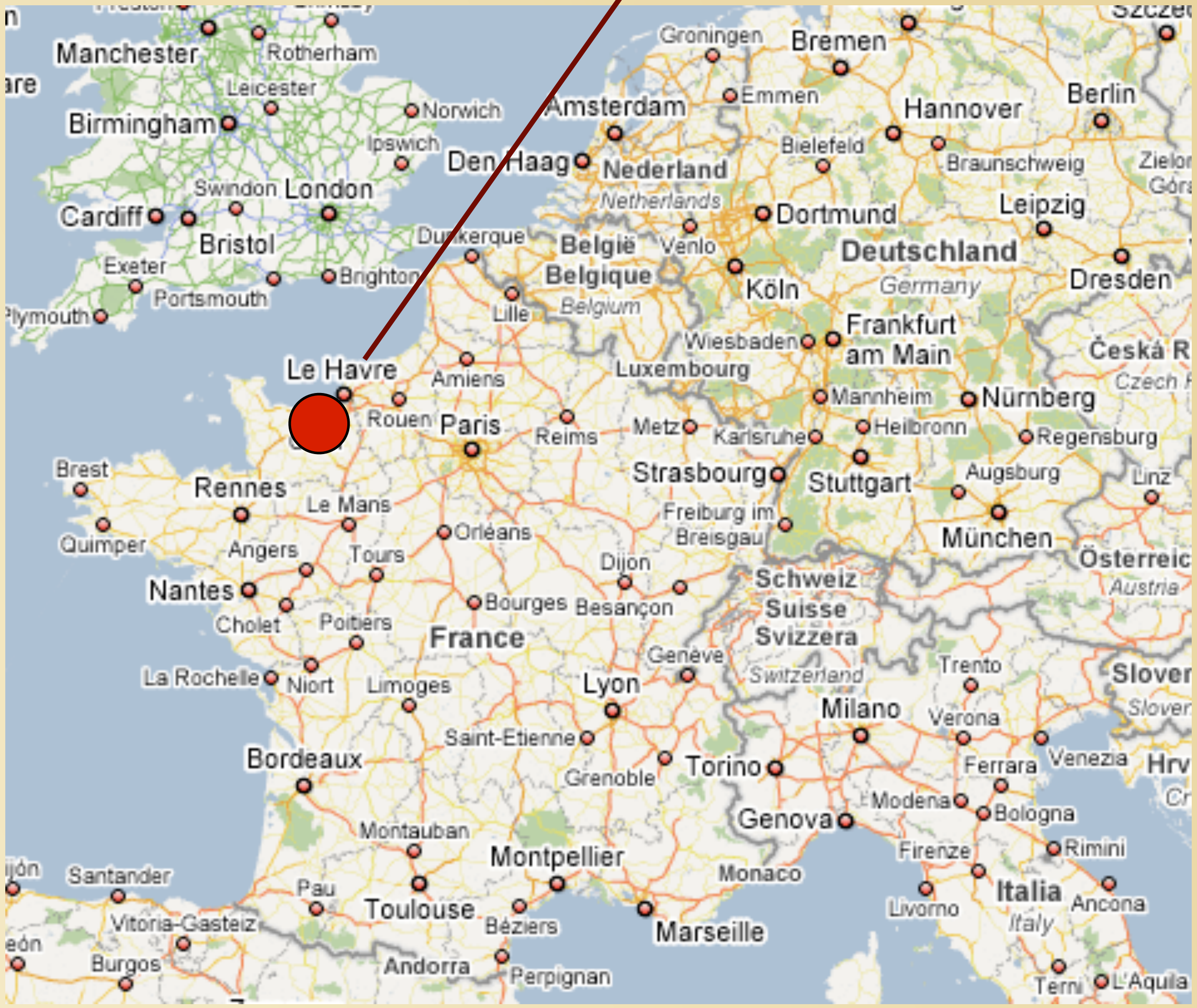
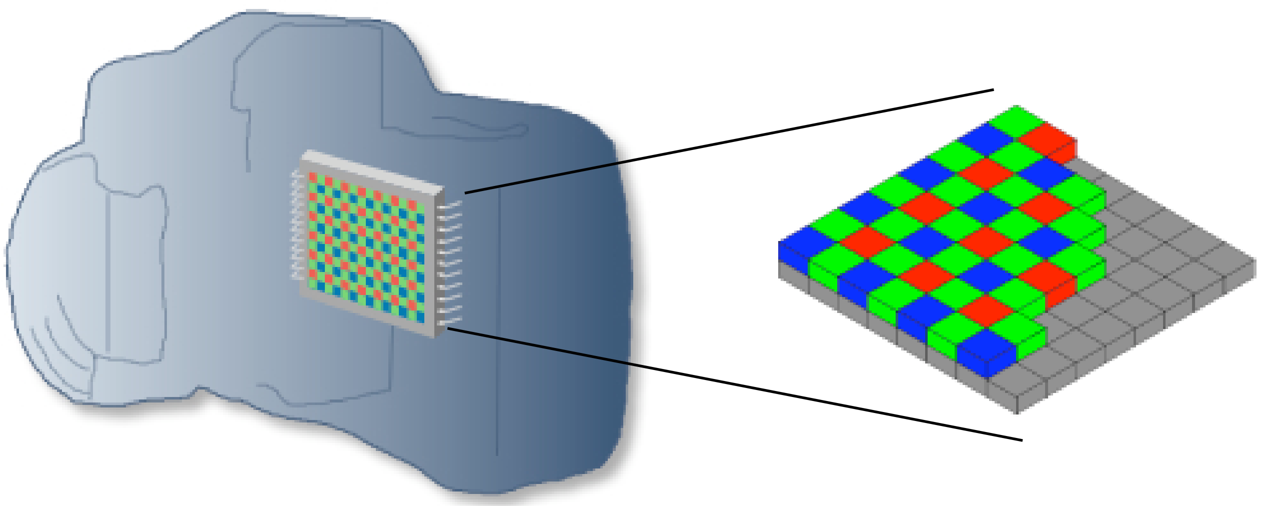
Reconstruction d'images couleurs par dématriçage/débruitage conjoint

