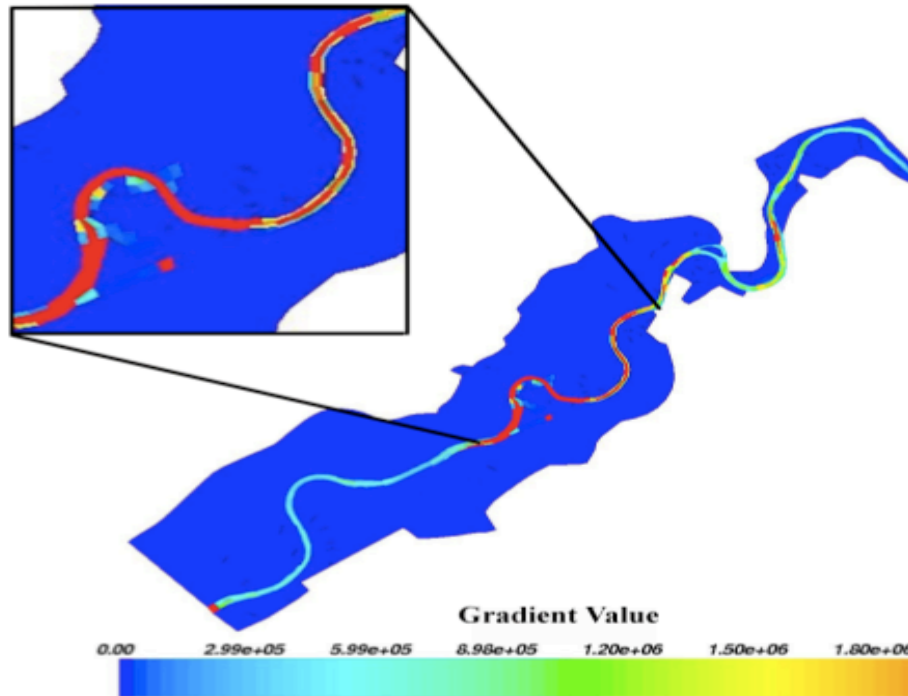


➔ The 4D-var process (VDA) improved greatly the hydraulic model calibrated « by hand »

# Sensitivity analysis without a-priori « Manning-area decomposition » i.e. no land-use decomposition

Refs.  
[Lai, Monnier] J. Hydrology'09  
[Hostache, Lai, Monnier, Puech]  
J. Hydrology'10

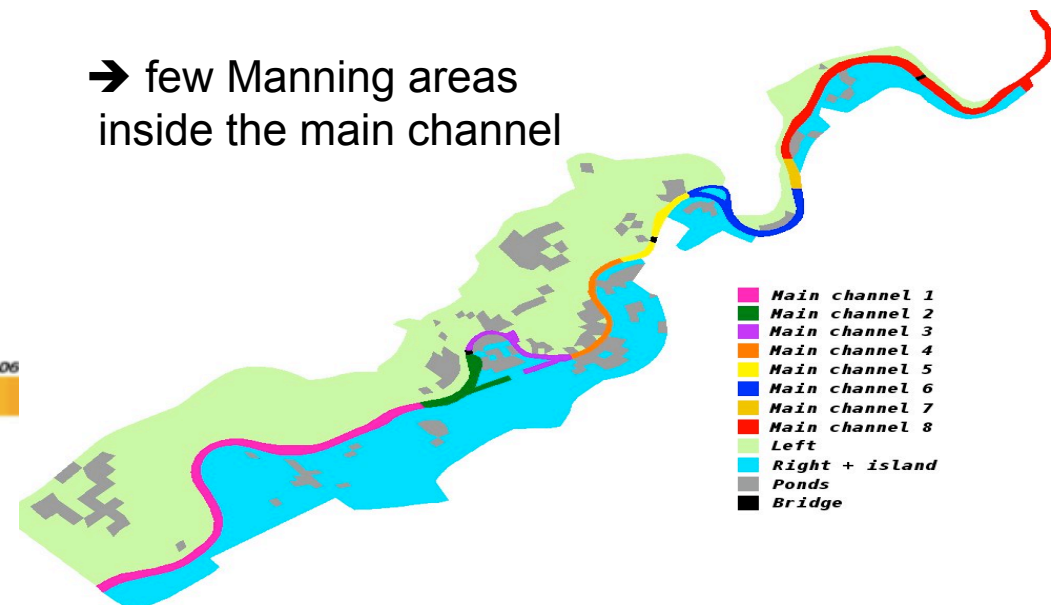
Local sensitivity analysis



**Result:**

sensitive Manning areas one needs to focus on

→ few Manning areas  
inside the main channel



**Preliminary sensitivity analysis runs:**

- 1) improve the understanding of the flow
- 2) lead to a more reliable definition of Manning-areas



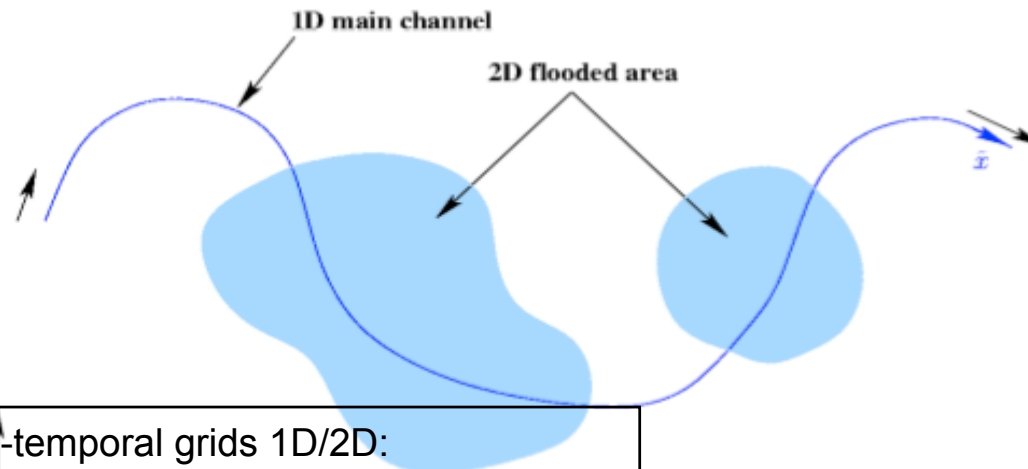
## Part 2. Coupling 1D-2D SWE and simultaneous assimilation

### Basic idea

- o Given a « global » flow model, superpose locally a « zoom » model on it, while keeping the existing geometry and mesh of the global model.  
Local zoom model: richer physics, finest grids.
- o In a **variational data assimilation context**, take advantage of the optimal control process and data in order to:
  - Couple both models (i.e. quantify the information)
  - Assimilate local data represented by the zoom into the global model→ The local zoom model can be viewed as a mapping operator.

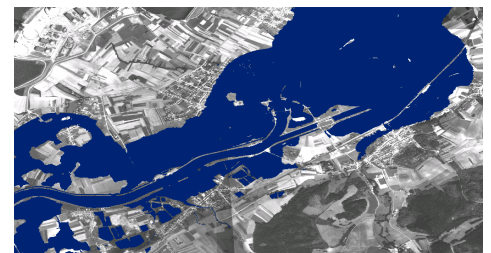
### Flood plain

Global model = 1D-net river branche(s). 1D SWE (St-Venant).  
Local model = flood plain. 2D SWE (St-Venant).



Typical ratios of spatio-temporal grids 1D/2D:

$DX \sim 10$   $Dt \sim 100$



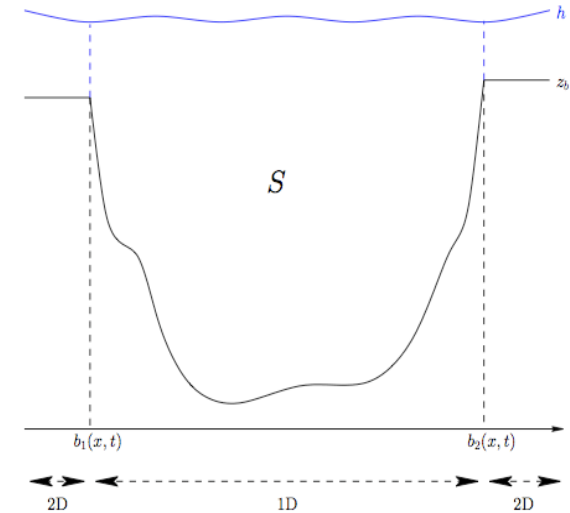
# Global model: 1D SWE with its 2D coupling source term

## Classical 1D St-Venant equations

$S$ : wet cross-section in main channel ;  $Q$ : discharge

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial \tilde{x}} = \Psi_1$$

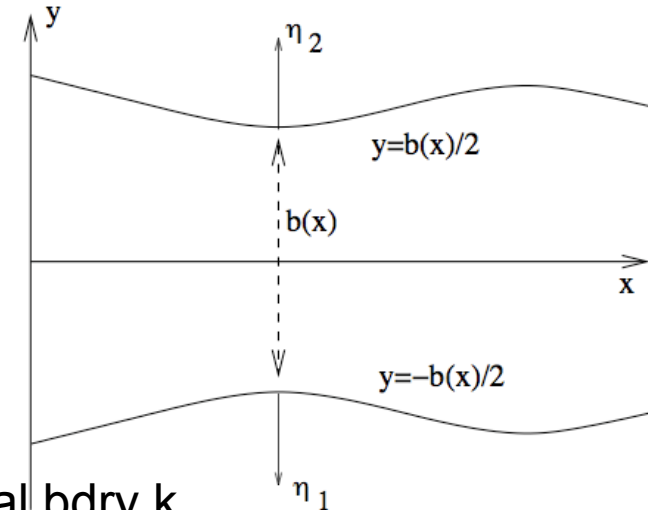
$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial \tilde{x}} \left( \frac{Q^2}{S} \right) + gS \frac{\partial (H + Z_b)}{\partial \tilde{x}} = \Psi_2$$



**If over-flowing and/or lateral filling, derivation from 3D Navier-Stokes eqns gives:**

If the canal width variations are small, if  $u$  is nearly constant over the cross section,  
if  $(u, v)$  do not depend on  $z$  on lateral boundaries,

$$\Psi_1 = -(q_{n1} + q_{n2}) ; \quad \Psi_2 = -(q_{n1}u_{t1} + q_{n2}u_{t2})$$



$$q_{nk} = \sum_{l=1}^2 \left[ \int_{z_b}^{z_b+h} u_l dz \right]_{b_k} \cdot n_k^l \quad : \text{normal discharges at lateral bdry } k$$

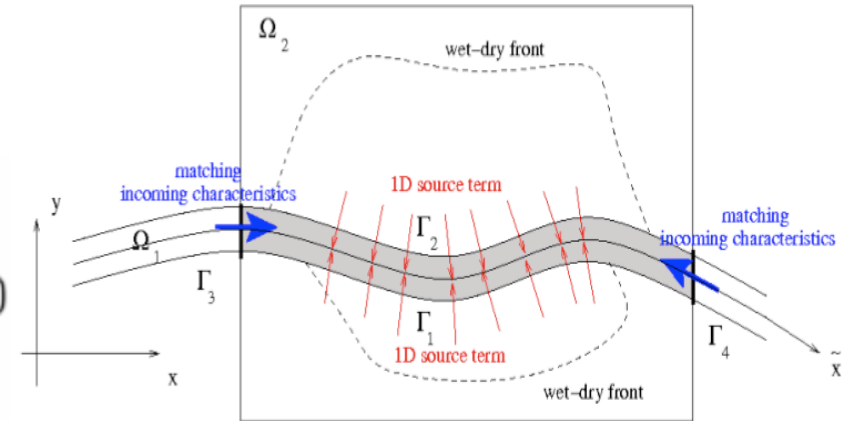
$u_{tk}$  : tangent component of the  $z$ -mean value of  $u$  at lateral bdry  $k$

## Local model: 2D SWE (non viscous)

$h$  : water elevation ;  $q$  : 2D discharge

$$\begin{cases} \partial_t h + \operatorname{div}(q) = 0 \\ \partial_t q + \operatorname{div}\left(\frac{1}{h} q \otimes q\right) + \frac{1}{2} g \nabla h^2 + gh \nabla z_b + \frac{\rho^2 \|q\|_2}{8 h^{7/3}} q = 0 \end{cases}$$

+ Initial Condition + Boundary Conditions.



