Congrès SMAI 2011, 5e Biennale Française des Mathématiques Appliquées

Guidel, Bretagne, 23-27 mai 2011

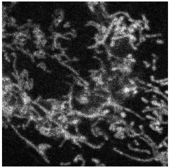
Débruitage non-local
Adaptation au type de bruit et aux structures de l'image

Charles Deledalle

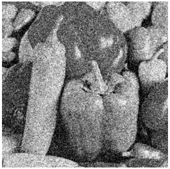
Collaborateurs: Loïc Denis, Vincent Duval, Jospeh Salmon, Florence Tupin

Institut Telecom, Telecom ParisTech, CNRS LTCI, Paris, France

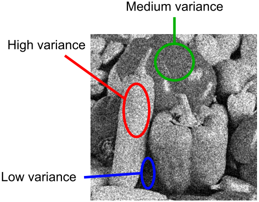
- Noise: fluctuations which corrupt a signal or an image,
- Examples of noise in imagery:
  - Gaussian noise:
    ex: optical imagery.
  - Poisson noise: due to low flux,
    ex: optical imagery, microscopy, astronomy.
  - Speckle noise: due to coherent summation of random phasors
    ex: SAR imagery, SONAR imagery, ultrasound imagery.



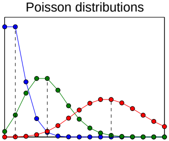
- Signal dependent noise
  - Noise variance is a function of the true image,



Gaussian noise



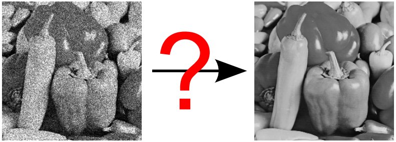
Poisson noise



  - Generally modeled by non-Gaussian distributions.

Overview of denoising approaches

Image denoising: find an estimation of the true image from the noisy image.



Noisy image

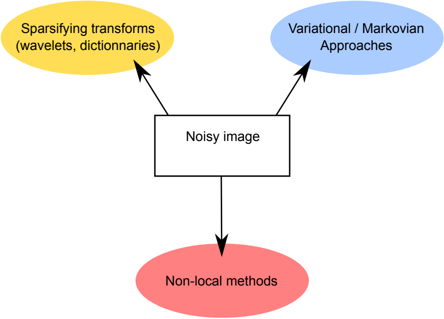
True image

How to denoise an image?

- Three main approaches,
- Lots of hybrid methods.

Problems of non-local approaches

- Designed for Gaussian noise,
- Adaptation to the local structures.



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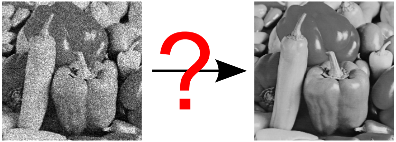

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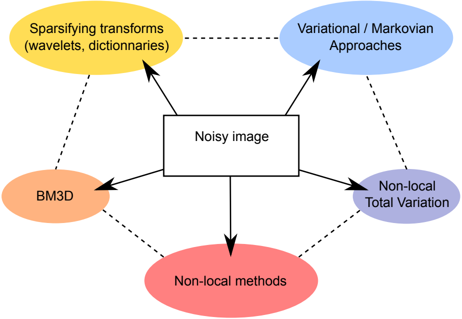

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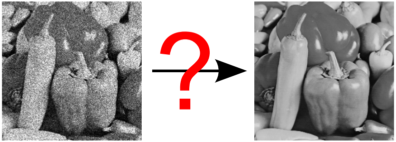

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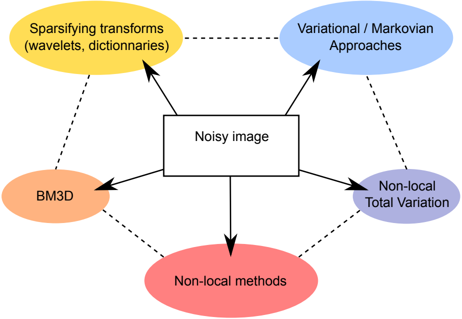

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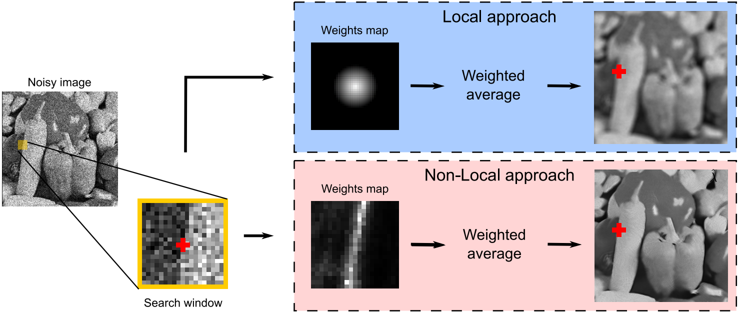
Non-local filtering

Non-local approach [Buades et al., 2005]

- Local filters: loss of resolution,
- Non-local filers: data-driven adaptive weights,
- Weights are based on patch similarity.



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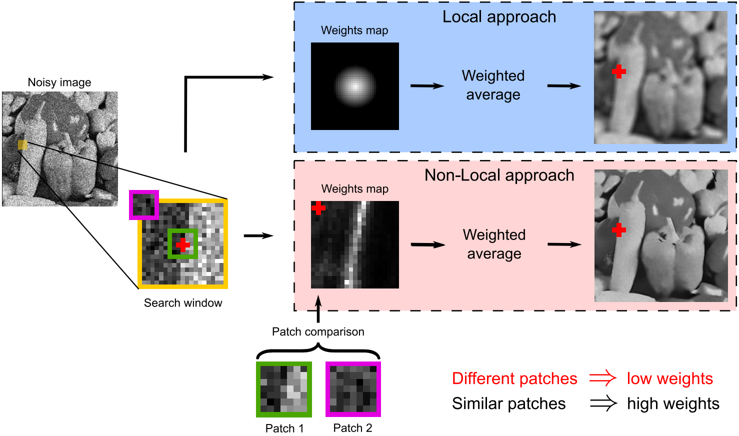

