Control Strategies for Solar Sail

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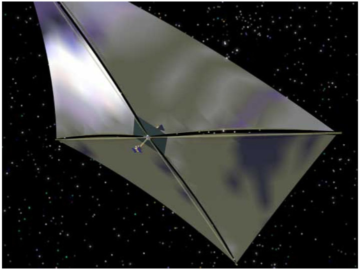
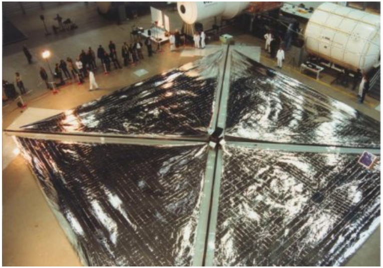
What is a Solar Sail ?

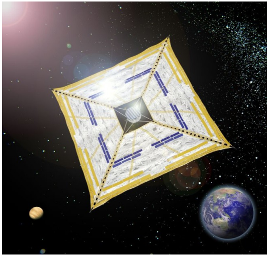
- Solar Sails a proposed form of propulsion system that takes advantage of the Solar radiation pressure to propel a spacecraft.
- The impact of the photons emitted by the Sun on the surface of the sail and its further reflection produce momentum on it.
- Solar Sails open a wide new range of possible missions that are not accessible by a traditional spacecraft.

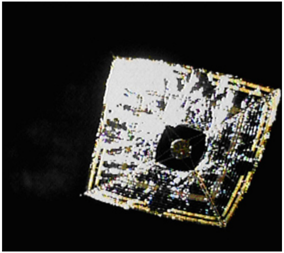




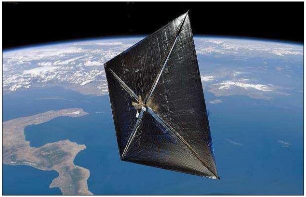
There have recently been two successful deployments of solar sails in space.

- IKAROS: in June 2010, JAXA managed to deploy the first solar sail in space.





- NanoSail-D2: in January 2011, NASA deployed the first solar sail that would orbit around the Earth.



The Solar Sail

We consider the solar sail to be flat and perfectly reflecting. Hence, the force due to the solar radiation pressure is in the normal direction to the surface of the sail.

The force due to the sail is defined by the *sail's orientation* and the *sail's lightness number*.

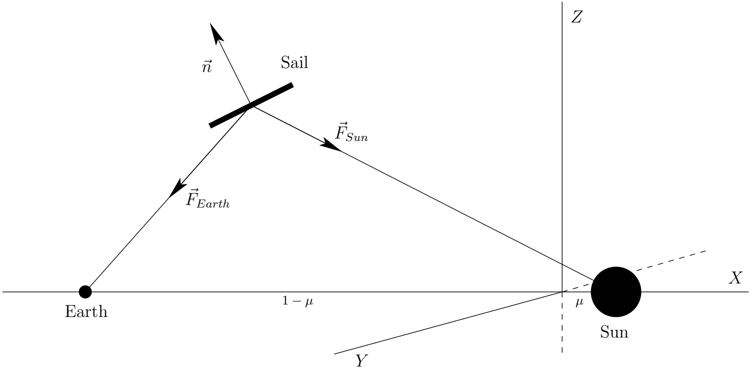
- The *sail's orientation* is given by the normal vector to the surface of the sail, $\vec{n}$. It is parametrised by two angles, α and δ.
- The *sail's lightness number* is given in terms of the dimensionless parameter β. It measures the effectiveness of the sail.

Hence, the force is given by:

$$\vec{F}_{sail} = \beta \frac{m_s}{r_{ps}^2} \langle \vec{r}_s, \vec{n} \rangle^2 \vec{n}.$$

The Dynamical Model

We use the Restricted Three Body Problem (RTBP) taking the Sun and Earth as primaries and including the solar radiation pressure due to the solar sail.