Conservative form of 2D SWE with topography and friction source terms:

$$\partial_t U + \partial_x F_1(U) + \partial_y F_2(U) = (S_g + S_f)(U)$$

### 2D $\rightarrow$ 1D information transfer

Open-boundaries  $\rightarrow$  continuity of incoming characteristics at interfaces:

$$W_k^{1D}(t) = \int_{\Gamma_k} w_k(t) ds \quad k = 1, 2 \text{ interfaces}$$

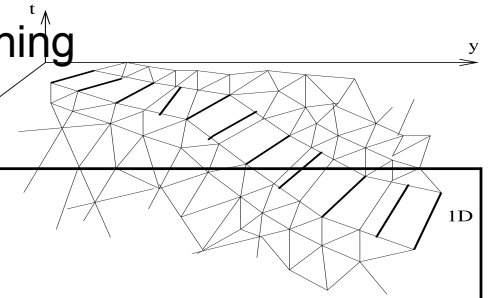
where the 2D characteristics are: (linearized 2D SWE, no topo, no friction)

$$w_1 = \mathbf{u} \cdot \mathbf{n} + \sqrt{\frac{g}{h_0}} h, \quad w_2 = \mathbf{u} \cdot \boldsymbol{\tau} \quad \text{and} \quad w_3 = \mathbf{u} \cdot \mathbf{n} - \sqrt{\frac{g}{h_0}} h,$$

associated to eigenvalues:  $\lambda_1 = \mathbf{u}_0 \cdot \mathbf{n} + c$  ,  $\lambda_2 = \mathbf{u}_0 \cdot \boldsymbol{\tau}$  ,  $\lambda_3 = \mathbf{u}_0 \cdot \mathbf{n} - c$  :

## Numerical schemes: superposed F.V. schemes 1D-2D

A-priori, grids are non-matching



### o Discretization for the source term $(\Psi_1, \Psi_2)$ in the 1D model

$q_n$  = component #1 of the numerical flux

$u_t$  : up-winding upon the sign of intermediate wave speed  $S^* \approx u_n$

→ dynamical over-flowing / filling lateral flows taken into account

**Numerical validation.** Schwarz 2Dover1D vs full 2D-model: perfect matched results

### o Globally well-balanced ? Since:

• An 1D topography term  $\frac{\partial Z_b}{\partial \bar{x}}$  appears in the 1D SWE,

• A 2D topography term  $(\frac{\partial z_b}{\partial x}, \frac{\partial z_b}{\partial y})$  appears in the 2D SWE (thus implicitly in the 2D term of 1D SWE too),

→ Is the resulting global FV scheme 1D-2D well-balanced ?

**Answer:** If both 1D conservative schemes (1D&2D models) are separately well-balanced, then the coupled global scheme 1D-2D is well-balanced too.

### o Ex. of explicit F.V. schemes possible:

HLL / HLLC, Roe, Rusanov

Part done with

E. Fernandez-Nieto, univ. Sevilla, Spain

Ref [Fernandez-Marin-Monnier] '10

## Algorithm of coupling: two approaches compared

We seek to **superpose** the 2D model (local zoom) over the 1D global model:  
2D SWE with fine grids over 1D SWE with coarse grids

### 1) Schwarz type algorithms

With a **Domain Decomposition** (D.D.) approach:

for SWE 1D-2D-1D see eg. [Miglio-Perroto-Saleri'05]

With a **Superposition** approach:

for SWE 1D-2D-1D, see the following num. tests, [Gejadze-Monnier'07]  
[Fernandez-Marin-Monnier]'10

### 2) A minimization / optimal control approach

→ the present **Joint Assimilation Coupling (JAC) algorithm(s)**

we assume to be in a context of variational data assimilation,

we take advantage of the existing optimal control process and data...

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**Principle of JAC algo.** A «relaxed» coupled problem (one way coupled model) is controlled:  
Control of the quantities (characteristics) at interfaces,  
Minimization of  $\Delta(\text{quantities})$  at interfaces.

#### o Some references related to the subject

- **Virtual-control method:** optimal control of conditions at interfaces/link with D.D.

See [Lions-Pironneau]'98 & '99, [Lions'00]

Heterogeneous coupling by virtual control, see [Gervasio-Lions-Quarteroni'01]etc

- **Augmented lagrangian approach:** see e.g. [LeTallec-Sassi]'96

- **Nested multi-d river models with a-posteriori** selection criteria

see [Amara-Capatina-Trujillo]'04 + Petrau PhD'09

## Our coupling algorithm: Joint Assimilation-Coupling (JAC)

Refs

[Marin, Monnier] '09

[Gejadze, Monnier] '07

### Principle:

- 1) One-way coupling term is relaxed (incoming charac. at interfaces),  
It is added into the cost function (extra term)
- 2) Data are used to quantify the coupling information

### → Augmented cost function:

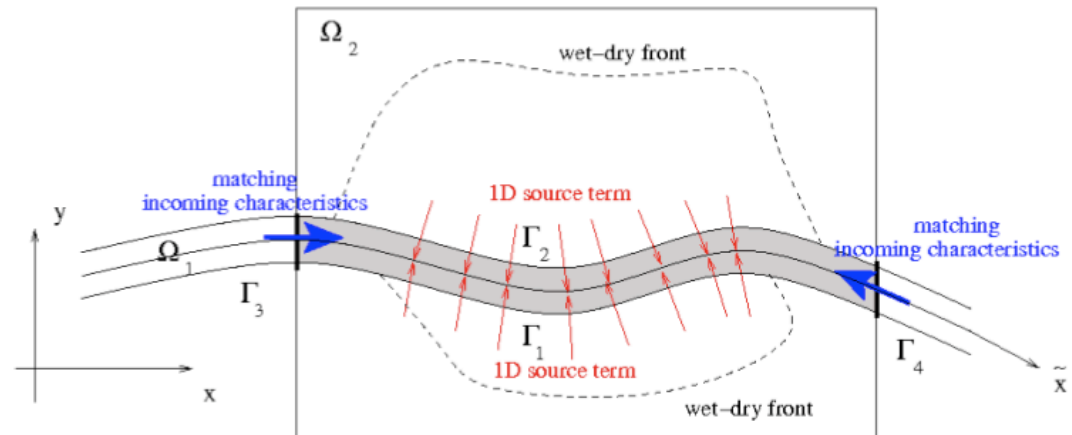
$$j_{total}(k) = j_{assim}(k) + j_{coupling}(k)$$

with

$$j_{coupling}(k) = \sum_{interfaces} \int_0^T [W_l^{1D} - \int_{interf} w_l ds]^2 dt \quad l = \# \text{ incoming charac.}$$

- If the term vanishes after minimization process then weak continuity of incoming characteristics at interfaces is obtained

An other version of JAC algorithm:  
the « sequential » JAC



# The « relaxed » JAC algorithm

## Principle:

We control the one way coupled model 2D → 1D

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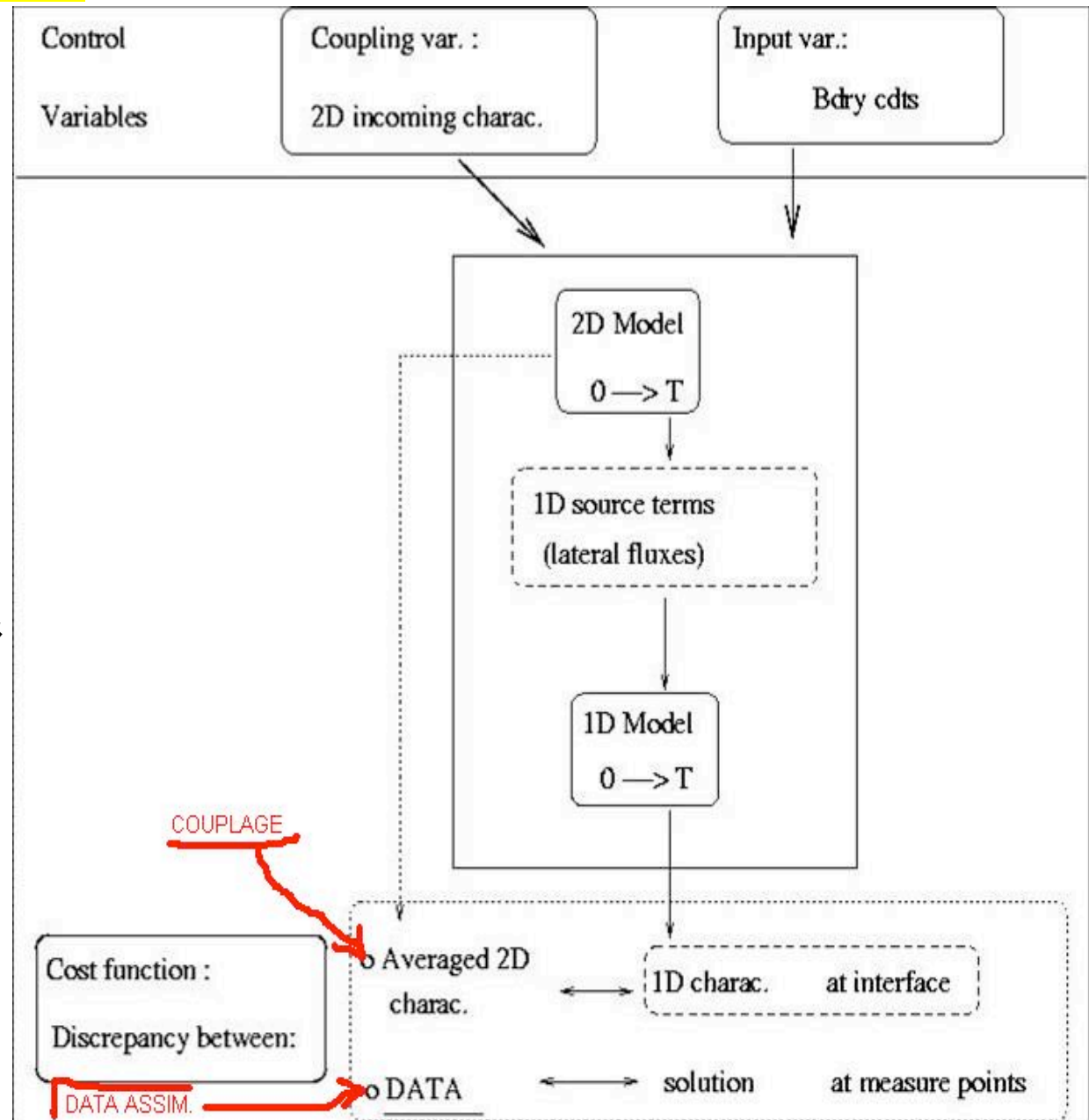
## Features:

- Multi-objectives optimization:
  - Need to balance « by hand » observation terms, regul. terms & coupling terms
- Convergence looks to be quite robust (academic test case)

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## Augmented cost function:

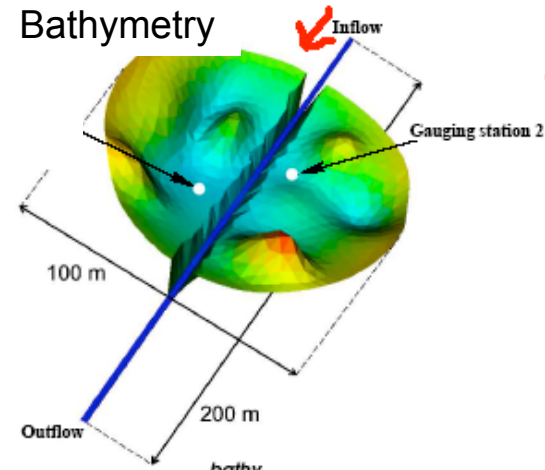
$$j_{tot}(k) = \alpha_{1D}j_{1D}(k) + \alpha_{2D}j_{2D}(k) + \alpha_{coupl}j_{coupl}(k)$$



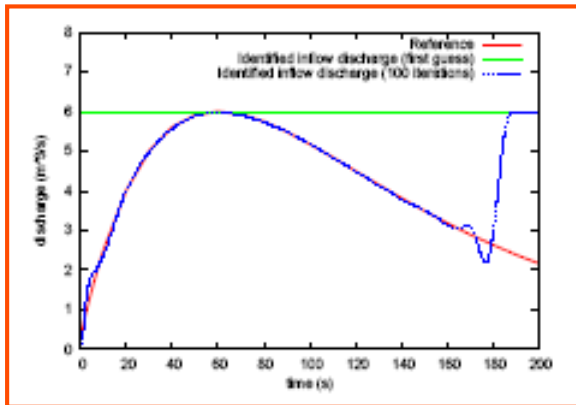
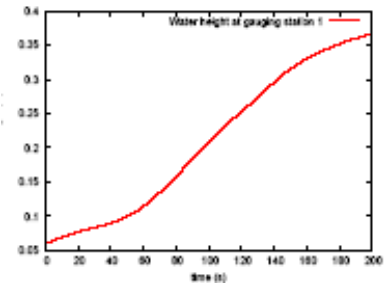
**Academic numerical test:  
Identification of  $Q_{in}$  in the 1D-model**