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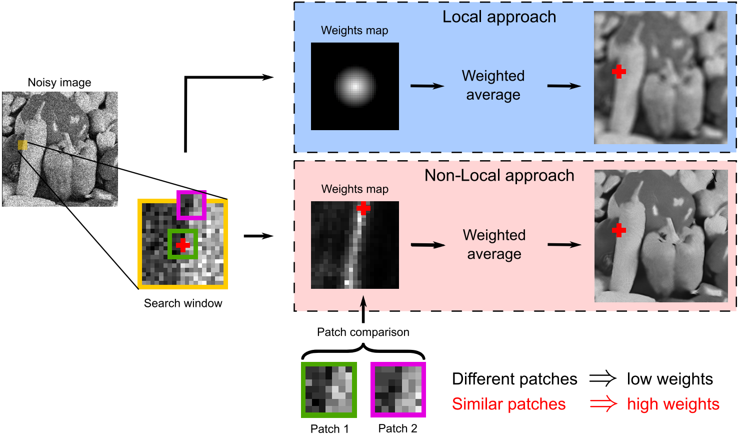
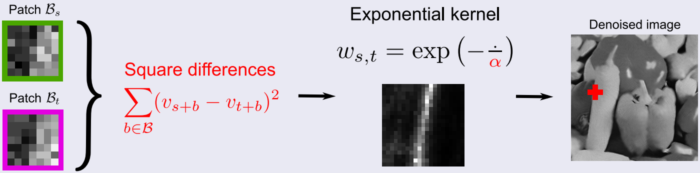


Non-local means [Buades et al., 2005]

- Define weights from the squared differencess between patches 1 and 2:



with ${}_{s+b}$ and ${}_{t+b}$ the b-th respective pixels in $\mathcal{B}_s$ and $\mathcal{B}_t$.

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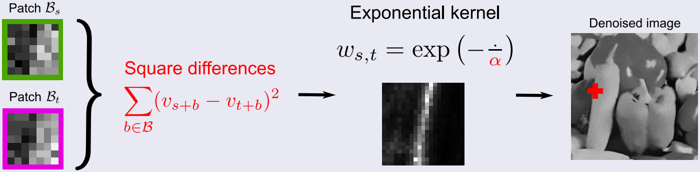


with ${}_{s+b}$ and ${}_{t+b}$ the b-th respective pixels in $\mathcal{B}_s$ and $\mathcal{B}_t$.

Beyond Gaussian noise?

- squared differences: adapted for Gaussian noise,
- Which criterion for non-Gaussian noise?
- How to choose the "optimal" parameters?

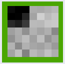
Loïc Denis

Florence Tupin

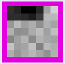
The rare patch effect

- Around edges with high contrast, almost all weights can be zeros:

Patch $\mathcal{B}_s$

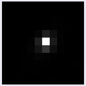


Patch $\mathcal{B}_t$



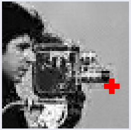
Square differences

$$\sum_{b \in \mathcal{B}} (v_{s+b} - v_{t+b})^2 \longrightarrow$$



$\longrightarrow$

Denoised image



with ${}_{s+b}$ and ${}_{t+b}$ the b-th respective pixels in $\mathcal{B}_s$ and $\mathcal{B}_t$.

- The rare patch effect leads to *a noise halo*.

Beyond the rare patch effect?

- Square patches non-adapted to heterogeneous area.
- How to use efficiently non square patches?
- How to choose the best patch shape?

Vincent Duval

Joseph Salmon

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Patch-similarities: how to replace the squared differences? [Deledalle et al., 2009]

- Weights have to select pixels with close true values,
- Compare patches $\Leftrightarrow$ test the hypotheses that patches have:

$\mathcal{H}_0$: same true values ,
$\mathcal{H}_1$: independent true values .

$$\mathrm{P}(\mathcal{H}_0 | \boxed{}_1, \boxed{}_2) = \frac{\mathrm{P}(\boxed{}_1, \boxed{}_2 | \mathcal{H}_0)}{\mathrm{P}(\boxed{}_1, \boxed{}_2 | \mathcal{H}_1)} \times \mathrm{P}(\mathcal{H}_0)$$

[Deledalle et al., 2009] Deledalle, C., Denis, L., and Tupin, F. (2009).
Iterative Weighted Maximum Likelihood Denoising with Probabilistic Patch-Based Weights.
IEEE Transactions on Image Processing, 18(12):2661–2672.

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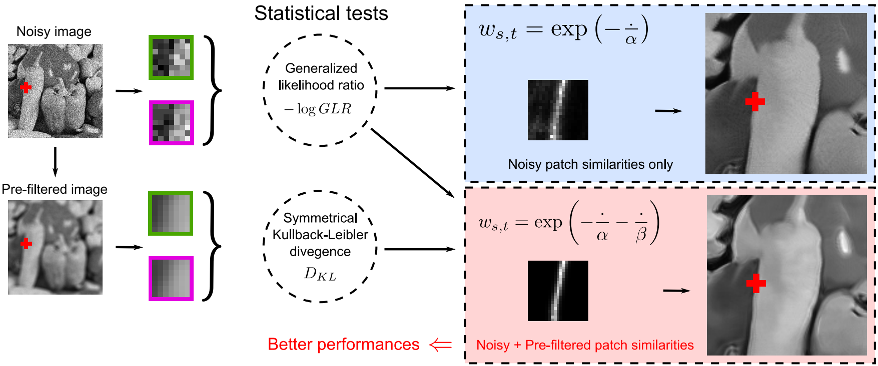