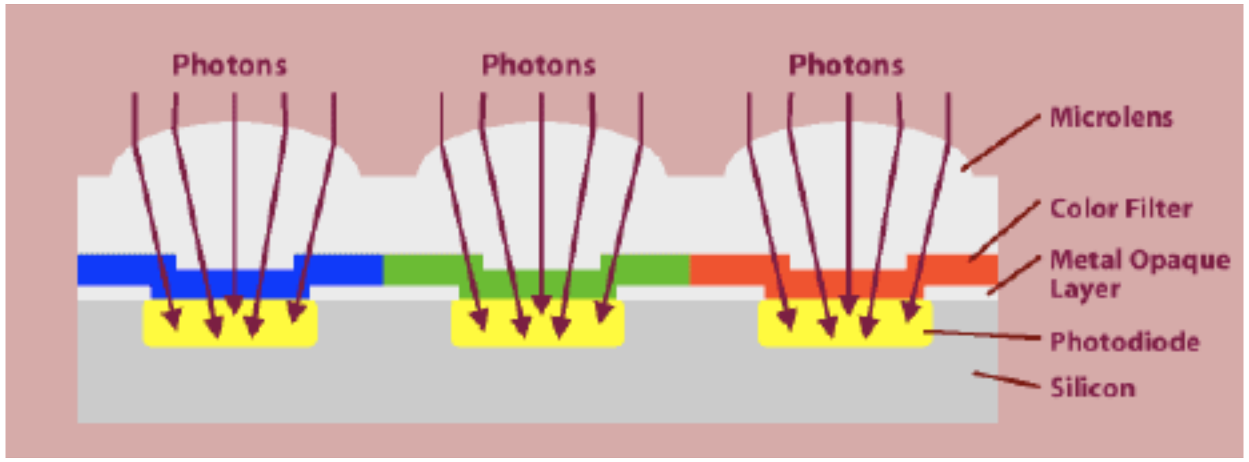
Laurent Condat



Color image acquisition with a single sensor

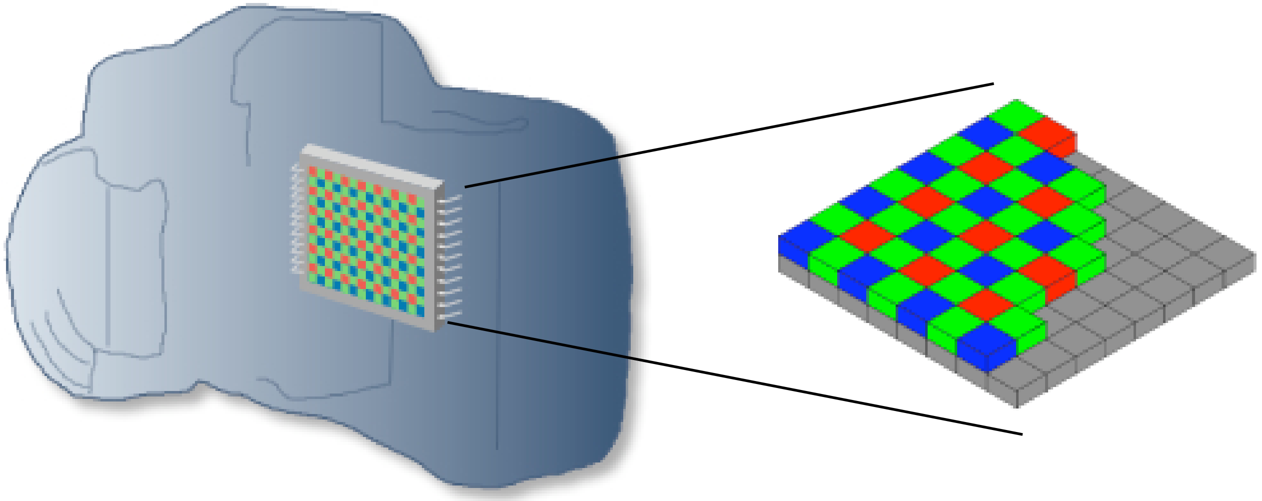


A (Bayer) color filter array (CFA)
is overlaid on the sensor



what you see