**Observations:** station 2 in the flooding area only  
→ read by the « local » zoom 2D model

**Problem:** identify the inflow discharge in the 1D « global » model

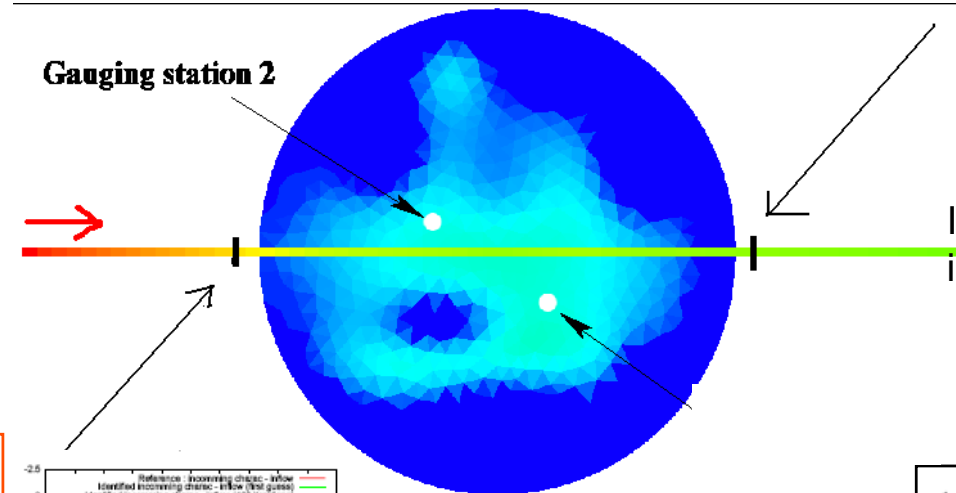


Obs:  $h$  at station #2

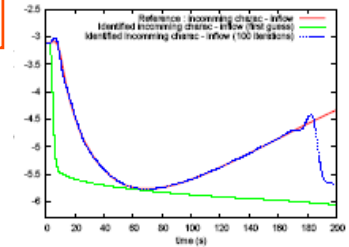
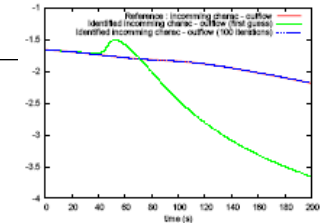


**Inflow discharge identified**

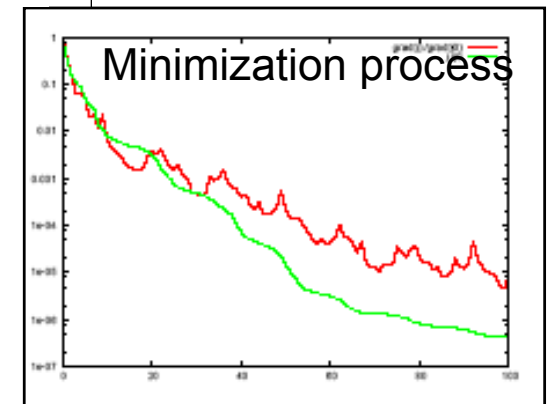
Ref. [Marin, Monnier] '09



Incom charac at outflow identified



Incom charac at inflow identified





# An other version: the « sequential » JAC algorithm

## Step 1

- Calibration (minimization) of the 2D zoom model only
- Save the resulting source term

## Step 2

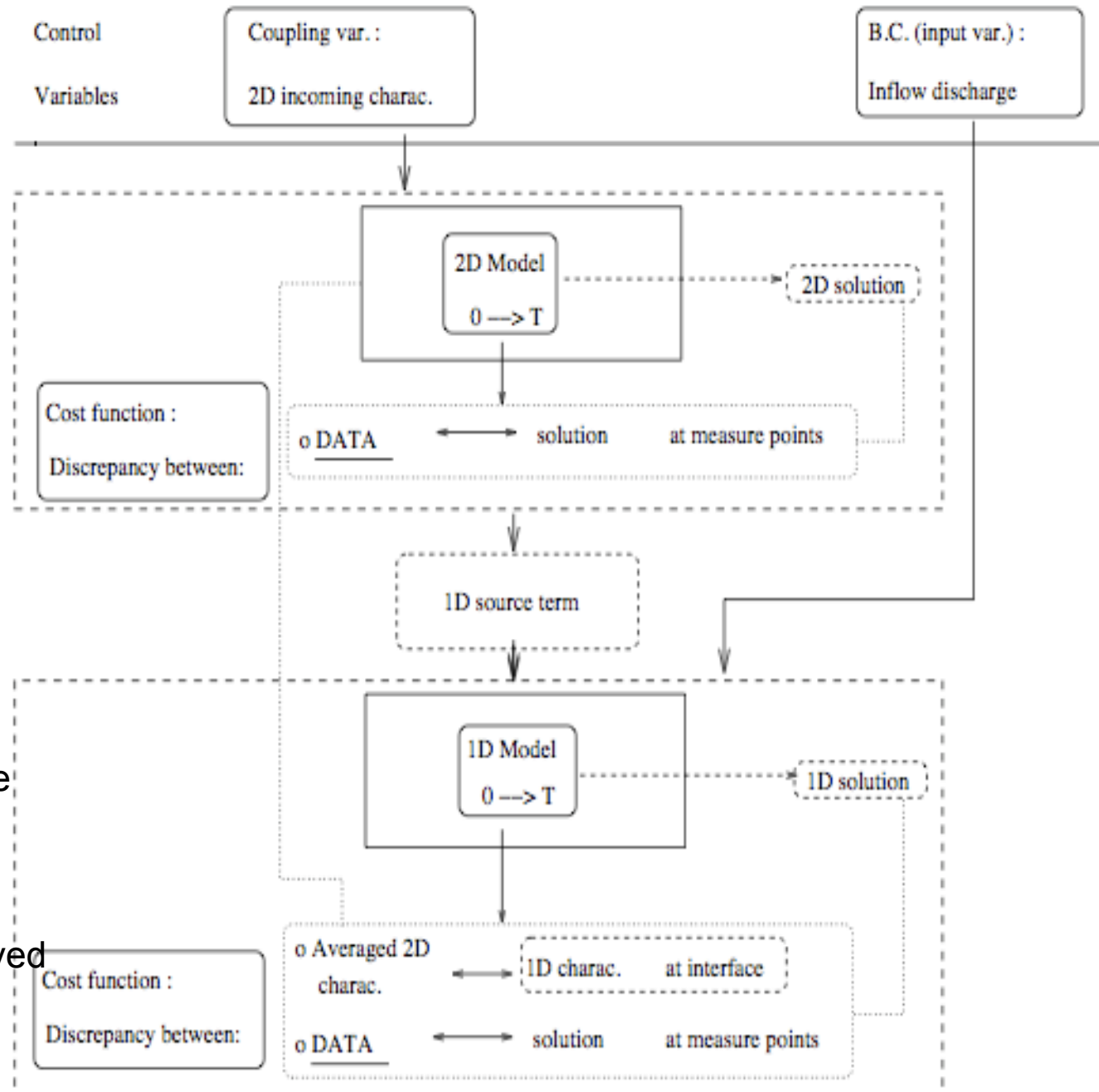
- Calibration (minimization) of the 1D global model

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## Features

- The adjoint codes are separated
- the two optimization problems are solved sequentially

But convergence is less robust, and a « blind period » must be removed between both steps...



# A comparison JAC vs Schwarz algorithm global in time

## Same coupling configuration

1D-2D non-matching grids (but constant slopes)

Ratio 1D/2D: space =10 , time =100

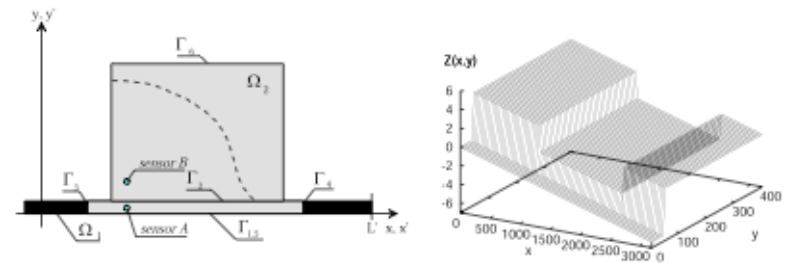


Fig. 3. Simplified problem layout and bathymetry used in numerical tests.

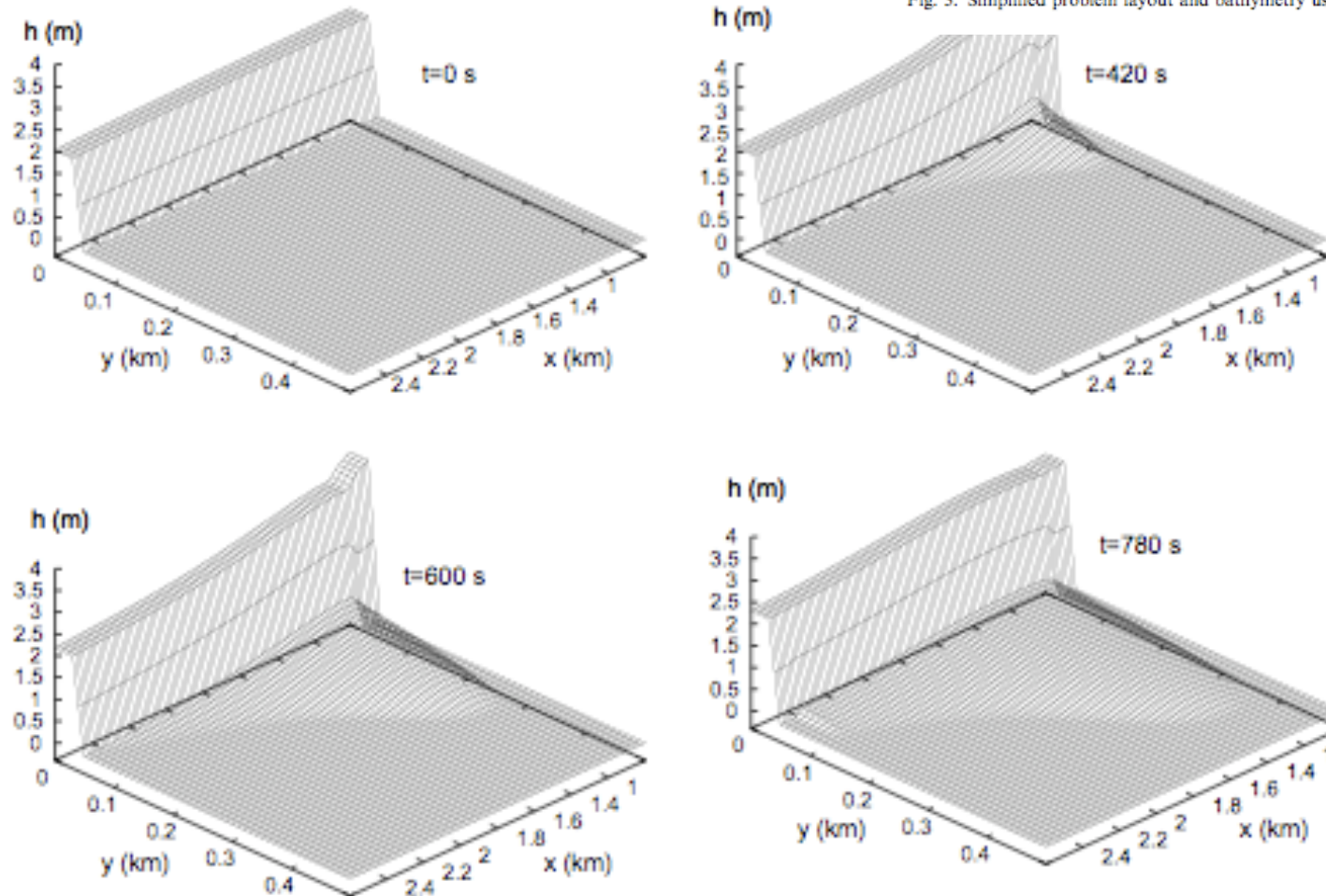


Fig. 4. Reference flow (surface elevation  $h$ ) for different times.