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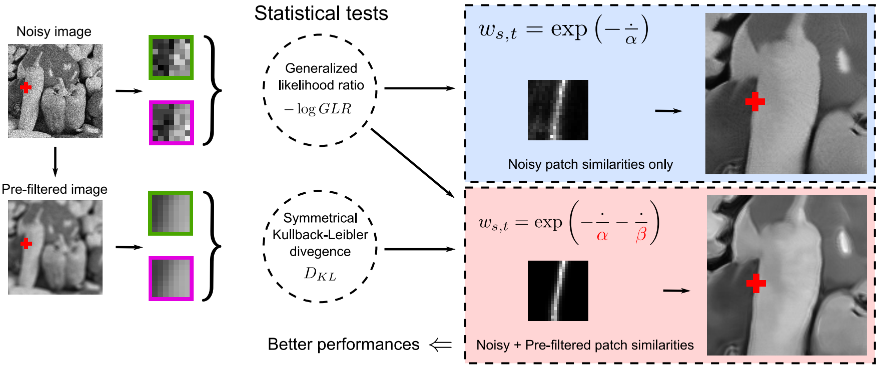
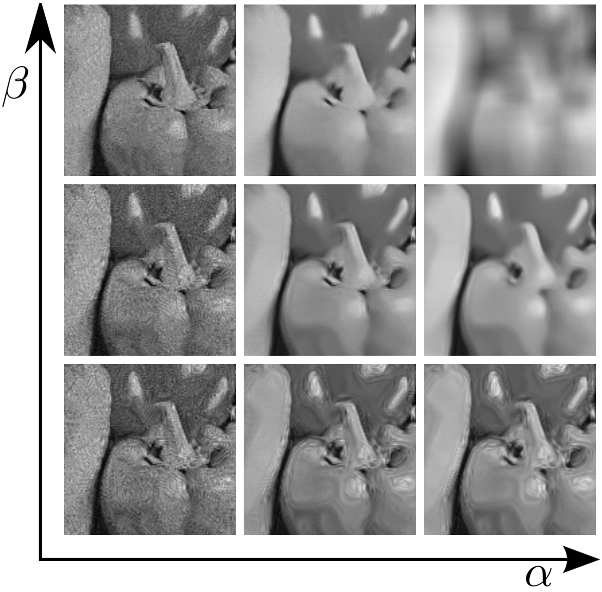
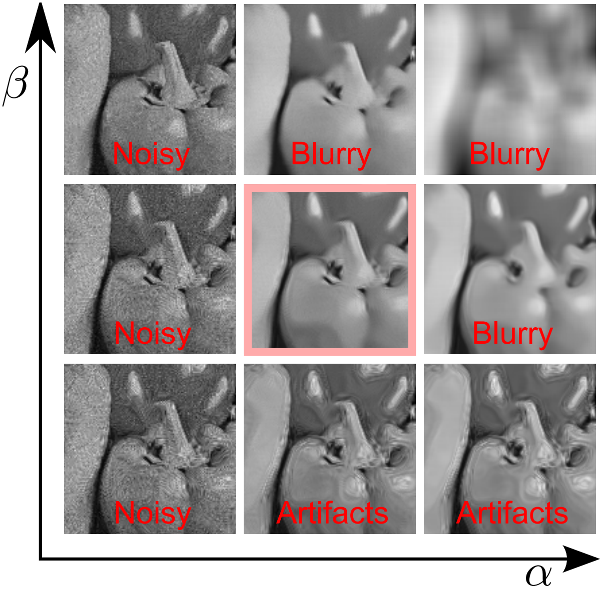


Next, how should we set the parameters α and β?

Non-local estimation – Influence of the parameters



How to choose the parameters?
(trade-off noisy/pre-filtered)



How to choose the parameters?
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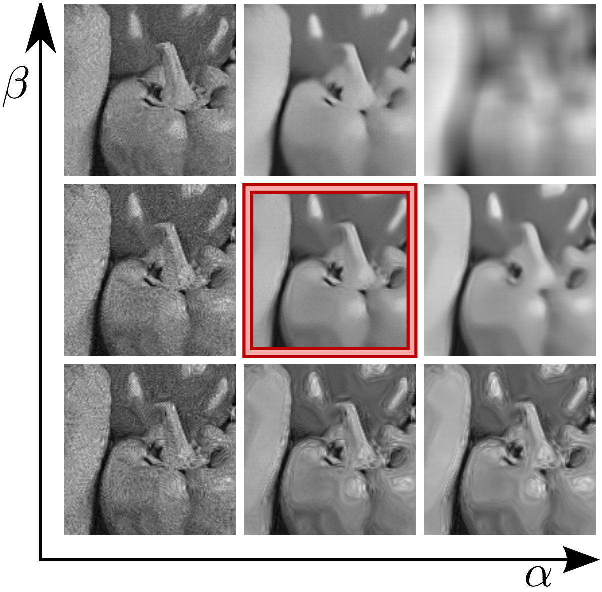
Visually?



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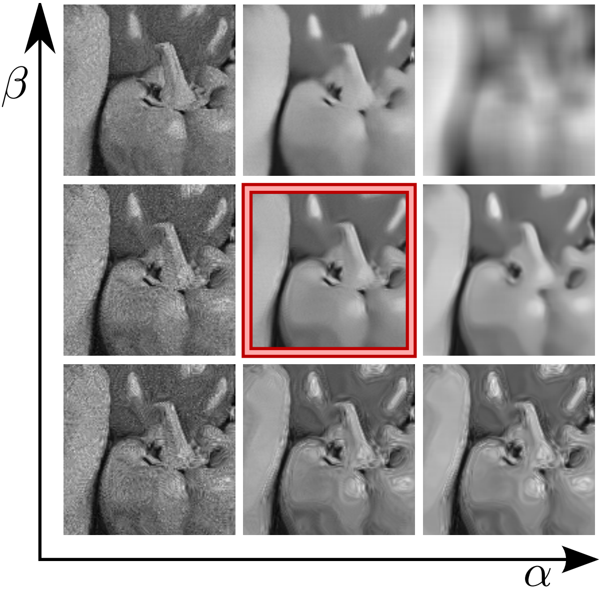
Visually?

Mean squared error (MSE)?



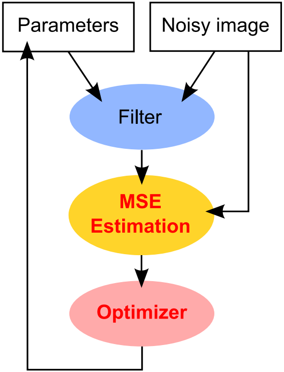
How to choose the parameters?
(trade-off noisy/pre-filtered)

Visually?

Mean squared error (MSE)?

How to estimate the MSE?