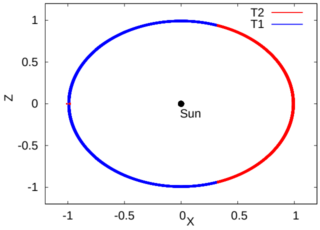


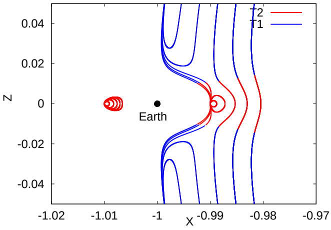
Equilibrium Points (I)

- The RTBP has 5 equilibrium points (L_i). For small β, these 5 points are replaced by 5 continuous families of equilibria, parametrised by α and δ.
- For a fixed small value of β, we have 5 disconnected family of equilibria around the classical L_i.
- For a fixed and larger β, these families merge into each other. We end up having two disconnected surfaces, S_1 and S_2. Where S_1 is like a sphere and S_2 is like a torus around the Sun.
- All these families can be computed numerically by means of a continuation method.

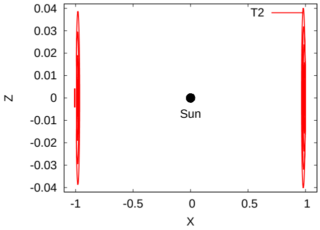
Equilibrium Points (II)

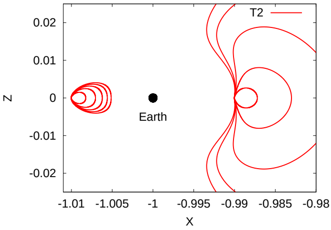
Equilibrium points in the XY plane





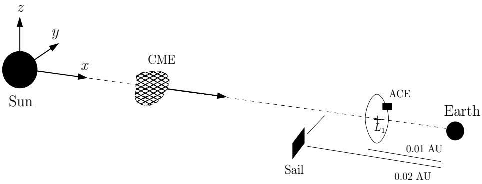
Equilibrium points in the XZ plane



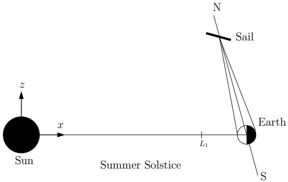


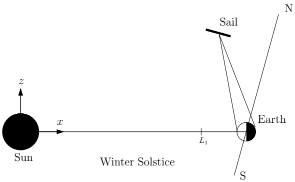
Interesting Missions Applications

Observations of the Sun provide information of the geomagnetic storms, as in the Geostorm Warning Mission.



Observations of the Earth's poles, as in the Polar Observer.





AIM of this TALK

One of the main goals of our work was to understand the geometry of the phase space and how it varies when the sail orientation is changed. Then use this information to derive strategies to control the trajectory of a Solar Sail.

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We will:

1. describe the dynamics of a solar sail around an equilibrium point (for a fixed sail orientation) and show the effects of variations on the sail orientation on the sail trajectory and show how to use this knowledge to derive a station keeping strategy around an equilibrium point.

2. we have two different ways to use this information. We will describe both strategies and apply them to the GeoStorm Mission.

3. finally we will discuss the robustness of these strategies when we include different sources of error.

Station Keeping Strategies Around Equilibria

Station Keeping for a Solar Sail

We want to design station keeping strategy to maintain a trajectory of a solar sail close to an unstable equilibrium point.

Instead of using *Control Theory Algorithms*, we will use *Dynamical System Tools* to find a station keeping algorithm for a Solar Sail.

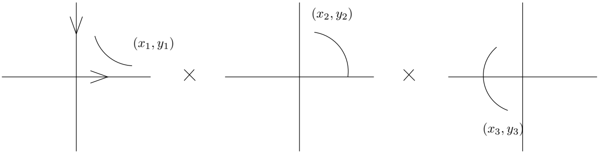
The main ideas are ...

- To focus on the linear dynamics around an equilibrium point and study how this one varies when the sail orientation is changed.
- To change the sail orientation (i.e. the phase space) to make the system act in our favour: keep the trajectory close to a given equilibrium point.