## A comparison with Schwarz algorithm, global in time

Refs. [Gejadze, Monnier] '07  
[Fernandez-Marin-Monnier]'10

Plot: h and u, Schwarz algo., 3 iterations (vs JAC algo., down)

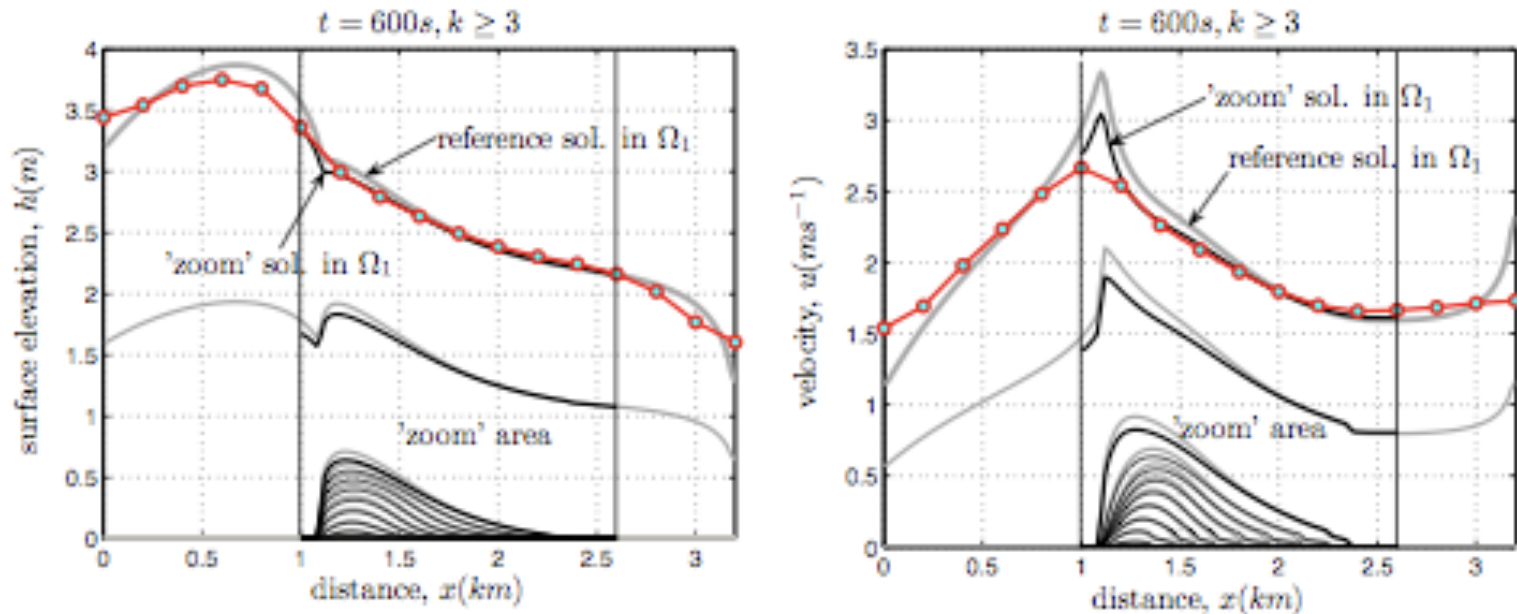
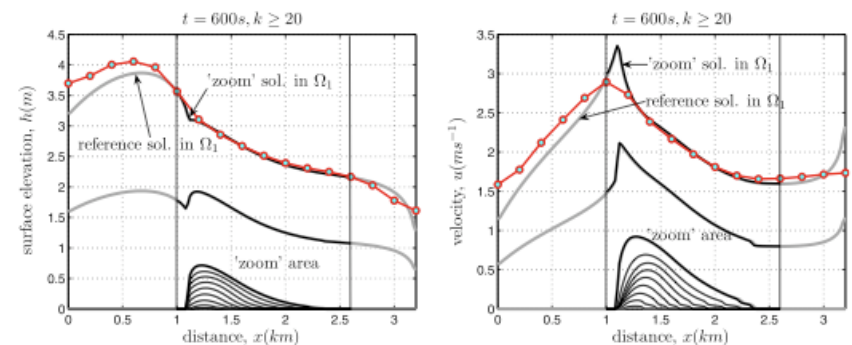


Fig. 7. WFR method, inconsistent discretization, using defect correction (13)

### Remarks

- This remains a **superposition** of the 2D model  
→ no « model decomposition » required



- **Accuracy:** similar to those obtained with JAC algorithm (using synthetic data)
- Obviously, Schwarz algorithm is much less time-consuming (no adjoint model, no minimization process) but **no calibration is done** (e.g. the 1D inflow b.c. must be given)

## In conclusion

- **4D-var - calibration of friction coefficients** (Manning-Strickler):  
One image (spatial distributed information) and preliminary sensitivity analysis lead to a better understanding of the flow...
- **Superposition 2D-1D SWE**: the integrity of the 1D-global model is preserved, the coupled solution is accurate (=full 2D model if same meshes).
- **In a variational data assimilation context, advantages of JAC algorithms**
  - Num. experiments show:
    - no significant extra computational-cost compared to 4D-var mono “full-model”,
    - accuracy similar to Schwarz approach (direct modeling),
    - quite robust convergence (toy test case...)
  - Weak continuity is natural if non-matching grids
  - The 2D zoom model can map local observations into the global model

**Drawbacks** of algorithms based on **optimal control & adjoint method**:

- Adjoint codes are required
- The optimization process is very time-consuming (~ 50-100 times the forward runs)

**This is a preliminary study**: no numerical analysis done, no real data considered; Nevertheless, both the superposition pcp & JAC algo seem to be interesting.

### References

#### 4D-var / Mosel river:

[Hostache, Lai, Monnier, Puech] J Hydrology'10  
[Lai-Monnier] J. Hydrology'09

**DassFlow software**: see webpage

#### JAC algorithm:

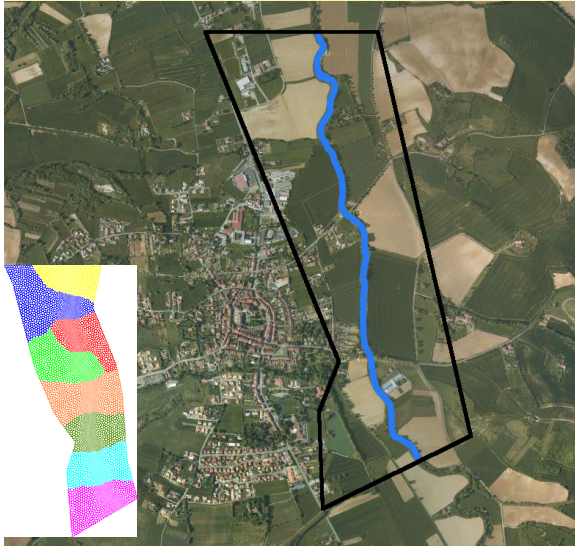
[Marin-Monnier] Math. Comput. Simul.'09  
[Gejadze-Monnier] CMAME '07

**Coupled FV scheme**: [Fernandez-Marin-Monnier]'10

# Expérimentations numériques en cours



Université  
de Toulouse



Lèze River (Toulouse, France)

## Lèze river. Collaboration IMFT - IMT

At Math Inst. of Toulouse (IMT):

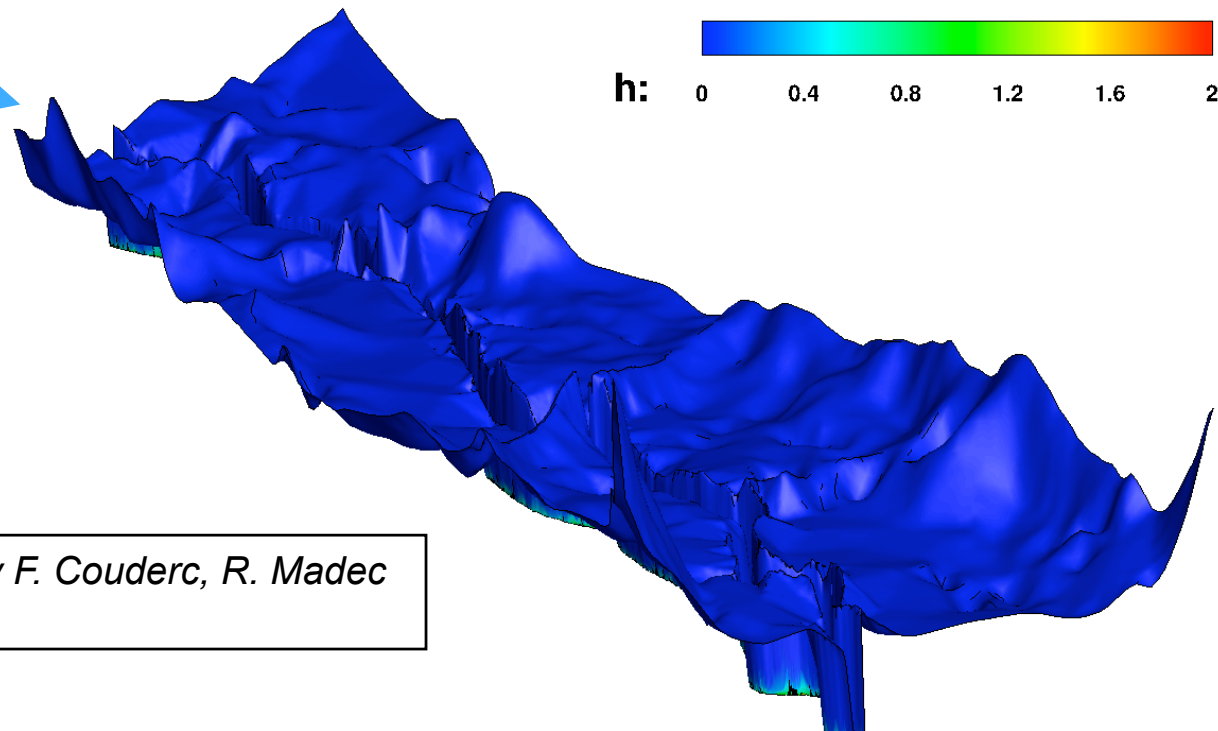
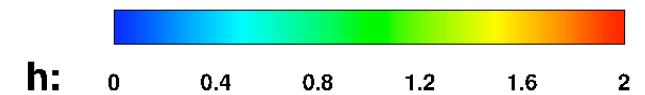
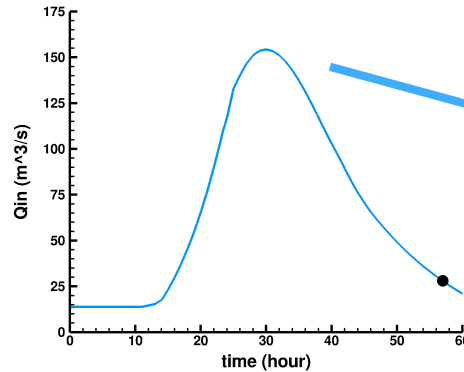
F. Couderc, R. Madec, J.M., JP. Vila

At Fluid Mech. Inst. of Toulouse (IMFT):

D. Dartus, K. Larnier, J. Chorda

## ANR AMAC 2010-13

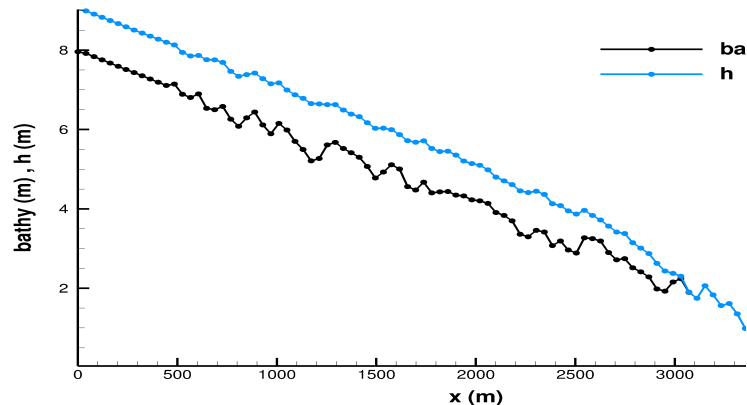
(IMFT, IMT, Schapi, Dreal31, Geode, LMTG)



*Num. results performed at IMT-Insa by F. Couderc, R. Madec  
DassFlow-Hydro software*

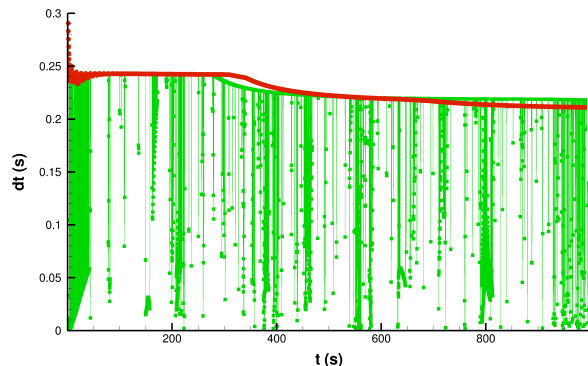
# Cas test front sec, topographie bruitée. Parmi nos questionnements actuels...

## Schémas explicites HLLC



1) Formulation [Toro book'01], Equilibre a la [Leveque'98]  
Heps front sec requis  $\rightarrow$  vit. de front Heps-dependant,  
mais aussi pbs de débordements éventuels:  
Moselle: OK, Lèze: pas physique...