Automatic setting of the denoising parameters



Unsupervised filtering

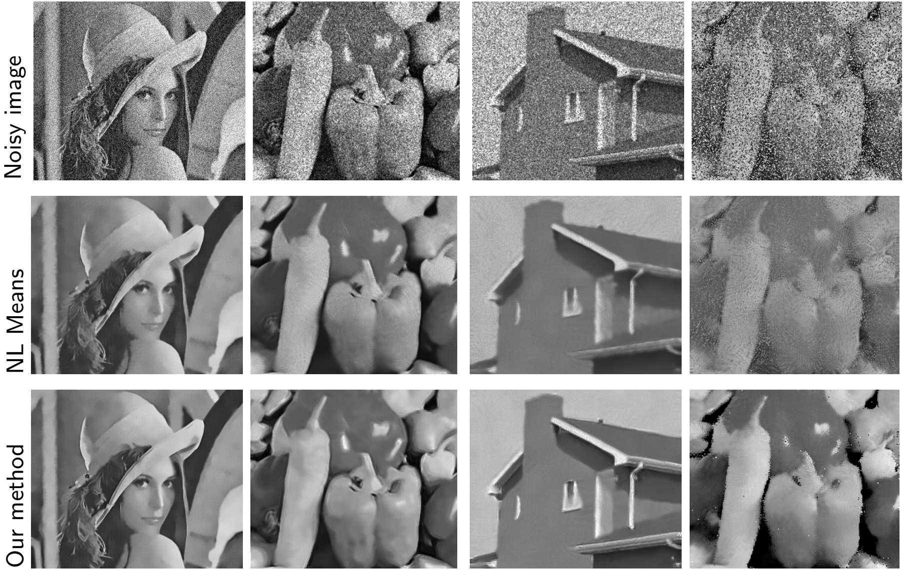
MSE estimators: unbiased risk estimators

Estimator	Gaussian	Poisson
General	SURE [Stein, 1981]	PURE [Chen, 1975]
Wavelet	SUREshrink [Donoho et al., 1995] SURE-LET [Blu et al., 2007]	PURE-LET [Luisier et al., 2010]
NL means	SURE based NL means [Van De Ville et al., 2009] Local-SURE NL means [Duval et al., 2010]	Poisson NL means [Deledalle et al., 2010a]

SURE: Stein's Unbiased Risk Estimator
PURE: Poisson Unbiased Risk Estimator

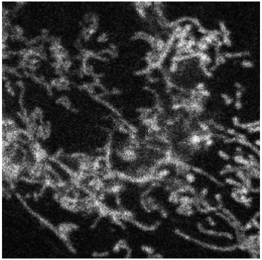
[Deledalle et al., 2010a] Deledalle, C., Tupin, F., and Denis, L. (2010a).
Poisson NL means: Unsupervised non local means for Poisson noise.
In *Image Processing (ICIP), 2010 17th IEEE International Conference on*, pages 801–804. IEEE.
Best student paper award

Results on simulations

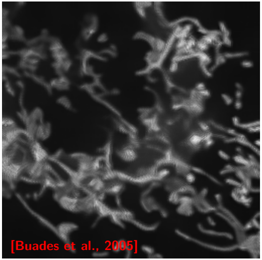


(a) Gaussien +0.87 dB (b) Poisson +1.13 dB (c) *Speckle* +4.00 dB (d) Impuls. +3.82 dB

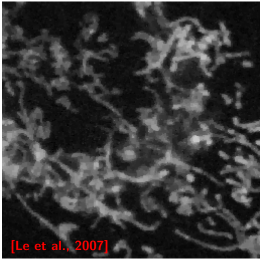
Comparisons with Poisson noise on true data [Deledalle et al., 2010a]



(a) Noisy image

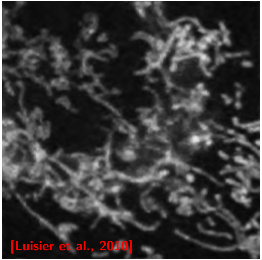


(b) NL means

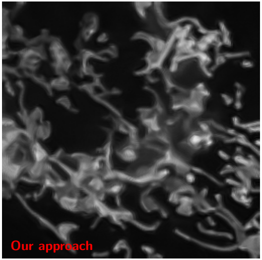


(c) Poisson-TV

Cardiac mitochondrion,
Confocal fluorescence microscopy,
Image courtesy of Y. Tourneur.



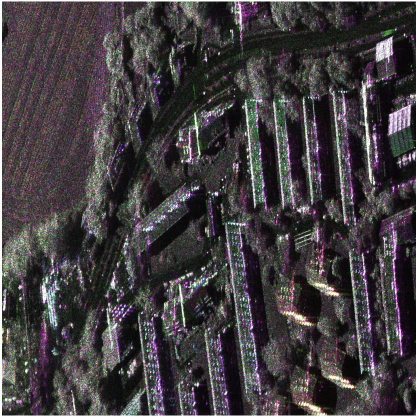
(e) PURE-LET



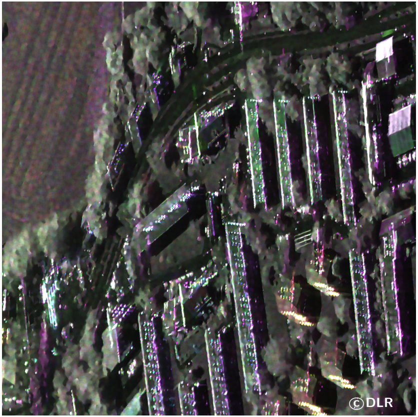
(f) Poisson NL means

Results on polarimetric SAR data [Deledalle et al., 2010b]

(Complex Wishart distributions)



(a) High-resolution noisy image



(b) Our estimation (4096 × 4096: 2 min 10)

[Deledalle et al., 2010b] Deledalle, C., Tupin, F., and Denis, L. (2010b).
Polarimetric SAR estimation based on non-local means.
In the proceedings of IGARSS, Honolulu, Hawaii, USA, July 2010.