Color image acquisition with a single sensor



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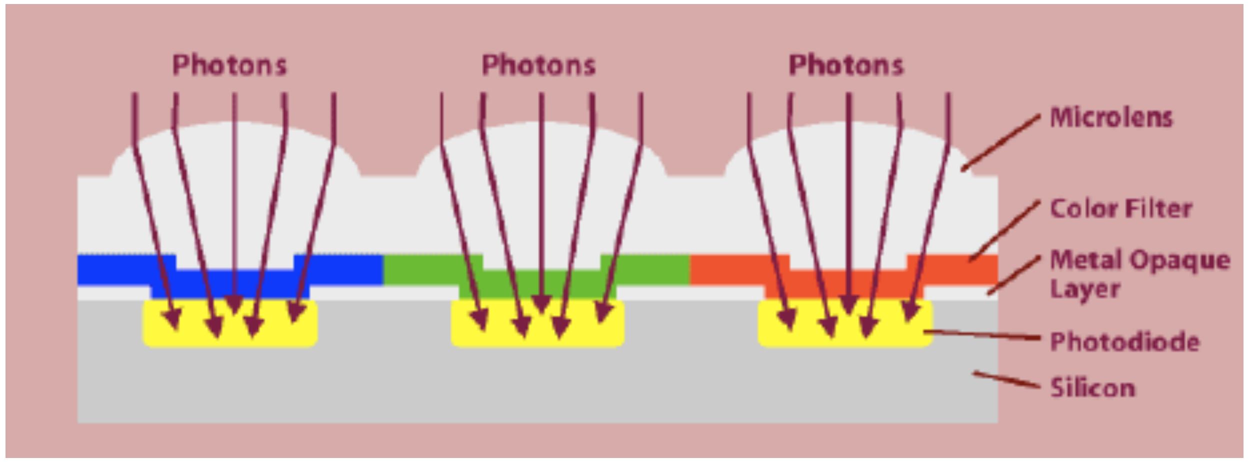
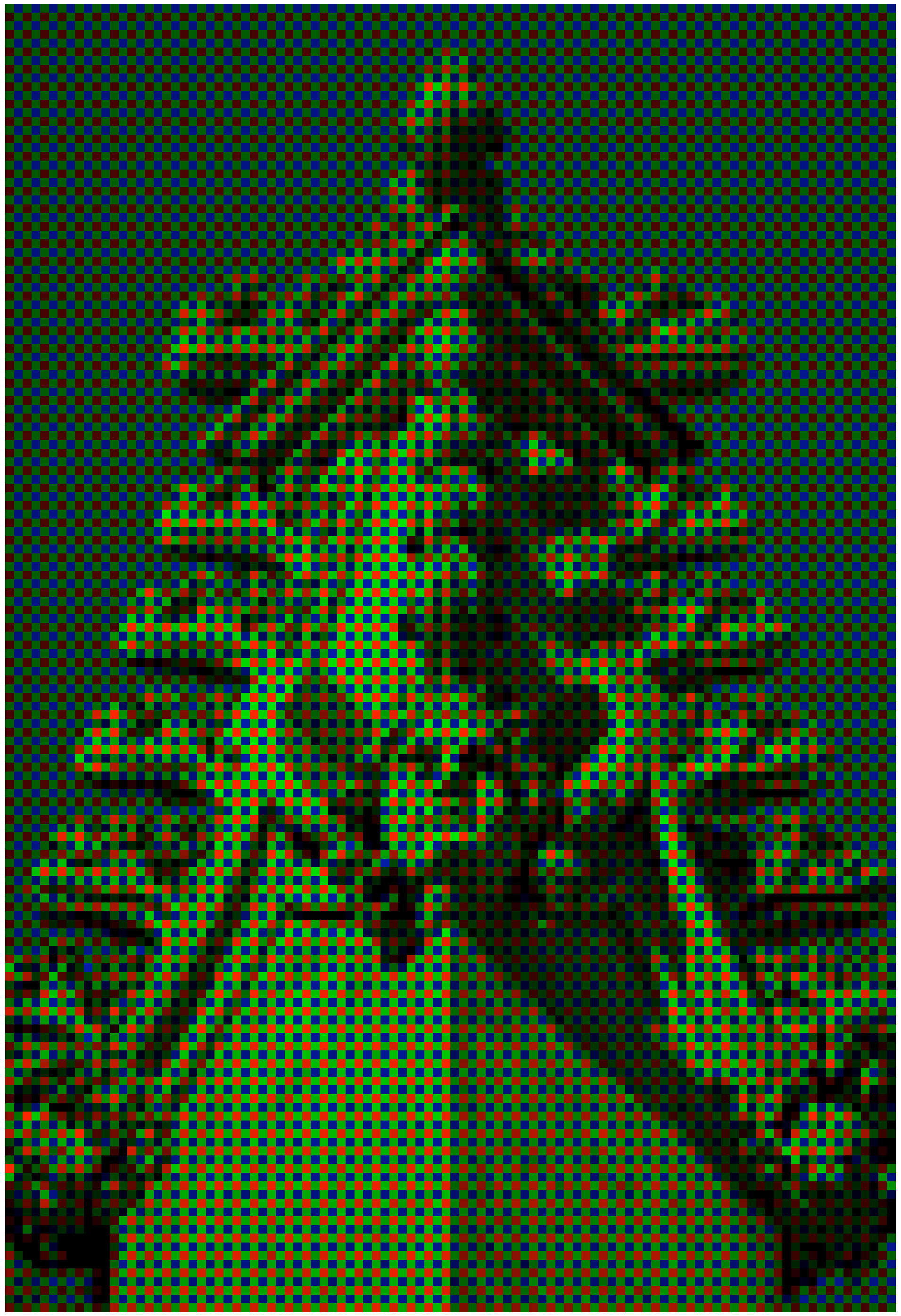
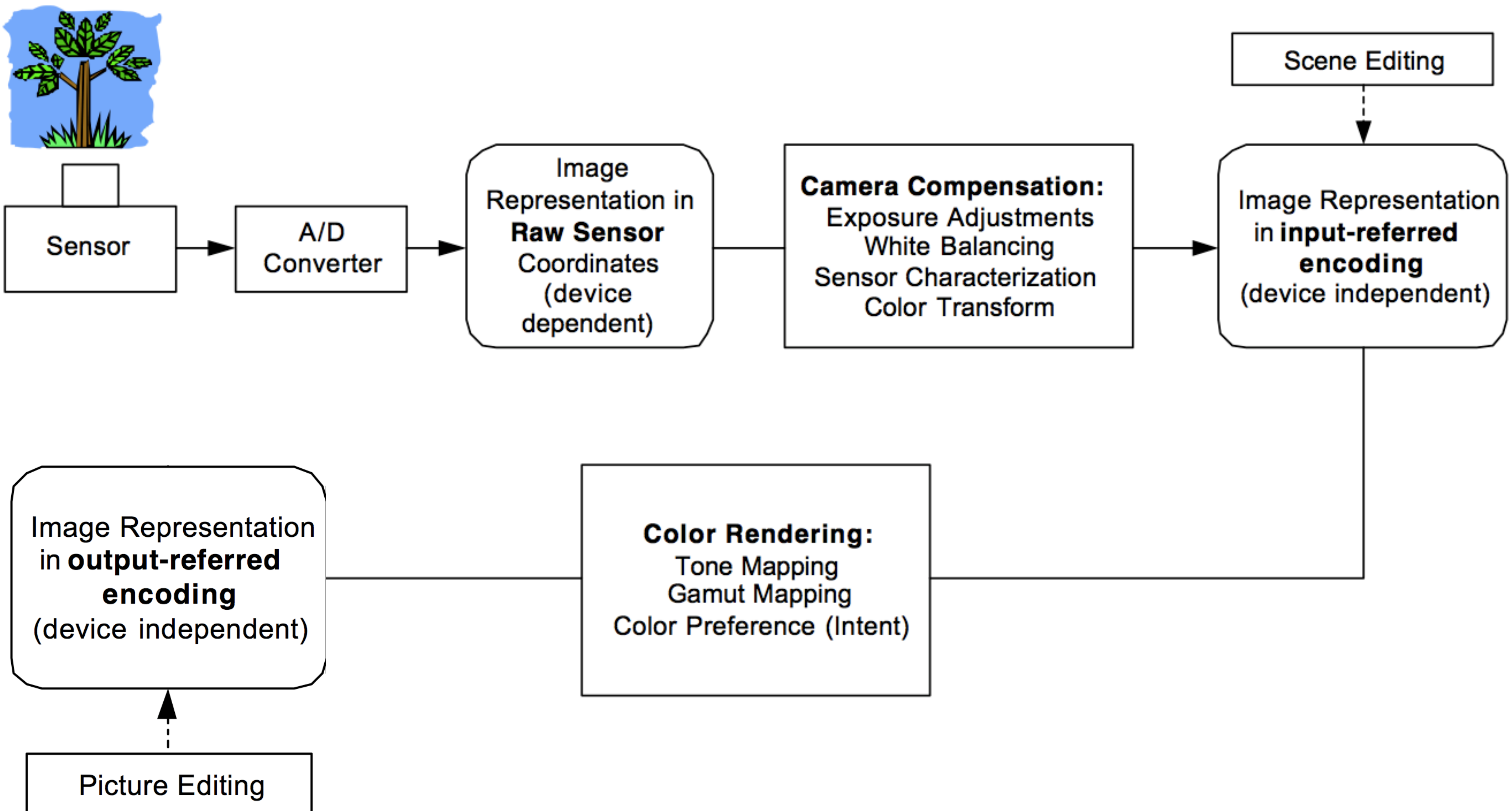