2) Formulation [Vila SIAM'86], équilibre semblable  
 $\rightarrow$  Heps=0 est ok  
 $\rightarrow$  min. CFL « stable » (cf figure)  
par contre le linéaire tangent devient instable.  
A suivre... (résultat de la semaine dernière)



$$\Delta t \frac{\max(\|u\| + c)}{\min(d_{L,R})} \leq 1$$

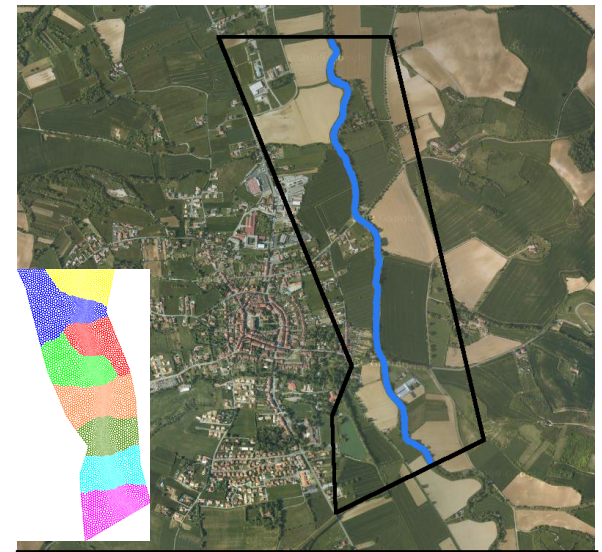
## Our software DassFlow (see webpage)

- o Forward models: SWEs. Also: transport, sedimentation (FV), Stokes ALE non-newtonian (FEM).
- o FV schemes: explicit HLLC or implicit Van Leer
  - $\rightarrow$  **FV Order 2 and semi-implicit under progress**
- o Adjoint code: automatic differentiation (TapeNade software, Inria)
- o From libraries: minimization (BFGS, Inria), linear algebra (Mumps, U. Toulouse)
- o MPI Fortran codes

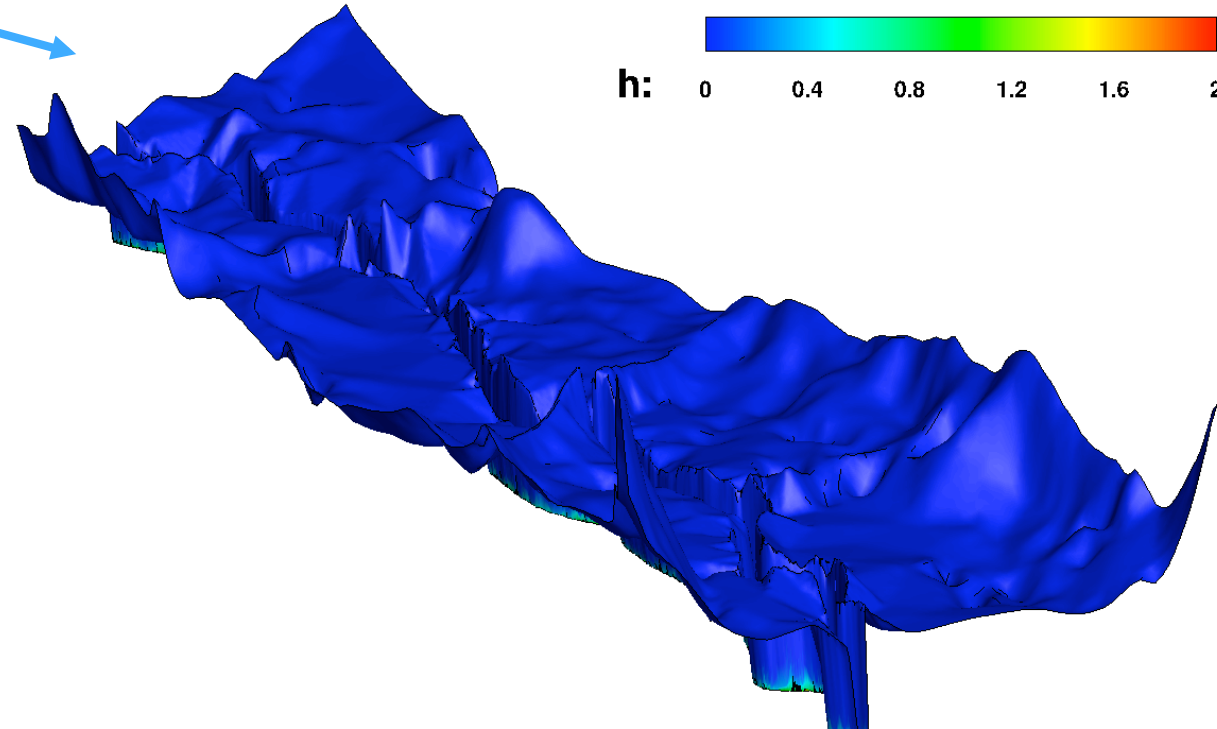
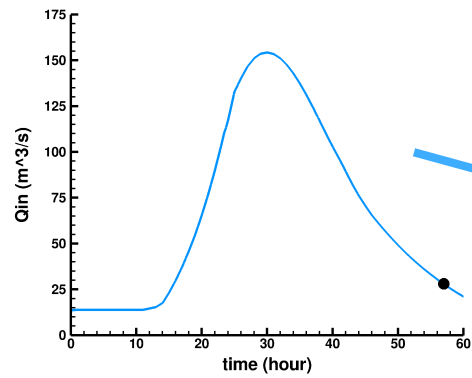


Université  
de Toulouse

**Merci pour votre attention**



Lèze River (Toulouse, France)

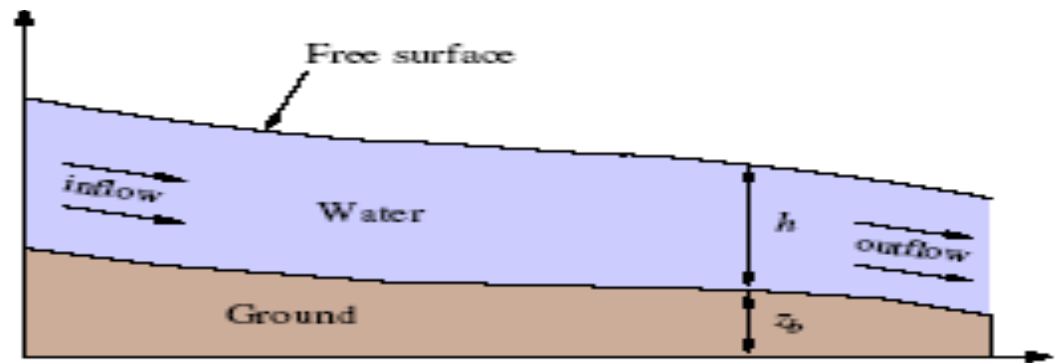






## Local model: 2D SWE (non viscous)

$h$  : water elevation ;  $\mathbf{q}$  : 2D discharge



$$\begin{cases} \partial_t h + \operatorname{div}(\mathbf{q}) = 0 \\ \partial_t \mathbf{q} + \operatorname{div}\left(\frac{1}{h} \mathbf{q} \otimes \mathbf{q}\right) + \frac{1}{2}g\nabla h^2 + gh\nabla z_b + g\frac{n^2\|\mathbf{q}\|_2}{h^{7/3}}\mathbf{q} = 0 \end{cases}$$

+ I.C. + B.C.

**Conservative form** of 2D SWE with topography and friction source term:

$$\partial_t U + \partial_x F_1(U) + \partial_y F_2(U) = (S_g + S_f)(U)$$

where

$$F_1(U) = \left(q_1, \frac{q_1^2}{2} + \frac{1}{2}gh^2, \frac{q_1 q_2}{h}\right)^T$$

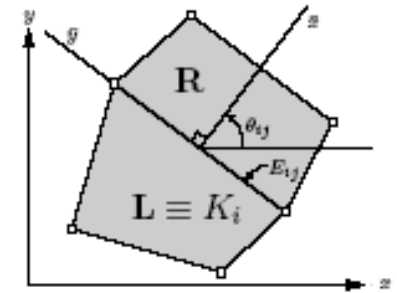
$$F_2(U) = \left(q_2, \frac{q_1 q_2}{h}, \frac{q_2^2}{2} + \frac{1}{2}gh^2\right)^T$$

$$S(U) = \begin{pmatrix} 0 \\ -gh \partial_x z_b - g\frac{n^2\|\mathbf{q}\|_2}{h^{7/3}} q_x \\ -gh \partial_y z_b - g\frac{n^2\|\mathbf{q}\|_2}{h^{7/3}} q_y \end{pmatrix}$$

## Finite volume schemes: 1D conservative schemes

For 2D SWE:

- we use the invariance rotation property:  $F(U) \cdot \eta = T_\eta^{-1} F_1(T_\eta U)$
- we neglect tangential terms,



then **2D SWE = 1D SWE + linear transport** (e.g. pollutant):

$$\partial_t(T_\eta U) + \partial_\eta F_1(T_\eta U) = (0, gh, 0)^T \partial_\eta z_b$$

We set:  $V = T_\eta U = [h, q_n, q_\tau]^T$      $F_1(V) = [q_n, \frac{q_n^2}{h} + g \frac{h^2}{2}, q_n q_\tau h]^T$

= x-component of the flux

→ **1D conservative schemes**

(1st or 2<sup>nd</sup> order)

$$\frac{1}{\Delta t} (V_i^{n+1} - V_i^n) + \frac{1}{\Delta x} (F_{i+1/2}^S - F_{i-1/2}^S)^n = (S_{topo})_i^n$$

Where  $(S_{topo})_i^n$  = standard centered approximation

$F_{i+1/2}^S$  = 1D numerical flux including correction due to the topography term for **well-balanced properties**

# Finite volume schemes: well-balanced properties

Numerical fluxes of the 1D scheme are associated to 1D local Riemann problems with source term:

$$\partial_t V + \partial_n F_1(V) = S_{topo}(h \partial_n z_b)$$

$$\text{with } V(x, 0) = V_L \text{ if } x_{\bar{n}} < 0 ; \quad V(x, 0) = V_R \text{ if } x_{\bar{n}} > 0.$$

**1D SWE: HLL scheme.** See [Chacon et al '04], [Dominguez-Fernandez-Martin'06]

→ Water at rest and steady-state solution are preserved (up to 2nd order in time)

**2D SWE: HLLC scheme** considers in addition the intermediate wave speed (shear wave)  $\lambda_2 = u$

$$[F_S^{hllc}]_3 = [F_S^{hll}]_1 \cdot u^*$$

It is defined from HLL as follows (see [Toro])

$$\text{avec } u^* = (V_L)_3 \text{ si } S^* \geq 0 \text{ et } u^* = (V_R)_3 \text{ si } S^* < 0$$

→ HLLC preserves water at rest + steady-state solutions

Ref. [Fernandez-Bresch-Monnier] Note CRAS'08

**Definition of the 2D coupling source term**

$q_n$

= component #1 of the numerical flux

$(\Psi_1, \Psi_2)$

$u_{tk}$

is approximated upon the sign of

$$S^* \approx u_n$$

(up-winding)

→ mix of over-flowing – filling flows is possible

**Finally, the global scheme** (coupled 1D-2D) preserves water at rest since the 2D coupling source term vanishes if velocity = 0

In collaboration with  
E. Fernandez-Nieto (Sevilla)