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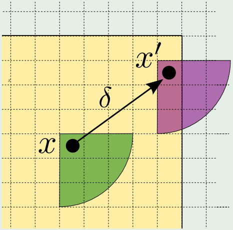
Adaptation to local image structures

- The rare patch effect depends on the patch size/shape:
  - Choose different size and shapes,
  - Calculate efficiently each associated estimate,
  - Combine properly the different estimates.

Squared differences for general patch shapes

If candidates $x' = x + \delta$, then the squared differences between the two general patches at x and x' is given by:

$$\underbrace{\sum_{\tau \in \Omega} S(\tau)(v(x+\tau) - v(x+\delta+\tau))^2}_{\text{Discrete convolution product}}$$



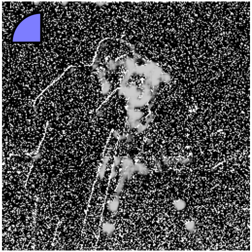
- For all displacements δ
  - Calculate the squared differences: $\forall x,\ \Delta_\delta(x) = (v(x) - v(x+\delta))^2$
  - Convolve Δ_δ by the shape $S(-\tau)$ using Fast Fourier Transform (FFT)
  - Deduce the weights associated to all candidates $x' = x + \delta$.

Complexity: search window size $\times$ image size $\times$ log(image size)

[Deledalle et al., 2011b] Deledalle, C., Duval, V., and Salmon, J. (2011b).
Non-local methods with shape-adaptive patches (NLM-SAP).
Journal of Mathematical Imaging and Vision.

Proper combination

- Estimate the local MSE associated to all estimators
  $\Rightarrow$ SURE has too large variance



Noisy SURE

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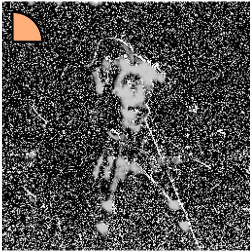

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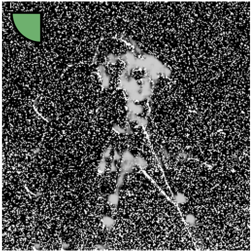

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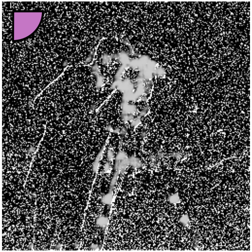

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- Estimate the local MSE associated to all estimators
  $\Rightarrow$ SURE has too large variance
- Regularize the estimation (e.g. with Yaroslavsky filter)

Regularized SURE

[Deledalle et al., 2011b] Deledalle, C., Duval, V., and Salmon, J. (2011b).
Non-local methods with shape-adaptive patches (NLM-SAP).
Journal of Mathematical Imaging and Vision.

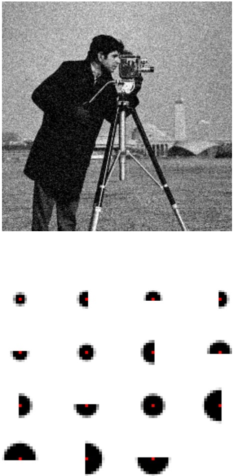
- Estimate the local MSE associated to all estimators
  $\Rightarrow$ SURE has too large variance
- Regularize the estimation (e.g. with Yaroslavsky filter)
- Combine the estimates using a convex aggregation

$$\hat{u}(x) = \frac{\displaystyle\sum_{\star =} \exp\left(-\frac{SURE_{\star}(x)}{\beta}\right) \hat{u}_{\star}(x)}{\displaystyle\sum_{\star =} \exp\left(-\frac{SURE_{\star}(x)}{\beta}\right)}$$

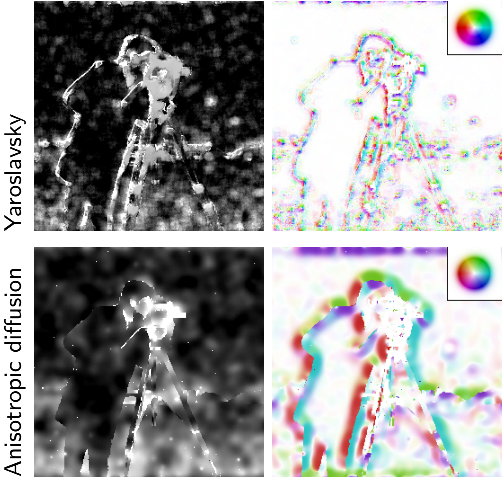
e.g. the Exponential Weighted Aggregation [Leung and Barron, 2006]

Regularized SURE

[Deledalle et al., 2011b] Deledalle, C., Duval, V., and Salmon, J. (2011b).
Non-local methods with shape-adaptive patches (NLM-SAP).
Journal of Mathematical Imaging and Vision.



(a) 15 pie sizes/shapes



(b) Regularized SURE (c) Patch orientations

Results and comparisons [Deledalle et al., 2011b]

(a) NL Means

(b) BM3D [Dabov et al., 2007]

(c) BM3D [Dabov et al., 2007]

(d) [Goossens et al., 2008]

(e) Our approach

(f) Our approach

30 seconds on 256×256 images with the online Matlab implementation

Conclusion and perspectives

Conclusions

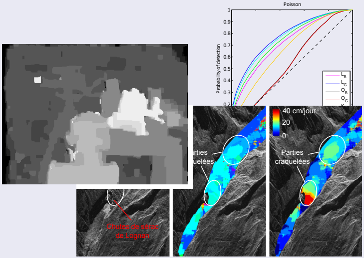
- Noise adaptation: patch similarity is linked to detection theory,
- Choose global parameters: use risk estimation (SURE, PURE...).

- Adaptation to local structures: patch shape/size varies inside images
- Choose local best shape/size: use regularized risk map (SURE, PURE...)

Perspectives

- Mix both methods,

- Extend to other problems using patches:
  - Change detection,
  - Stereo vision,
  - Object tracking.



Questions?

deledalle@telecom-paristech.fr

http://perso.telecom-paristech.fr/~deledall/
→ More details, articles and pieces of software available.

[Buades et al., 2005] Buades, A., Coll, B., and Morel, J. (2005).
A Non-Local Algorithm for Image Denoising.
Computer Vision and Pattern Recognition, 2005. CVPR 2005. IEEE Computer Society Conference on, 2.