Station Keeping for a Solar Sail

We focus on the two previous missions, where the equilibrium points are unstable with two real eigenvalues, $\lambda_1 > 0, \lambda_2 < 0$, and two pair of complex eigenvalues, $\nu_{1,2} \pm \mathrm{i}\omega_{1,2}$, with $|\nu_{1,2}| << |\lambda_{1,2}|$.

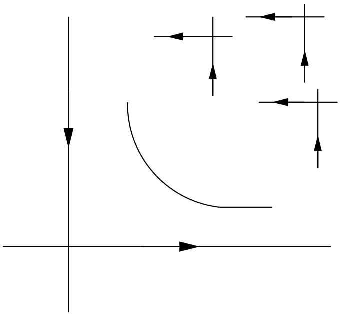
- To start we can consider that the dynamics close the equilibrium point is of the type saddle $\times$ centre $\times$ centre.
- From now on we describe the trajectory of the sail in three reference planes defined by each of the eigendirections.



- For small variations of the sail orientation, the equilibrium point, eigenvalues and eigendirections have a small variation. We will describe the effects of the changes on the sail orientation on each of these three reference planes.

Effects of Variations on the Orientation (I)

In the saddle projection of the trajectory:

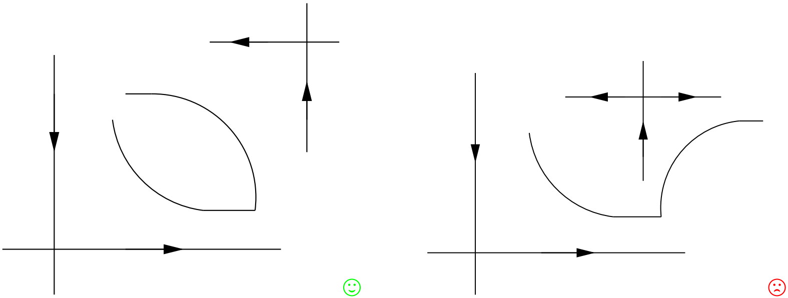


- When we are close to the equilibrium point, p_0, the trajectory escapes along the unstable direction.
- When we change the sail orientation the equilibrium point is shifted.

Effects of Variations on the Orientation (II)

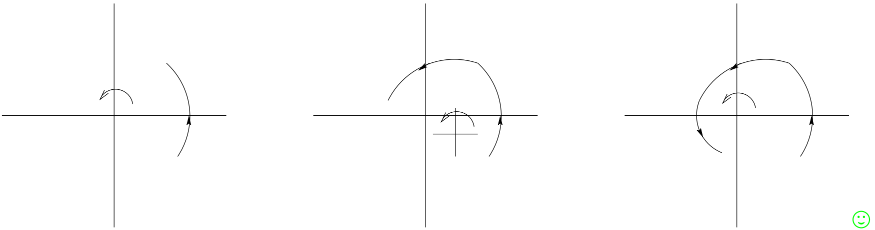
In the saddle projection of the trajectory:

- Now the trajectory will escape along the new unstable direction.
- We want to find a new sail orientation (α, δ) so that the trajectory will come close to the stable direction of p_0.



Effects of Variations on the Orientation (III)

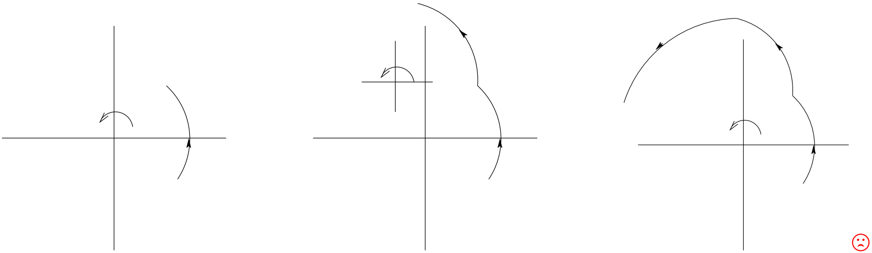
In the centre projection of the trajectory:



A sequence of changes on the sail orientation implies a sequence of rotations around different equilibrium points on the centre projection, which can result of an unbounded grouth.