Ref.  
[Fernandez-Marin-Monnier]'10

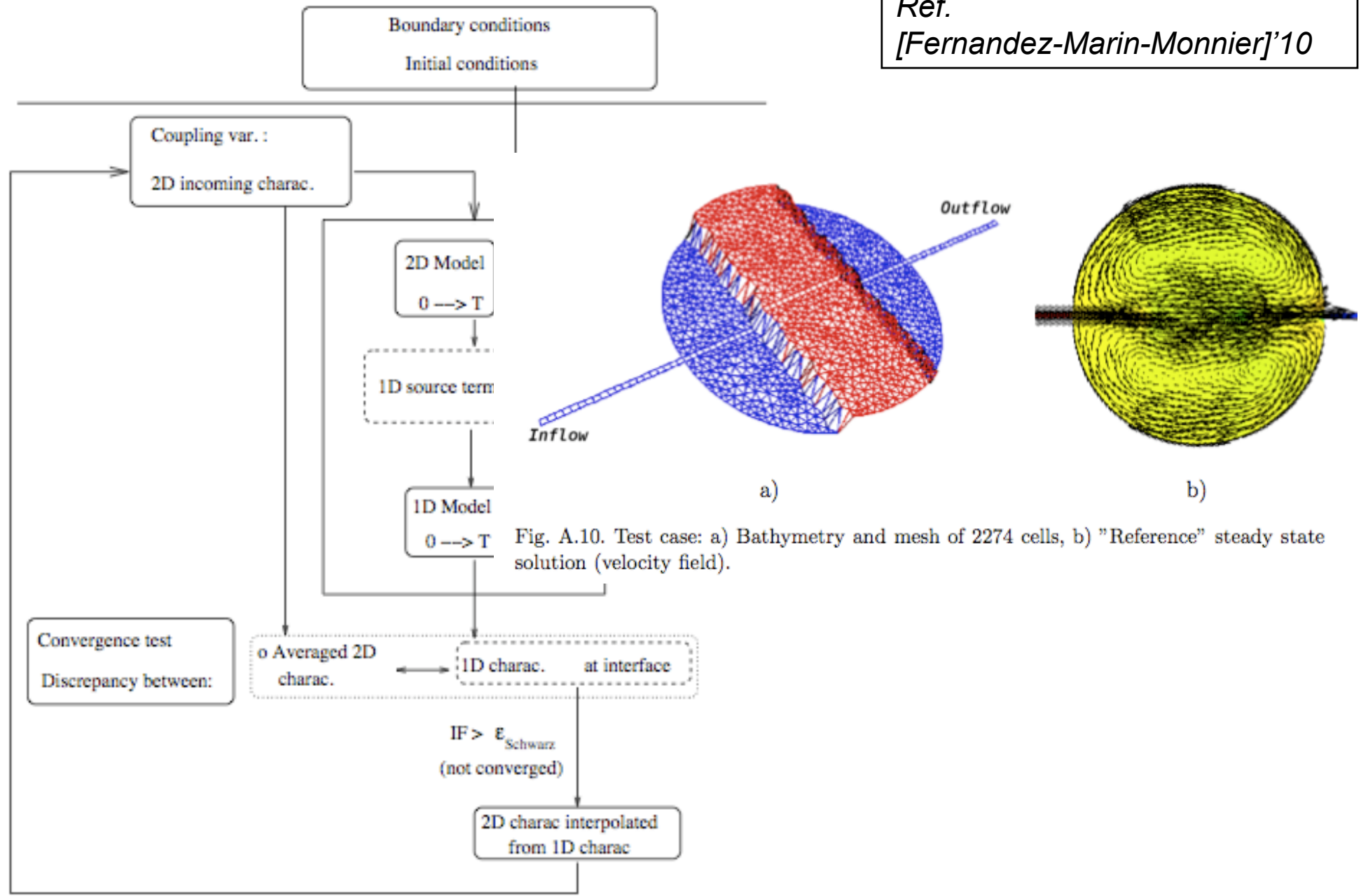


Fig. A.10. Test case: a) Bathymetry and mesh of 2274 cells, b) "Reference" steady state solution (velocity field).

Fig. A.9. Coupling algorithm based on a Schwarz method.

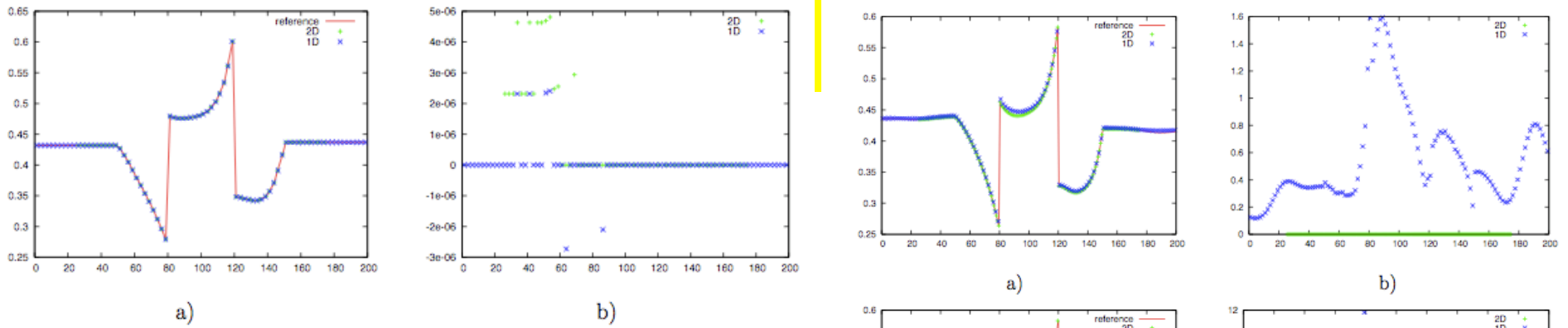


Fig. A.11. Matching grids case. Comparison of velocity values ( $u$ ) in the 1D main channel (common area) and after Schwarz algorithm convergence. a) 2D reference solution and values computed by the 1D model and by the local 2D zoom model; b) Differences in percent.

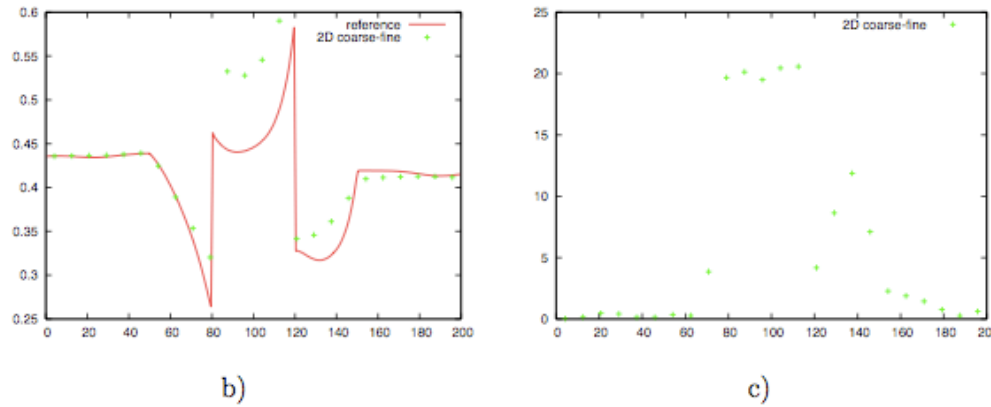
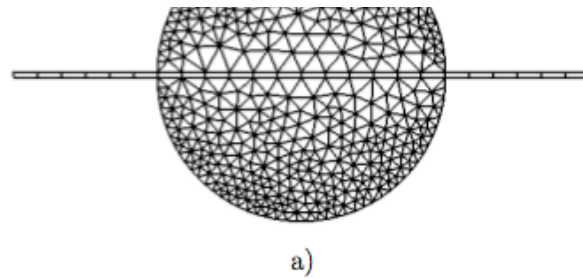


Fig. A.13. Superposition vs full 2D. a) The full 2D mesh is defined from the 1D mesh in the channel ( $R = 1$ ). b) Velocity  $u$  in the main channel: "full 2D" solution ( $R = 1$ , legend "2D coarse-fine") and the 2D coupled solution with  $R_{spac} = 10$  (legend "reference"). c) Differences in percent.

Comparison of velocity values ( $u$ ) in the 1D main channel (common area) and after Schwarz algorithm convergence. a)  $R_{spac} = 2$ . 2D reference solution and values computed by the 1D model and by the local 2D zoom model; b) Differences in percent; c) Differences in percent; d) Differences in percent.

# Flat topographies, steady-state solution, 2 points of observation

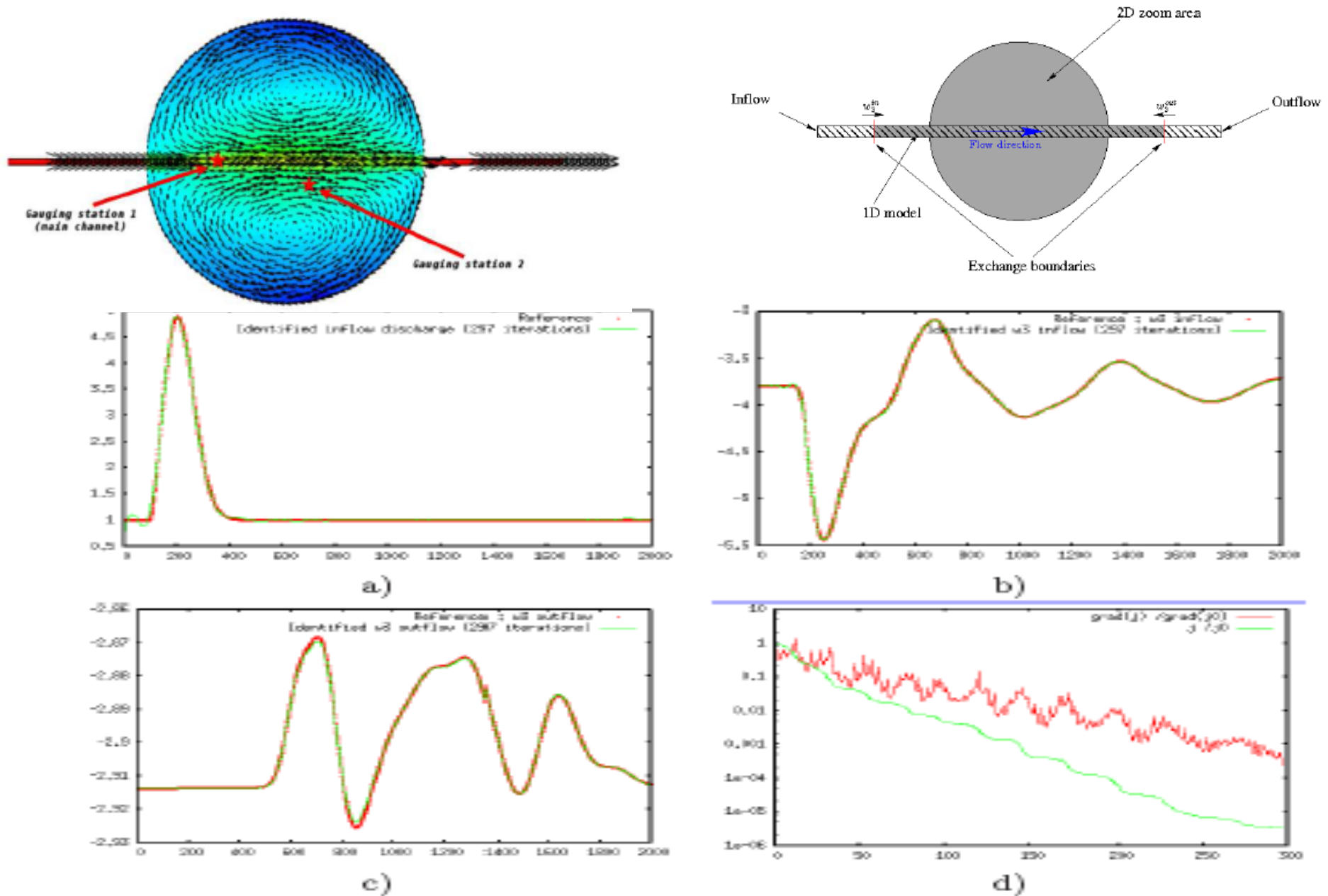
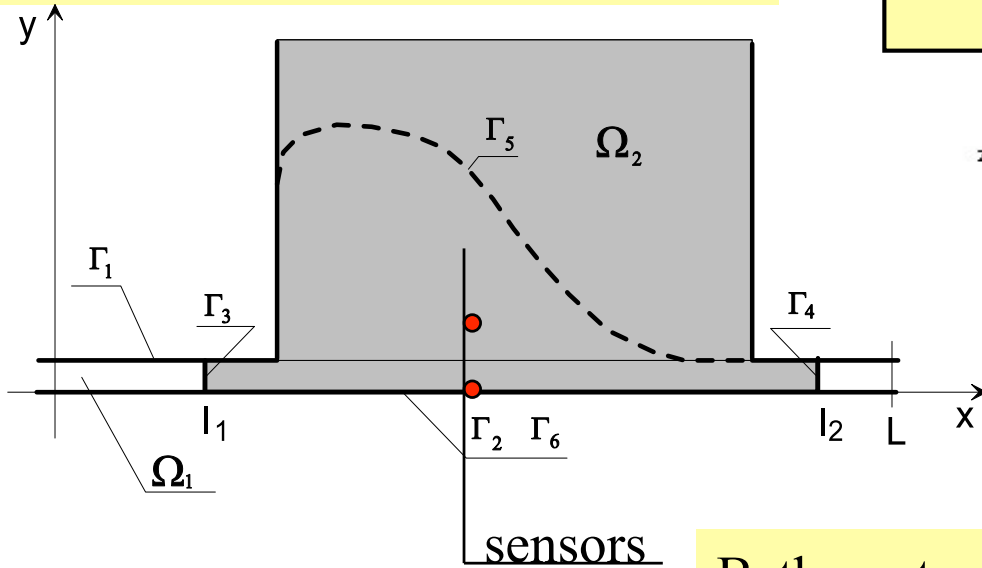