[Chen, 1975] Chen, L. (1975).
Poisson approximation for dependent trials.
The Annals of Probability, 3(3):534–545.

[Dabov et al., 2007] Dabov, K., Foi, A., Katkovnik, V., and Egiazarian, K. (2007).
Image denoising by sparse 3-D transform-domain collaborative filtering.
IEEE Transactions on image processing, 16(8):2080.

[Deledalle et al., 2009] Deledalle, C., Denis, L., and Tupin, F. (2009).
Iterative Weighted Maximum Likelihood Denoising with Probabilistic Patch-Based Weights.
IEEE Transactions on Image Processing, 18(12):2661–2672.

[Deledalle et al., 2011a] Deledalle, C., Denis, L., and Tupin, F. (2011a).
NL-InSAR : Non-Local Interferogram Estimation.
IEEE Transactions on Geoscience and Remote Sensing, 49(4):1441–1452.

[Deledalle et al., 2011b] Deledalle, C., Duval, V., and Salmon, J. (2011b).
Non-local methods with shape-adaptive patches (nlm-sap).
Journal of Mathematical Imaging and Vision.

[Deledalle et al., 2010a] Deledalle, C., Tupin, F., and Denis, L. (2010a).
Poisson NL means: Unsupervised non local means for Poisson noise.
In *Image Processing (ICIP), 2010 17th IEEE International Conference on*, pages 801–804. IEEE.

[Deledalle et al., 2010b] Deledalle, C., Tupin, F., and Denis, L. (2010b).
Polarimetric SAR estimation based on non-local means.
In the proceedings of IGARSS, Honolulu, Hawaii, USA, July 2010 (accepted for publication).

[Goossens et al., 2008] Goossens, B., Luong, H., Pižurica, A., and Philips, W. (2008).
An improved non-local denoising algorithm.
In *Proc. Int. Workshop on Local and Non-Local Approximation in Image Processing (LNLA'2008), Lausanne, Switzerland.*

[Kay, 1998] Kay, S. (1998).
Fundamentals of Statistical signal processing, Volume 2: Detection theory.
Prentice Hall PTR.

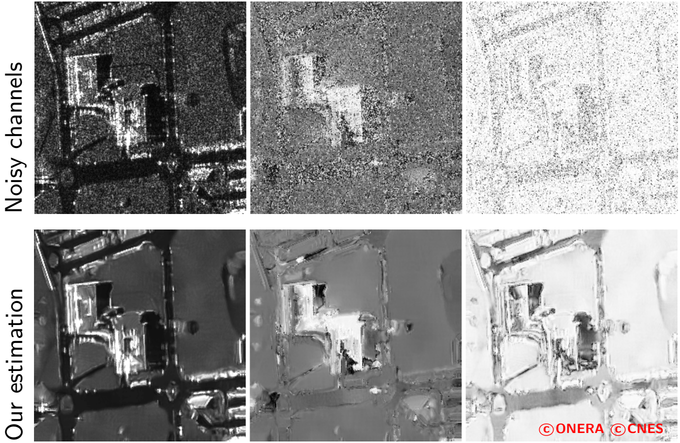
[Le et al., 2007] Le, T., Chartrand, R., and Asaki, T. (2007).
A variational approach to reconstructing images corrupted by Poisson noise.
J. of Math. Imaging and Vision, 27(3):257–263.

[Leung and Barron, 2006] Leung, G. and Barron, A. R. (2006).
Information theory and mixing least-squares regressions.
52(8):3396–3410.

[Luisier et al., 2010] Luisier, F., Vonesch, C., Blu, T., and Unser, M. (2010).
Fast interscale wavelet denoising of Poisson-corrupted images.
Signal Processing, 90(2):415–427.

[Stein, 1981] Stein, C. (1981).
Estimation of the mean of a multivariate normal distribution.
The Annals of Statistics, pages 1135–1151.

[Van De Ville and Kocher, 2009] Van De Ville, D. and Kocher, M. (2009).
SURE-Based Non-Local Means.
IEEE Signal Processing Letters, 16(11):973–976.



[Deledalle et al., 2011a] Deledalle, C., Denis, L., and Tupin, F. (2011a).
NL-InSAR : Non-Local Interferogram Estimation.
IEEE Transactions on Geoscience and Remote Sensing, 49(4):1441–1452.

Results on polarimetric SAR data [Deledalle et al., 2010b]
(Wishart distributions)


(a) Low-resolution noisy image

(b) Our estimation

[Deledalle et al., 2010b] Deledalle, C., Tupin, F., and Denis, L. (2010b).
Polarimetric SAR estimation based on non-local means.
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Patch-similarities: how to replace the squared differences? [Deledalle et al., 2009]

- Weights have to select pixels with close true values,
- Compare patches $\Leftrightarrow$ test the hypotheses that patches have:

$\mathcal{H}_0$: same true values ,
$\mathcal{H}_1$: independent true values .

$$\mathrm{P}(\mathcal{H}_0 | \blacksquare_1, \blacksquare_2) = \frac{\mathrm{P}(\blacksquare_1, \blacksquare_2 | \mathcal{H}_0)}{\mathrm{P}(\blacksquare_1, \blacksquare_2 | \mathcal{H}_1)} \times \mathrm{P}(\mathcal{H}_0)$$

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1. Similarity between noisy patches

- Based on detection theory, we propose to evaluate the generalized likelihood ratio (GLR) of both hypotheses given the noisy patches [Kay, 1998].