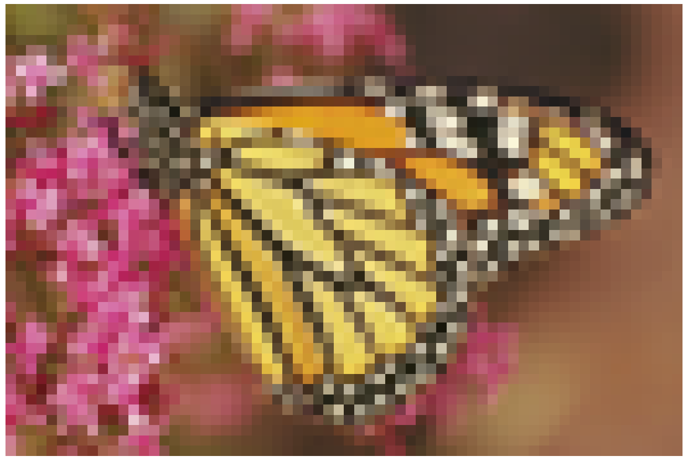
what your camera sees

The workflow of digital photography



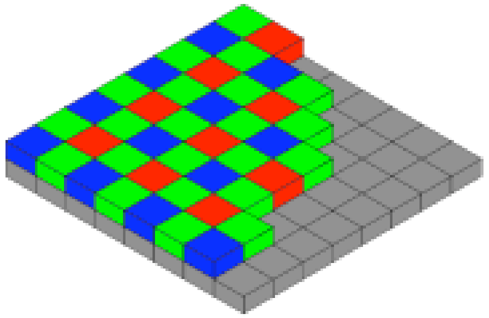
[Introduction to Color Processing in Digital Cameras, Süsstrunk]

Simplified acquisition model

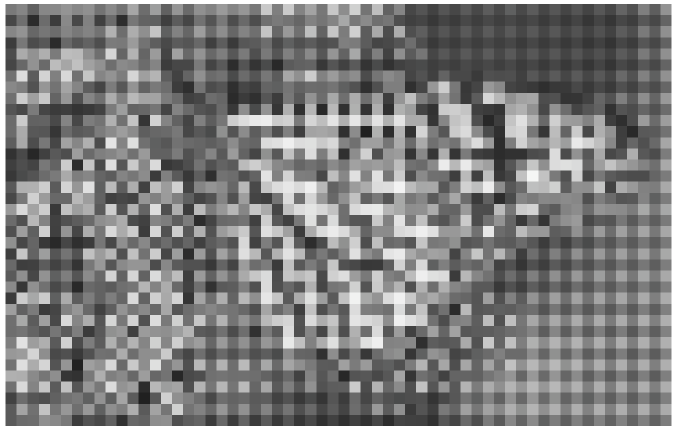


color image

$\mathbf{u} = (\mathbf{u}[\mathbf{k}])_{\mathbf{k}\in\mathbb{Z}^2}$



mosaicking + noise



noisaicked image

$v = (v[\mathbf{k}])_{\mathbf{k}\in\mathbb{Z}^2}$

with $\mathbf{u}[\mathbf{k}] = \begin{bmatrix} u^R[\mathbf{k}] \\ u^G[\mathbf{k}] \\ u^B[\mathbf{k}] \end{bmatrix}$

$$v[\mathbf{k}] = u^{X[\mathbf{k}]}[\mathbf{k}] + \varepsilon[\mathbf{k}], \quad \forall \mathbf{k} \in \mathbb{Z}^2, \text{ where } X[\mathbf{k}] \in \{R, G, B\}$$

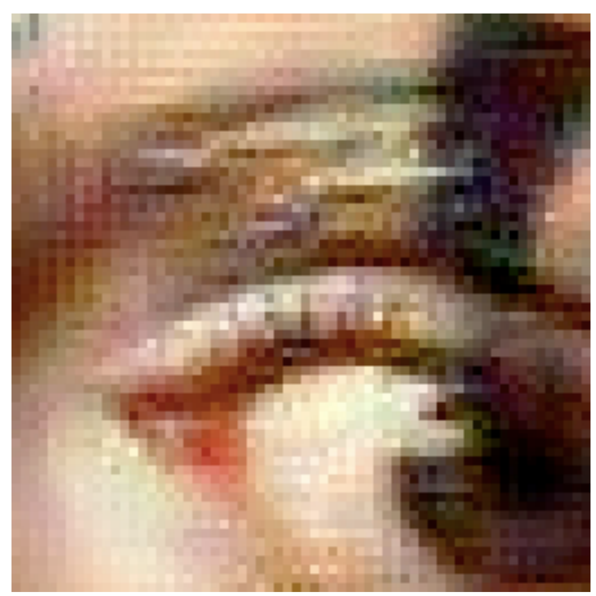
$$\varepsilon[\mathbf{k}] \sim \sigma\,\mathcal{N}(0,1)$$

Naive approaches

Original image

Demosaicked image

Demosaicking
+ denoising



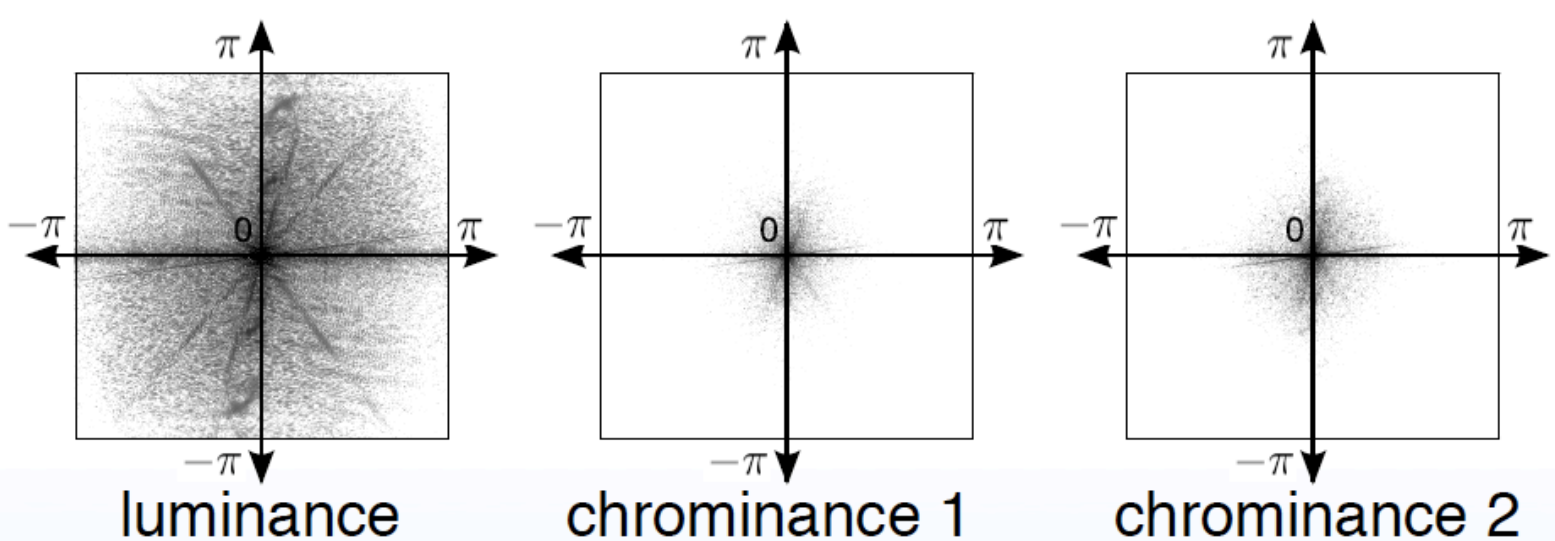
Denoising
+ demosaicking

Joint demosaicking/
denoising [Hirakawa, 2006]