FIG. 6 – Identification of  $q_{in}$ ,  $w_3^{in}$ , and  $w_3^{out}$  using the JAC algorithm with 2 observation points. ( $\alpha_{1D} = 0$ ,  $\alpha_{2D} = 3$  and  $\alpha_{coupling} = 1$ )

# Numerical test. Example 1.

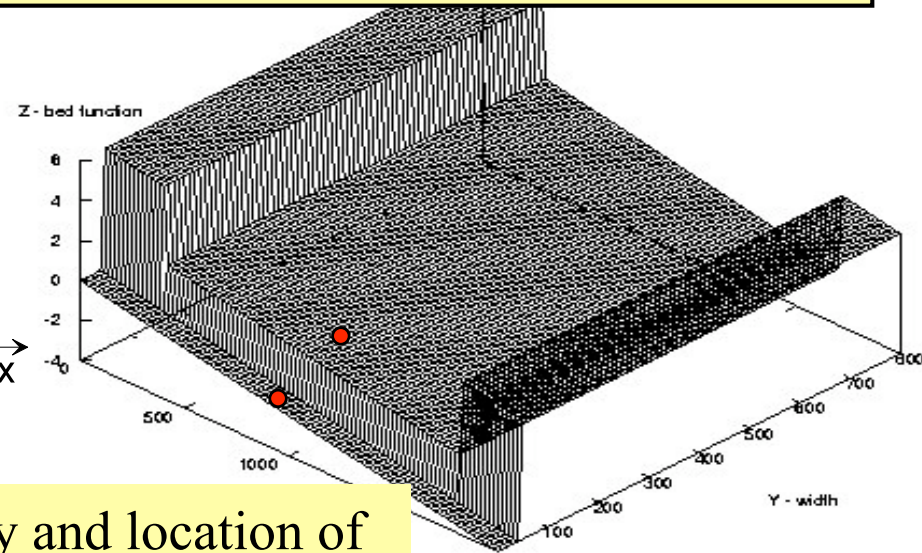
$L=2000\text{m}$ ,  $l_1=200$ ,  $l_2=1800$



Weak coupling at the 2 interfaces

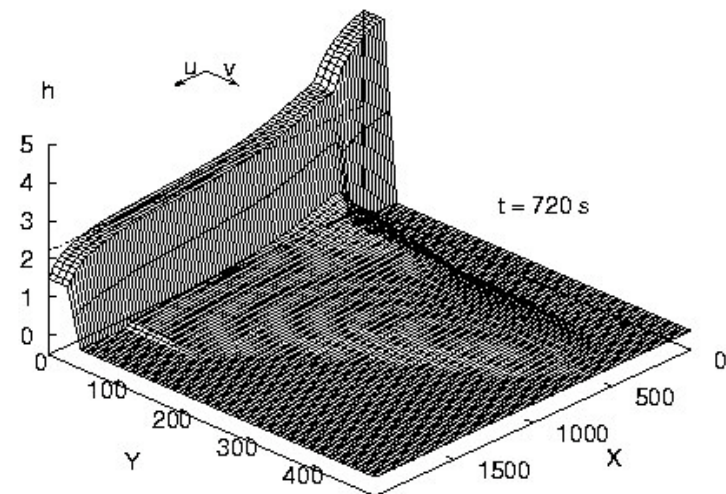
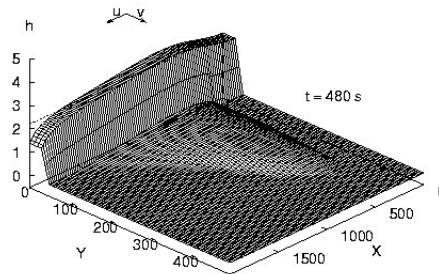
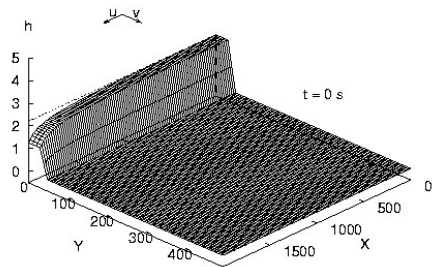
+

Data assimilated at 2 points  
(time series of elevation  $h$ )



Bathymetry and location of the 2 sensors

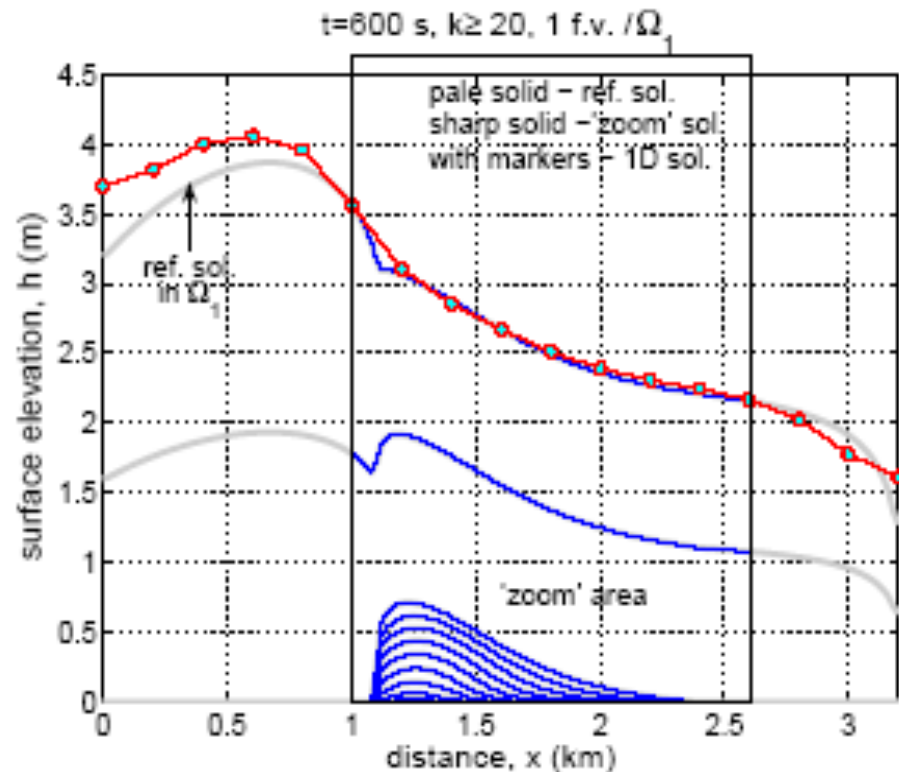
## The flow (overflowing)



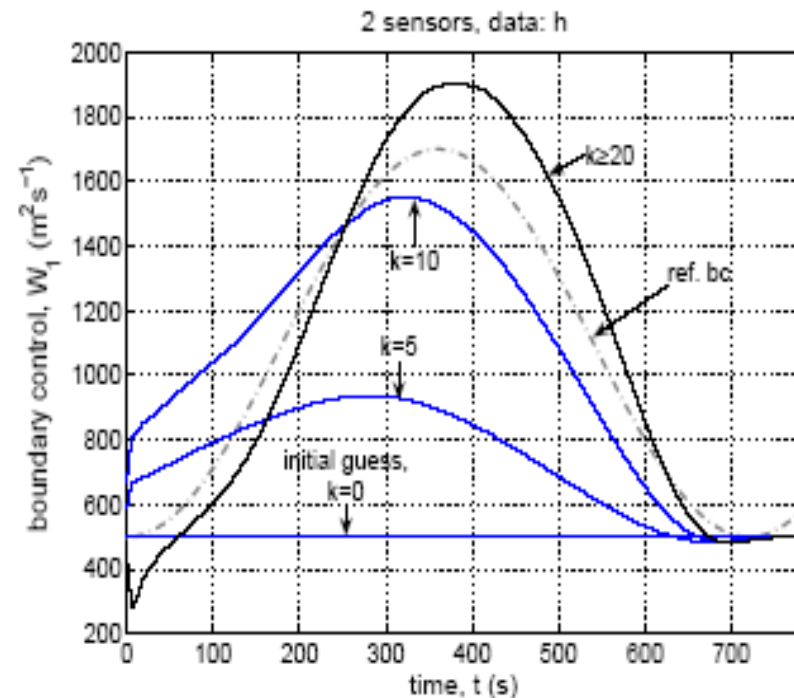
# Numerical results: JAC algorithm

Identification of 1D inflow b.c. while coupling the 1D-2D inconsistent models (Ratio 1D/2D space  $\sim 1/10$ , time  $\sim 1/100$ )

Computed elevation  $h$   
(after  $k=20$  iterations)



1D inflow bc identified  
(after  $k$  iterations)



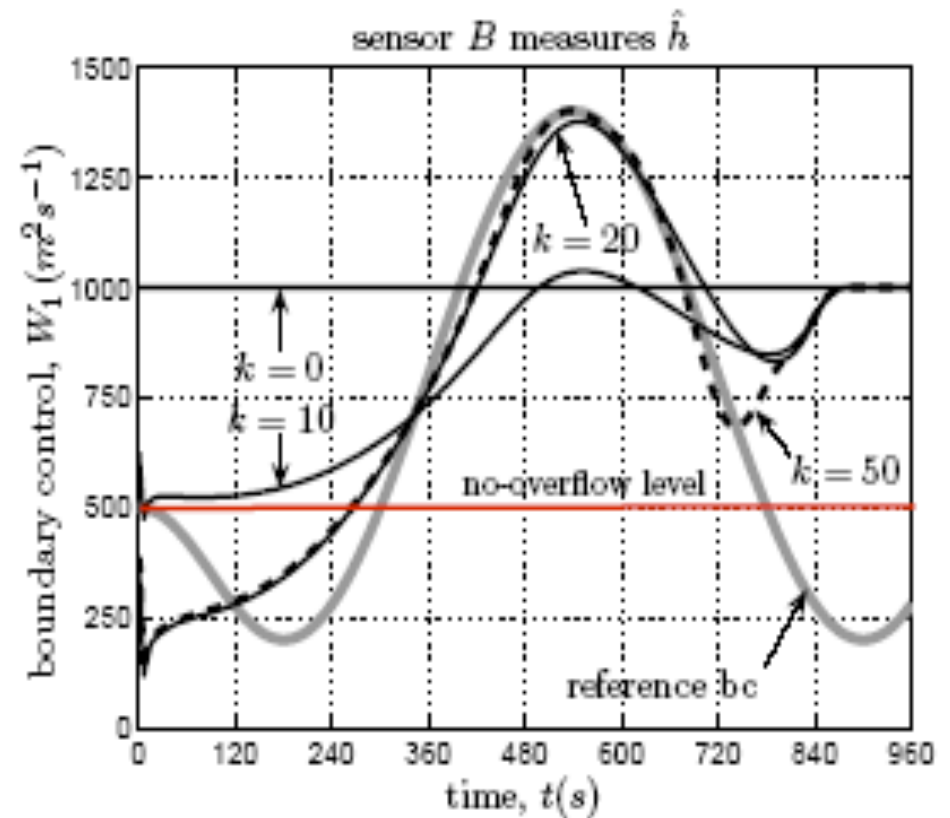
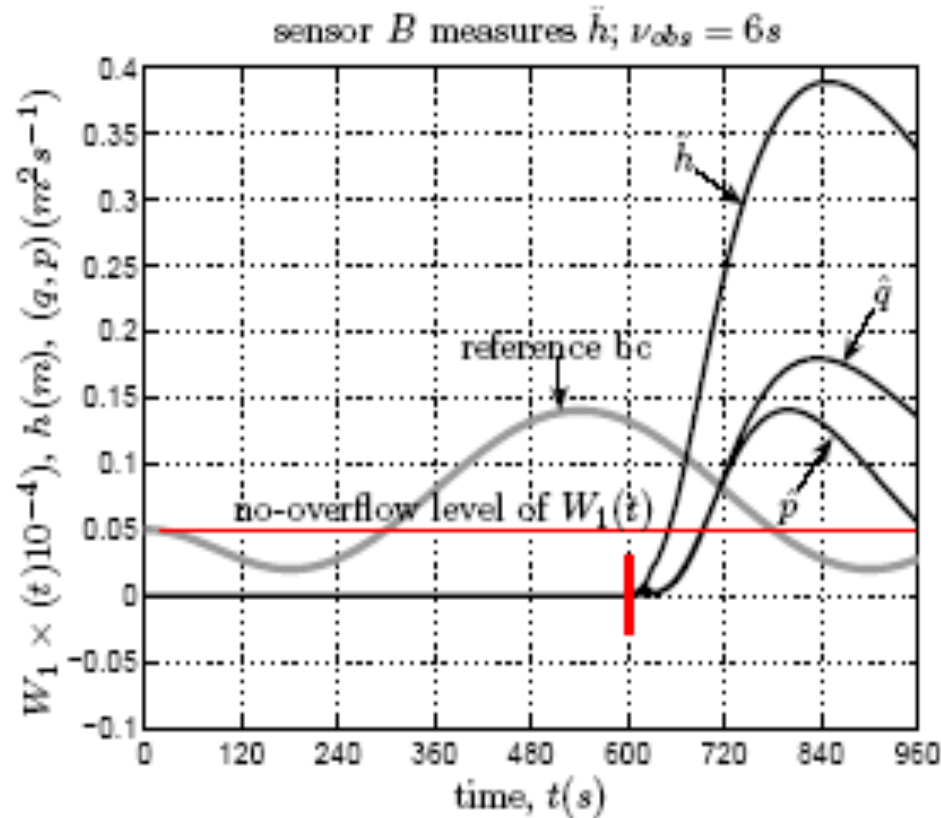
The 1D & 2D solution match perfectly with the reference solution within the zoom area (and within the main channel if consistent grids)