→ For speckle noise, we obtain the following criterion:

$$-\log GLR(v_1, v_2) = 2 \log \left(\frac{v_1}{v_2} + \frac{v_1}{v_2} \right) - 2 \log 2$$

→ For Poisson noise, we obtain the following criterion:

$$-\log GLR(k_1, k_2) = k_1 \log k_1 + k_2 \log k_2 - (k_1 + k_2) \log \left(\frac{k_1 + k_2}{2} \right) .$$

Non-local estimation under non-Gaussian noise

Patch-similarities: how to replace the squared differences? [Deledalle et al., 2009]

- Weights have to select pixels with close true values,
- Compare patches ⇔ test the hypotheses that patches have:

$\mathcal{H}_0$: same true values ,
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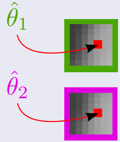
2. Similarity between pre-filtered patches

- We propose to refine weights by using the similarity between pre-filtered patches.
  Idea motivated by [Polzehl et al., 2006, Brox et al., 2007, Goossens et al., 2008, Louchet et al., 2008]
- A statistical test for the hypothesis $\mathcal{H}_0$ can be given by the symmetrical Kullback-Leibler divergence:

→ For Poisson noise, we obtain the following criterion:

$$D_{KL}(\hat{\theta}_1 \| \hat{\theta}_2) = \left(\hat{\theta}_1 - \hat{\theta}_2\right) \log \frac{\hat{\theta}_1}{\hat{\theta}_2}.$$



Patch-similarities: how to replace the squared differences? [Deledalle et al., 2009]

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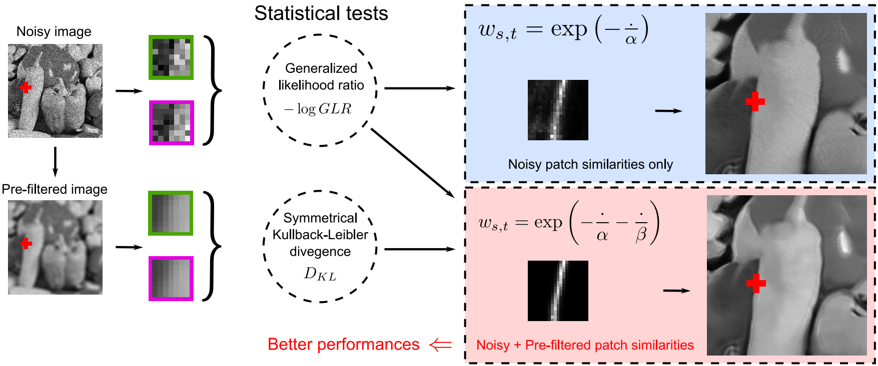
$$\mathrm{P}(\mathcal{H}_0 | \square_1, \square_2) = \frac{\mathrm{P}(\square_1, \square_2 | \mathcal{H}_0)}{\mathrm{P}(\square_1, \square_2 | \mathcal{H}_1)} \times \mathrm{P}(\mathcal{H}_0)$$



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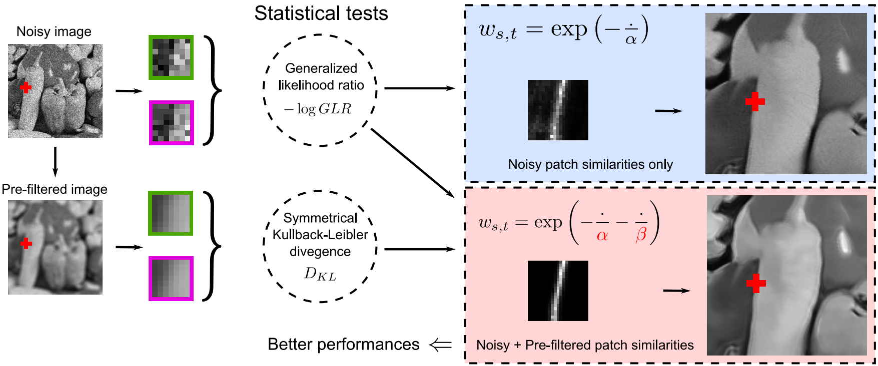


Next, how should we set the parameters α and β?

Automatic setting of the denoising parameters

PURE in Poisson NL means

- Based on the same ideas as *SURE based NL means*:
  - PURE is obtained in closed-form for Poisson NL means,
  - with almost same computation time.

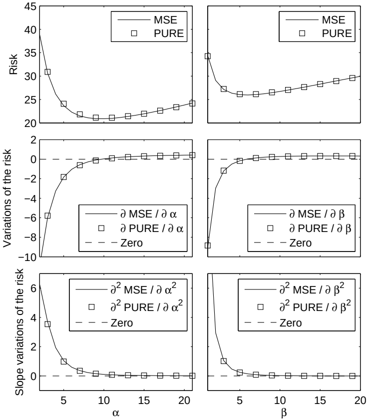
Selection of the parameters

- Optimum α and β obtained iteratively using Newton's method:

$$\begin{pmatrix} \alpha^{n+1} \\ \beta^{n+1} \end{pmatrix} = \begin{pmatrix} \alpha^{n} \\ \beta^{n} \end{pmatrix} - H^{-1}\nabla$$

  with H the Hessian and ∇ the gradient.
- The first and second order differentials of PURE are also obtained in closed-forms.

Find the best denoising level using similarities of noisy and pre-filtered patches!



MSE and PURE and their first and second order variations with respect to the parameters α and β

Risk minimisation

Choose the parameters α et β minimising the mean squared error (MSE):

$$E\left[\frac{1}{N}\|\lambda - \hat{\lambda}\|^2\right] = \frac{1}{N}\sum_s \left(\lambda_s^2 + E\left[\hat{\lambda}_s^2\right] - E\left[\lambda_s \hat{\lambda}_s\right]\right)$$

- $\sum_s \lambda_s^2$ idependent of the paramaters,
- $\sum_s E\left[\hat{\lambda}_s^2\right]$ can be estimated from $\hat{\lambda}$,
- How to estimate $\sum_s E\left[\lambda_s \hat{\lambda}_s\right]$?

Poisson unbiased risk estimator (PURE) [Chen, 1975, Luisier et al., 2010]

- If k is damaged by Poisson noise and $\hat{\lambda} = h(k)$ then

$$E\left[\lambda_s \hat{\lambda}_s\right] = E\left[k_s \overline{\lambda}_s\right]$$

with $\overline{\lambda} = h(\overline{k})$ and $\overline{k}$ defined by $\overline{k}_t = \begin{cases} k_t - 1 & \text{if } t = s \\ k_t & \text{otherwise} \end{cases}$

- PURE is given by: $$R(\hat{\lambda}) = \frac{1}{N}\sum_s \left(\lambda_s^2 + \hat{\lambda}_s^2 - 2k_s\overline{\lambda}_s\right).$$

Risk minimisation

Choose the parameters α et β minimising the mean squared error (MSE):

$$E\left[\frac{1}{N}\|\lambda - \hat{\lambda}\|^2\right] = \frac{1}{N}\sum_s \left(\lambda_s^2 + E\left[\hat{\lambda}_s^2\right] - E\left[\lambda_s \hat{\lambda}_s\right]\right)$$

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- How to estimate $\sum_s E\left[\lambda_s \hat{\lambda}_s\right]$?