Ad hoc approaches

- Hirakawa *et al*. "Joint demosaicing and denoising", *IEEE TIP*, 2006

Luminance / chrominance basis



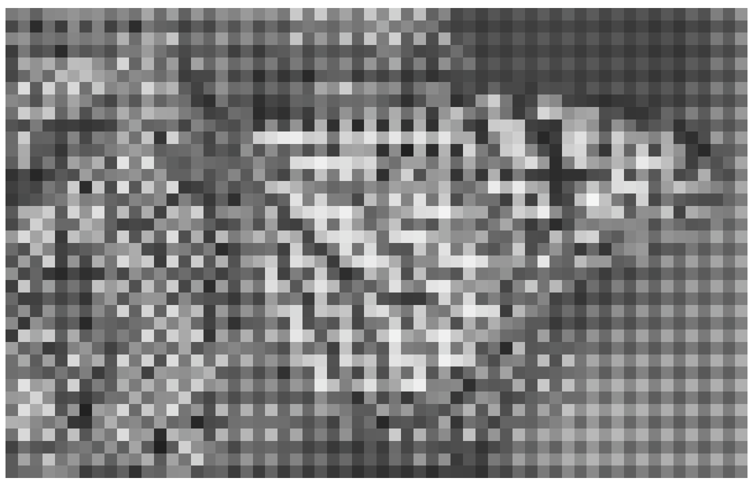
$$\mathbf{L} = \frac{1}{\sqrt{3}}[1,1,1]^{\mathrm{T}}$$

$$\mathbf{C}^{G/M} = \frac{1}{\sqrt{6}}[-1,2,-1]^{\mathrm{T}}$$

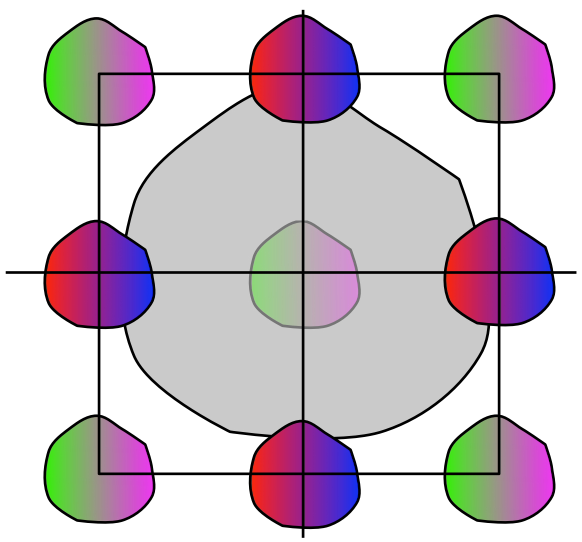
$$\mathbf{C}^{R/B} = \frac{1}{\sqrt{3}}[1,0,-1]^{\mathrm{T}}$$

Frequency interpretation of Bayer sampling

[Alleysson *et al.*, *IEEE TIP*, 2005]



Fourier transform →



$$\hat{v}(\boldsymbol{\omega}) = \frac{1}{\sqrt{3}}\hat{u}^{L}(\boldsymbol{\omega}) + \frac{1}{\sqrt{24}}\hat{u}^{G/M}(\boldsymbol{\omega}) + \frac{\sqrt{6}}{4}\hat{u}^{G/M}(\boldsymbol{\omega} - [\pi,\pi]^{\mathrm{T}}) + \frac{\sqrt{2}}{4}\hat{u}^{R/B}(\boldsymbol{\omega} - [0,\pi]^{\mathrm{T}}) - \frac{\sqrt{2}}{4}\hat{u}^{R/B}(\boldsymbol{\omega} - [\pi,0]^{\mathrm{T}}) + \hat{\varepsilon}(\boldsymbol{\omega})$$

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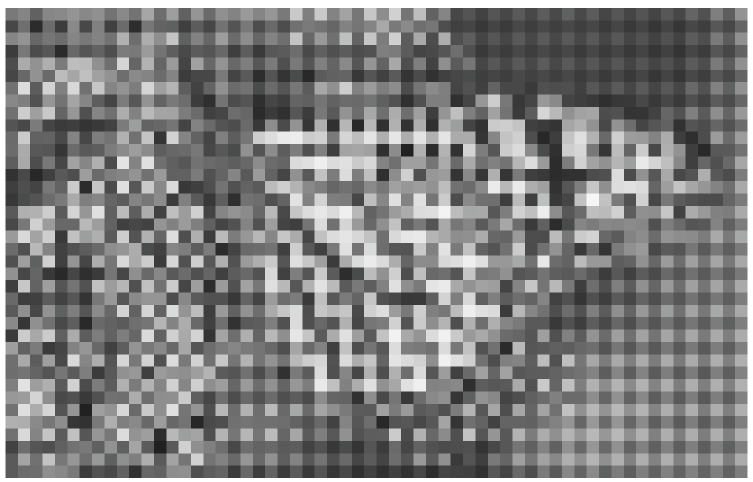
[Alleysson *et al.*, *IEEE TIP*, 2005]



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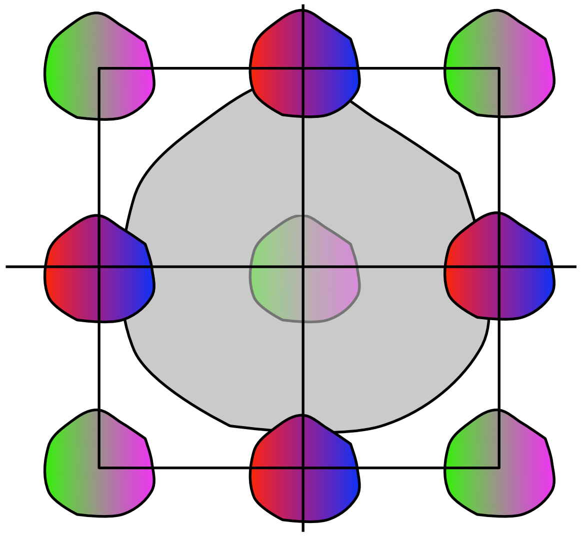


$$v[\mathbf{k}] = \frac{1}{\sqrt{3}} u^L[\mathbf{k}] + \frac{1}{\sqrt{24}} u^{G/M}[\mathbf{k}] + \frac{\sqrt{6}}{4}(-1)^{k_1+k_2} u^{G/M}[\mathbf{k}] + \frac{\sqrt{2}}{4}(-1)^{k_2} u^{R/B}[\mathbf{k}] - \frac{\sqrt{2}}{4}(-1)^{k_1} u^{R/B}[\mathbf{k}] + \varepsilon[\mathbf{k}]$$