The unknown 1D bc is not precisely retrieved (but it is if consistent grids)



## Example 2: assimilation of data available only in the zoom area (time series of elevation $h$ )

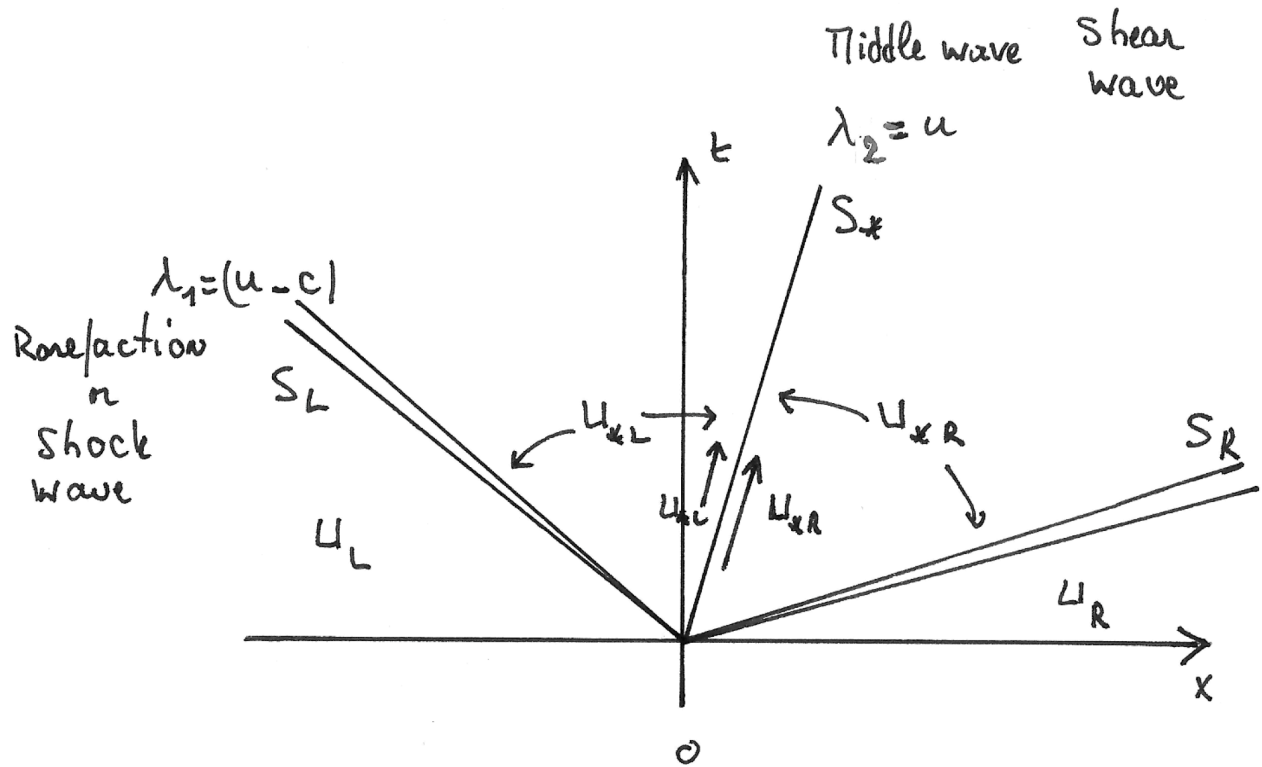
(Ratio 1D/2D space  $\sim 1/10$ , time  $\sim 1/100$ )



Reference BC &  
reading by the dry field sensor B

Identified inflow BC  
(after  $k$  iterates)

The 2D local zoom model allows to calibrate the 1D net-global model using data available into the zoom area only

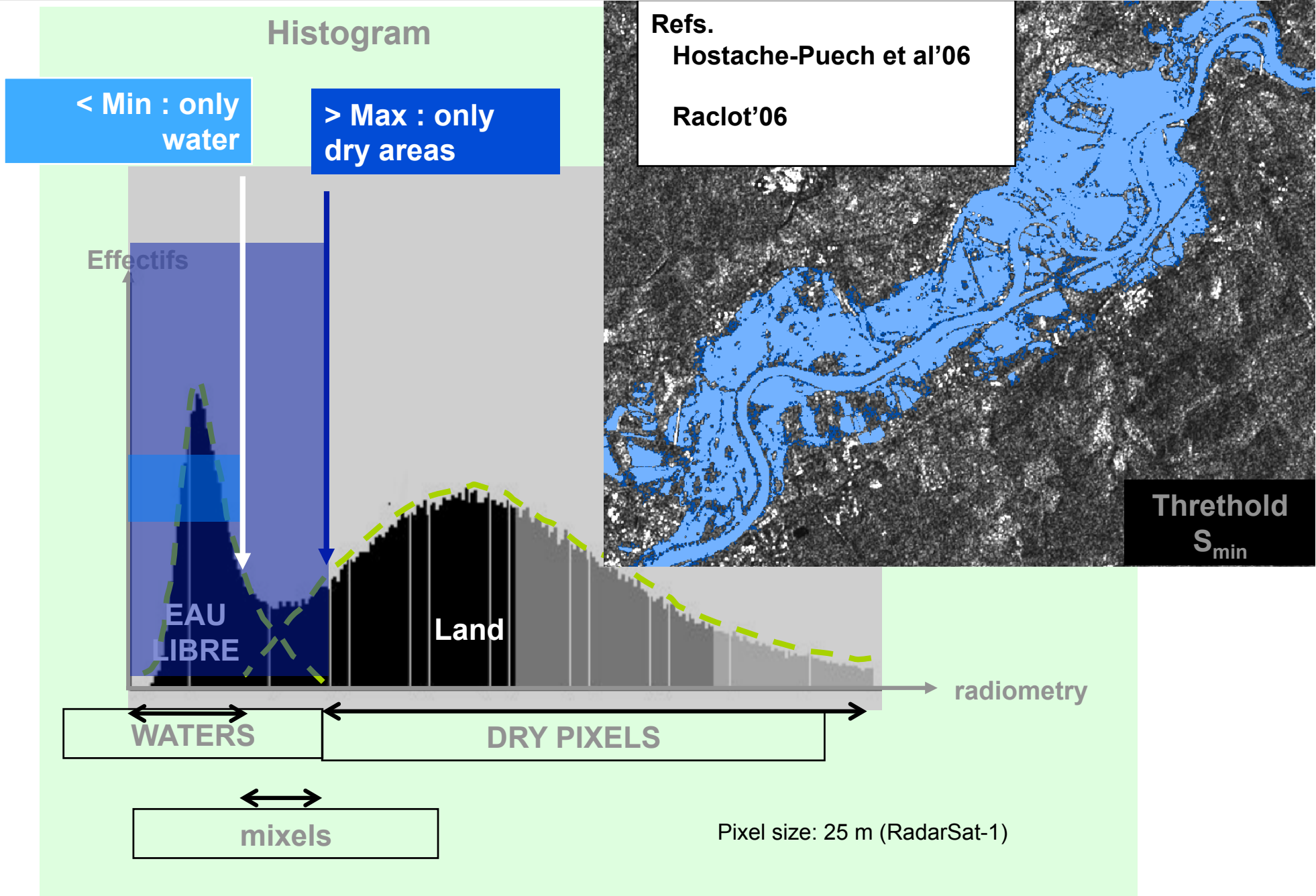


Solve Riemann  
 exact:  $S_* = U_x$

$\lambda_3 = (u + c)$   
 Rarefaction or Shock

# SAR image analysis

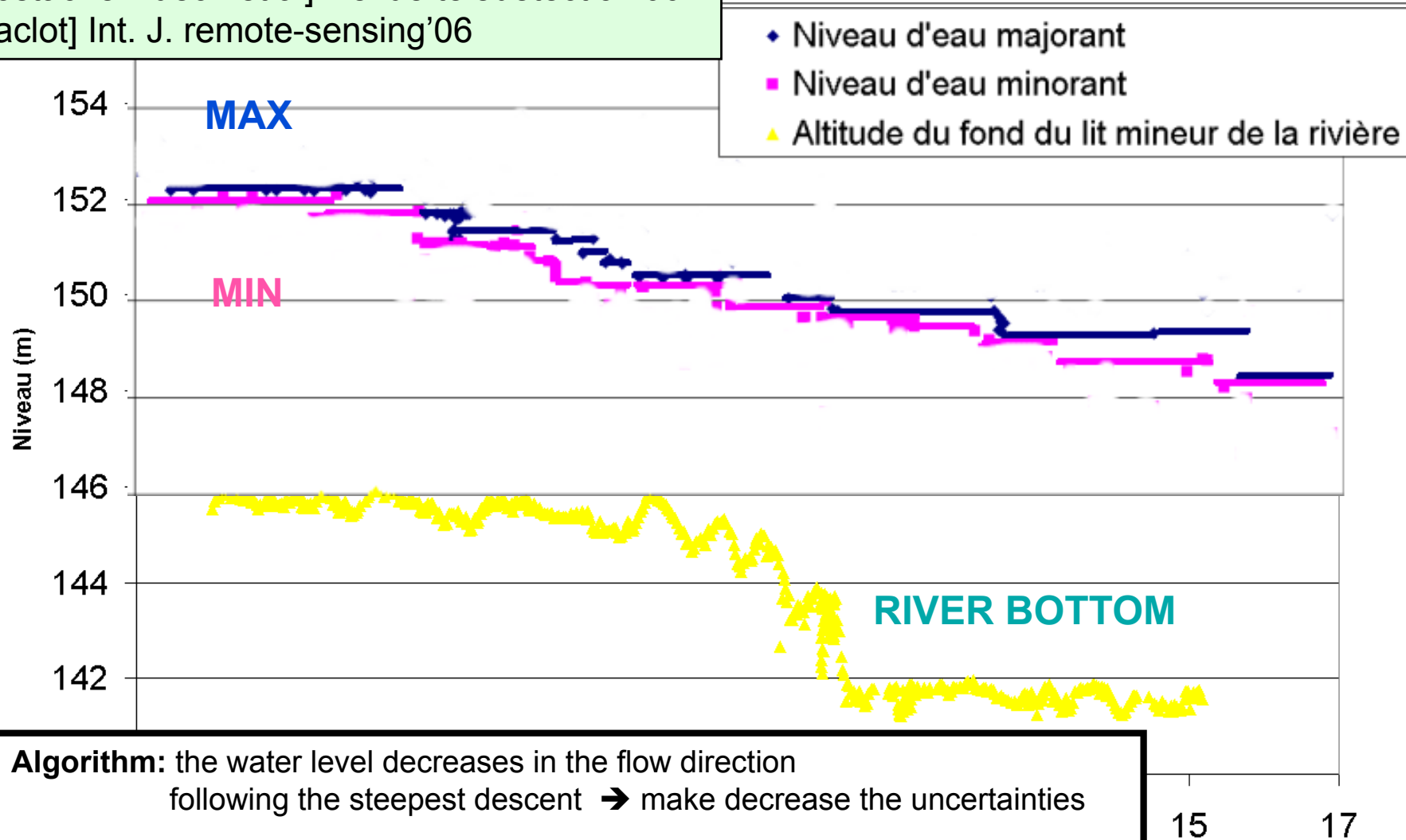
Step 1: Extracting waters, a fuzzy mapping...



# Image analysis

Step 4: relevant H values obtained after satisfying “hydraulic constraints”

Refs. [Puech-Raclot] Hydro. processes'02  
[Hostache-Puech et al] Revue teledetection'06  
[Raclot] Int. J. remote-sensing'06



**Algorithm:** the water level decreases in the flow direction

following the steepest descent → make decrease the uncertainties

**Final result:** H-values at “reliable image blocks” with mean uncertainty +/- 15 cm