PURE - Proof

Let be k a r.v. following a Poisson distribution and $h(.)$ a function:

$$\begin{aligned}
E\left[kh(k-1)\right] &= \sum_{k=1}^{\infty} kh(k-1)\frac{\lambda^k e^{-\lambda}}{k!} \\
&= \lambda \sum_{k=1}^{\infty} h(k-1)\frac{\lambda^{k-1} e^{-\lambda}}{(k-1)!} \\
&= E\left[\lambda h(k)\right]
\end{aligned}$$

PURE in Poisson NL means

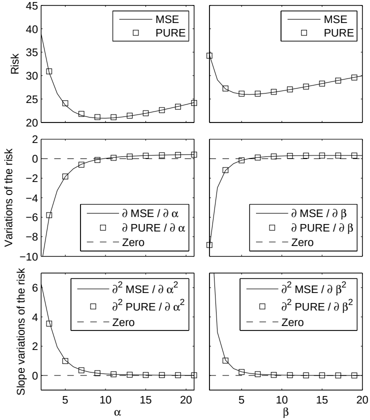
- [Van De Ville and Kocher, 2009] use an estimator of the risk for NL means and Gaussian noise.
- Based on the same ideas:
  - We obtained PURE in closed-form,
  - with almost same computation time.

Selection of the parameters

- We propose to optimize α and β iteratively using Newton's method:

$$\begin{pmatrix} \alpha^{n+1} \\ \beta^{n+1} \end{pmatrix} = \begin{pmatrix} \alpha^{n} \\ \beta^{n} \end{pmatrix} - H^{-1}\nabla$$

  with H the Hessian and ∇ the gradiant.
- The first and second order differentials of PURE are also obtained in closed-forms.



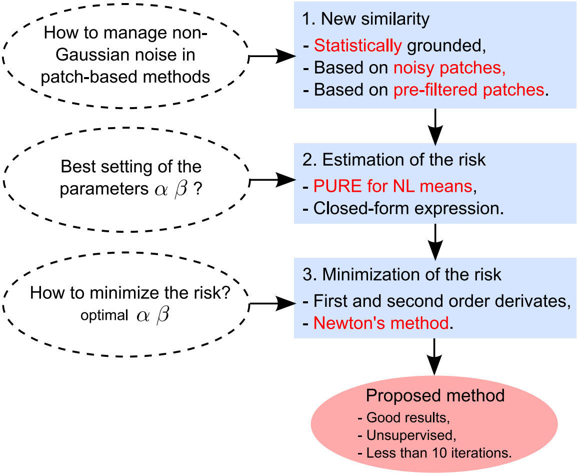
MSE and PURE and their first and second order variations with respect to the parameters α and β

Numerical results - PSNRs

Peppers (256 $\times$ 256)				
Noisy	3.14	13.14	17.91	23.92
MA filter	19.20	20.93	21.11	21.16
PURE-LET [Luisier et al., 2010]	19.33	24.29	27.27	30.79
NL means [Buades et al., 2005]	18.12	23.33	26.98	30.64
Poisson NL means	**19.90**	**25.32**	**28.07**	**31.06**
α_{opt}	(209)	(13.6)	(10.05)	(9.21)
β_{opt}	(0.72)	(1.31)	(2.76)	(7.64)
$\#$*iterations*	(13.5)	(8.02)	(7.03)	(6.90)
Cameraman (256 $\times$ 256)				
Noisy	3.28	13.27	18.03	24.05
MA filter	18.71	20.15	20.29	20.33
PURE-LET [Luisier et al., 2010]	19.67	24.32	26.87	**30.36**
NL means [Buades et al., 2005]	18.17	23.53	26.77	29.39
Poisson NL means	**19.89**	**25.07**	**27.42**	29.47
α_{opt}	(62.1)	(9.48)	(8.81)	(7.34)
β_{opt}	(0.51)	(1.19)	(3.57)	(16.19)
$\#$*iterations*	(11.0)	(6.80)	(7.60)	(11.3)

PSNR values averaged over ten realisations using different methods on images damaged by Poisson noise with different levels of degradation. The averaged optimal parameters and the averaged number of iterations of the proposed Poisson NL means are given.

Contributions on noise adaptation



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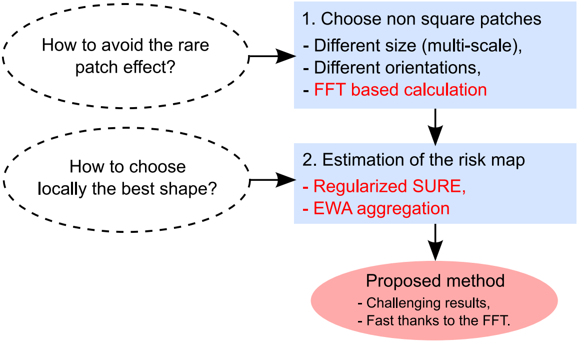


Contributions on signal adaptation

How to avoid the rare patch effect?

1. Choose non square patches
- Different size (multi-scale),
- Different orientations,
- FFT based calculation

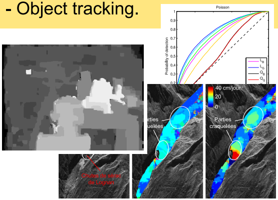
How to choose locally the best shape?

2. Estimation of the risk map
- Regularized SURE,
- EWA aggregation

Proposed method
- Challenging results,
- Fast thanks to the FFT.

Our perspectives

Extend to other tasks:
- Change detection,
- Stereo vision,
- Object tracking.



Mix both contributions:

Rare patch effect is a big problem for SAR imagery or HDR images,

Extend to "sparse-patch" filtering:

Take inspiration of recent works such as BM3D or NLSM.