Linear demosaicking by frequency selection

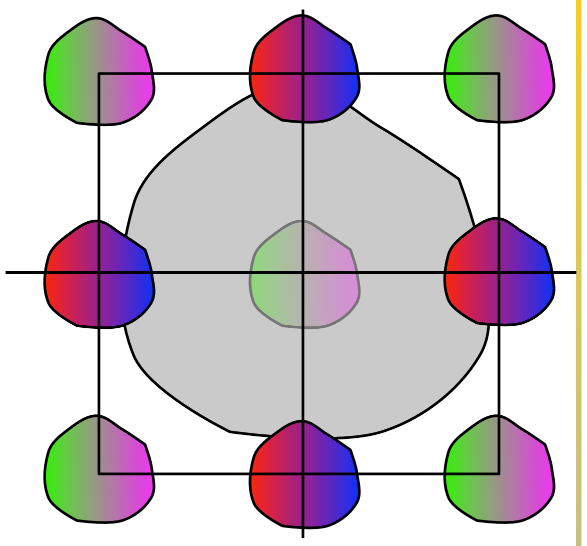
- Chrominance obtained by modulation + lowpass filtering

$$d^{G/M} = \frac{4}{\sqrt{6}} v_{\pi,\pi} * h_{G/M} \text{ where } v_{\pi,\pi}[\mathbf{k}] = (-1)^{k_1+k_2} v[\mathbf{k}]$$

$$d_H^{R/B} = -2\sqrt{2}\, v_{\pi,0} * h_{R/B} \text{ where } v_{\pi,0}[\mathbf{k}] = (-1)^{k_1} v[\mathbf{k}]$$

$$d_V^{R/B} = 2\sqrt{2}\, v_{0,\pi} * (h_{R/B})^{\mathrm{T}} \text{ where } v_{0,\pi}[\mathbf{k}] = (-1)^{k_2} v[\mathbf{k}]$$

$$d^{R/B} = \tfrac{1}{2}(d_H^{R/B} + d_V^{R/B})$$



- Luminance as the residual

$$\frac{1}{\sqrt{3}} d^L = v[\mathbf{k}] - \left(\frac{1}{\sqrt{24}} + \frac{\sqrt{6}}{4}(-1)^{k_1+k_2}\right) d^{G/M}[\mathbf{k}] - \frac{\sqrt{2}}{4}\left((-1)^{k_2} - (-1)^{k_1}\right) d^{R/B}[\mathbf{k}]$$

[Dubois, *IEEE SPL*, 2005]

Linear demosaicking: behavior under noise

$$v[\mathbf{k}] = v_0[\mathbf{k}] + \varepsilon[\mathbf{k}] \qquad \varepsilon[\mathbf{k}] \sim \mathcal{N}(0, \sigma^2)$$

- Let d_0 be the demosaicked image in absence of noise

$$\mathbf{d}[\mathbf{k}] = \mathbf{d}_0[\mathbf{k}] + \mathbf{e}[\mathbf{k}]$$

- The demosaicked color noise $\mathbf{e}$ is such that:
  - $e^{G/M}$, $e^{R/B}$, e^L are independent Gaussian noise realizations
  - $e^{G/M}$ is stationary with spectral density $\frac{8}{3}\sigma^2|\hat{h}_{G/M}(\boldsymbol{\omega})|^2$
  - $e^{R/B}$ is stationary with spect. dens. $2\sigma^2\left(|\hat{h}_{R/B}(\omega_1,\omega_2)|^2 + |\hat{h}_{R/B}(\omega_2,\omega_2)|^2\right)$
  - e^L is not stationary and not white

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  - e^{L} is not stationary and not white

→ The basis $\mathbf{L}, \mathbf{C}^{G/M}, \mathbf{C}^{R/B}$ is appropriate to address the problem

Strategy by frequency selection + denoising

[Condat, *IEEE ICIP*, 2010]

1) Estimate the denoised chrominance $d^{G/M} \approx u^{G/M}$ and $d^{R/B} \approx u^{R/B}$ by modulation and lowpass filtering

2) Subtract it to v

$$\frac{1}{\sqrt{3}} d^L[\mathbf{k}] \left(\approx \frac{1}{\sqrt{3}} u^L[\mathbf{k}] + \varepsilon[\mathbf{k}]\right) :=$$
$$v[\mathbf{k}] - \left(\frac{1}{\sqrt{24}} + \frac{\sqrt{6}}{4}(-1)^{k_1+k_2}\right) d^{G/M}[\mathbf{k}] - \frac{\sqrt{2}}{4}\left((-1)^{k_2} - (-1)^{k_1}\right) d^{R/B}[\mathbf{k}]$$

3) denoise d^L

MMSE chrominance filters

→ The chrominance should be denoised before estimating the luminance

- Wiener-like FIR chrominance filters of size $N \times N$ optimal for a learning image base: linear systems of size $N^2 \times N^2$ to solve:

$$\mathbf{A}_{G/M}\mathbf{h}_{G/M} = \mathbf{b}_{G/M}$$
$$\mathbf{A}_{R/B}\mathbf{h}_{R/B} = \mathbf{b}_{R/B}$$

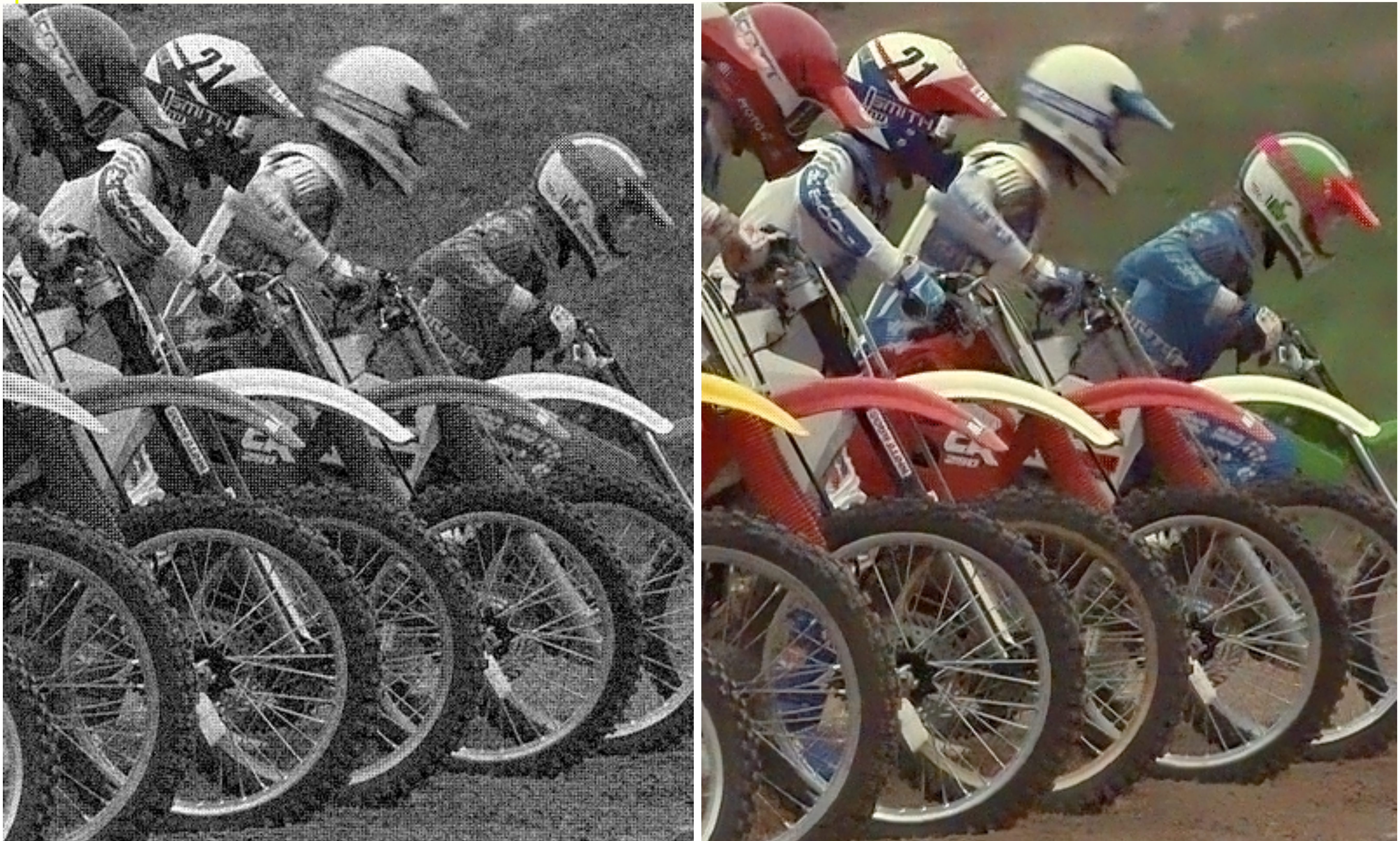
[Dubois, *IEEE ICIP*, 2006]

- In presence of noise:

$$(\mathbf{A}_{G/M} + \tfrac{8}{3}\sigma^2\mathbf{I})\mathbf{h}_{G/M} = \mathbf{b}_{G/M}$$
$$(\mathbf{A}_{R/B} + 4\sigma^2\mathbf{I})\mathbf{h}_{R/B} = \mathbf{b}_{R/B}$$

[Condat, *IEEE ICIP*, 2010]

Results $\sigma = 20$



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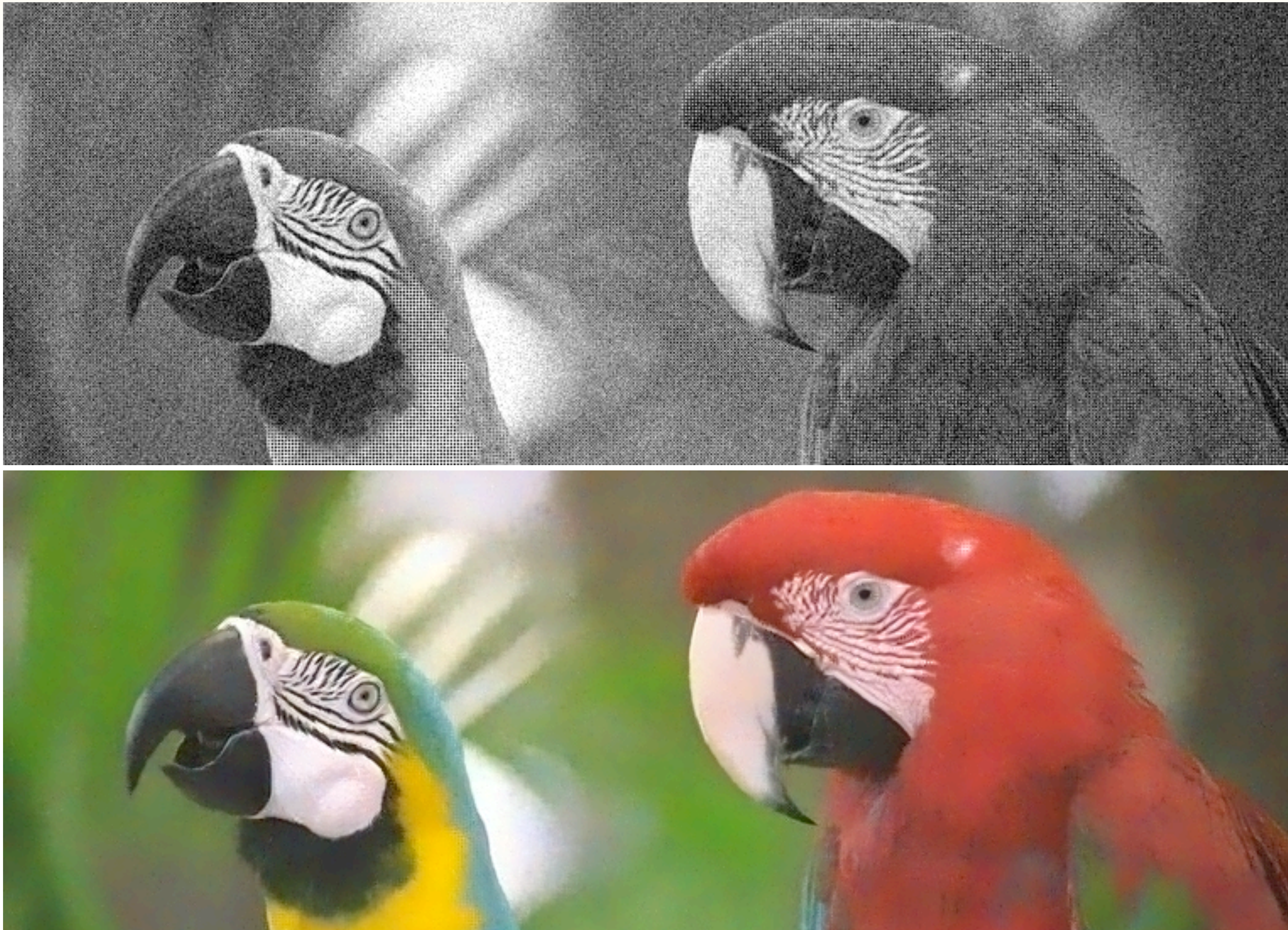

Results

Original Image

$\sigma = 20$

Results

Hirakawa *et al., IEEE TIP,* 2006

$\sigma = 20$

Results

Zhang *et al., IEEE TIP,* 2007

Results

$\sigma = 20$

Zhang *et al.*, *IEEE TIP*, 2009

$\sigma = 20$

Results

Paliy *et al.*, *Int. J. Im. Sys. and Tech.*, 2007

$\sigma = 20$

Results

Proposed

A variational interpretation

[Condat, *GRETSI*, 2009]
[Condat, *ICIP*, 2009]

- We can show that demosaicking by frequency selection (with particular filters) solves the following variational problem :

$$\mathbf{d} = \text{argmin}_{\mathbf{a}} \; \underline{\mu}\|\nabla a^L\|_{\ell_2}^2 + \|\nabla a^{G/M}\|_{\ell_2}^2 + \|\nabla a^{R/B}\|_{\ell_2}^2 \quad s.t. \;\; a^{X[\mathbf{k}]} = v[\mathbf{k}], \; \forall \mathbf{k}$$

- Key point: the chrominance energy is more penalized: $\mu \approx 0.05$ (even lower in the noisy case)
- Remark 1: the solution does not depend on the choice of the chrominance basis.
- Remark 2: this generic approach can be used with every CFA

Improvement: minimize the TV

- Variational formulation with a new color TV:

$$\mathbf{d} = \operatorname{argmin}_{\mathbf{a}} \; \|\mathbf{a}\|_{\mathrm{TV}} := \mu \left\| \sqrt{(\nabla_x a^L)^2 + (\nabla_y a^L)^2} \right\|_{\ell_1} +$$

$$\left\| \sqrt{(\nabla_x a^{G/M})^2 + (\nabla_x a^{R/B})^2 + (\nabla_y a^{G/M})^2 + (\nabla_y a^{R/B})^2} \right\|_{\ell_1}$$

$$s.t. \;\; a^{X[\mathbf{k}]} = v[\mathbf{k}], \; \forall \mathbf{k}$$

+ denoise d^L in the noisy case

TV minimization: classical strategies

- Problem:

$$\mathbf{d} = \mathrm{argmin}_{\mathbf{a}} \|\mathbf{a}\|_{\mathrm{TV}} + \imath_{\mathcal{M}\mathbf{a}=v}$$

- Classical splitting approaches
  - Douglas-Rachford → TV-denoising inner problem
  - $\mathbf{d} = \mathrm{argmin}_{\mathbf{a},\mathbf{b}} \big(\|\mathbf{b}\|_{\ell_1} + \imath_{\mathcal{M}\mathbf{a}=v}\big) + \imath_{\nabla \mathbf{a}=\mathbf{b}}$

    + Douglas-Rachford → Poisson equation to solve
  - $\mathbf{d} = \mathrm{argmin}_{\mathbf{a},\mathbf{b}} \big(\|\mathbf{b}\|_{\ell_1} + \imath_{\mathcal{M}\mathbf{a}=v}\big) + \imath_{\nabla \mathbf{a}=\mathbf{b}}$

    + Predictor-corrector proximal multiplier (PCPM) method of Chen and Teboulle (1994) → very slow

Primal-dual minimization strategies

- More recently: several papers on primal-dual approaches, combine the advantages of PCPM and augmented Lagrangian methods:
  - Zhu and Chan, 2008, “An efficient primal-dual hybrid gradient algorithm for total variation image restoration”
  - Esser, Zhang and Chan, 2009, “A general framework for a class of first order primal-dual algorithms for TV minimization”
  - Zhang, Burger, Osher, 2009, “A unified primal-dual algorithm framework based on Bregman iteration”
  - Chambolle and Pock, 2010, “A first-order primal-dual algorithm for convex problems with applications to imaging”

Primal-dual algorithm

- Initialization: with the frequency selection method.
- Choose $\alpha > 0$, $\beta = 1/(8.01\alpha)$, $\mathbf{b} = (0)$
- Iterate
  - $\forall X \in \{R, G, B\},\ \mathbf{b}^X_{(n+1)} = \mathbf{b}^X_{(n)} + \alpha \nabla \bar{d}^X_{(n)}$
  - $\forall \mathbf{k} \in \mathbb{Z}^2,\ \mathbf{b}^L_{(n+1)}[\mathbf{k}] = \dfrac{\mathbf{b}^L_{(n+1)}[\mathbf{k}]}{\max(1, |\mathbf{b}^L_{(n+1)}[\mathbf{k}]|/\mu)}$
  - $\forall \mathbf{k} \in \mathbb{Z}^2,\ \forall X \in \{G/M, R/B\},\ \mathbf{b}^X_{(n+1)}[\mathbf{k}] = \dfrac{\mathbf{b}^X_{(n+1)}[\mathbf{k}]}{\max(1, \sqrt{|\mathbf{b}^{G/M}_{(n+1)}[\mathbf{k}]|^2 + |\mathbf{b}^{R/B}_{(n+1)}[\mathbf{k}]|^2})}$
  - $\forall X \in \{R, G, B\},\ d^X_{(n+1)} = d^X_{(n)} + \beta \operatorname{div} \mathbf{b}^X_{(n+1)}$
  - $\forall \mathbf{k} \in \mathbb{Z}^2,\ d^{X[\mathbf{k}]}_{(n+1)}[\mathbf{k}] = v[\mathbf{k}]$
  - $\bar{\mathbf{d}}_{(n+1)} = 2\mathbf{d}_{(n+1)} - \mathbf{d}_{(n)}$

$\sigma = 20$

Result

Method by frequency selection

$\sigma = 20$

Result

Method by TV minimization

Take away messages

- Two steps process:
  - demosaick so that the chrominance is well recovered
    $\rightarrow$ the noise goes automatically in the luminance
  - denoise the luminance
- The proposed linear demosaicking approach is simple, fast and efficient
  - complexity: two convolutions + grayscale denoising only
- TV minimization approach: even better.
  - Efficient primal-dual strategy: optimal $O(1/n)$ convergence, 50 iterations enough