Effects of Variations on the Orientation (III)

In the centre projection of the trajectory:



A sequence of changes on the sail orientation implies a sequence of rotations around different equilibrium points on the centre projection, which can result of an unbounded grouth.

Schematic idea of the Station Keeping Algorithm

We look at the sails trajectory in the reference system $\{x_0; \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5, \vec{v}_6\}$, so $z(t) = x_0 + \Sigma_i s_i(t)\vec{v}_i$.

During the station keeping algorithm:

1. when $\alpha = \alpha_0, \delta = \delta_0$: if $|s_1(t)| \geq \varepsilon_{max}$ $\Rightarrow$ choose new sail orientation $\alpha = \alpha_1, \delta = \delta_1$.
2. when $\alpha = \alpha_1, \delta = \delta_1$: if $|s_1(t)| \leq \varepsilon_{min}$ $\Rightarrow$ restore the sail orientation: $\alpha = \alpha_0, \delta = \delta_0$.
3. Go Back to 1.

1st Idea for finding $\alpha_{new}, \delta_{new}$

We will choose a the position of the new equilibrium point (i.e. a new sail orientation) so that projection of the trajectory on the saddle will come back and the two centre projections remain bounded ?

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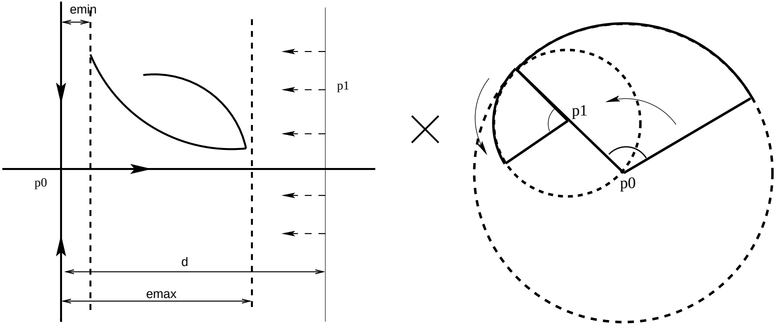


The constants ε_{min}, ε_{max} and d will depend on the mission requirements and the dynamics around the equilibrium point.

Remarks

- We do not know explicitly the position of the equilibrium points $p(\alpha, \delta)$. But we can compute the linear approximation of this function:

$$p(\alpha, \delta) = p(\alpha_0, \delta_0) + Dp(\alpha_0, \delta_0) \cdot (\alpha - \alpha_0, \delta - \delta_0)^T.$$

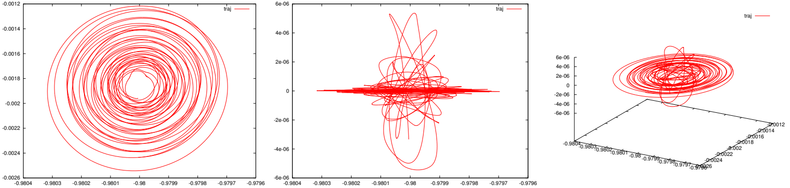
- There are some restrictions of the position of the new equilibria when we change α and δ. We have 2 unknowns and at least 6 conditions that must be satisfied.

- We will change the sail orientation so that the position of the new fixed point is as close as possible to the desired new equilibrium point and in the correct side in the saddle projection.

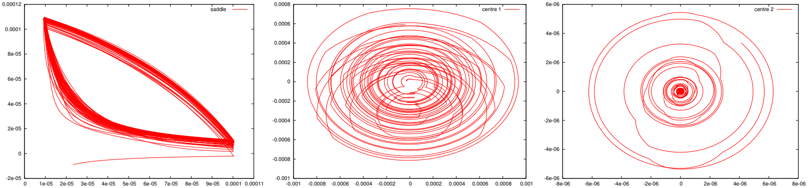
- To decide the new sail orientation we will assume that the eigenvalues and eigendirections do not vary when the sail orientation is changed.

Results for the Geostorm Mission (RTBPS)

XY and XZ and XYZ Projections

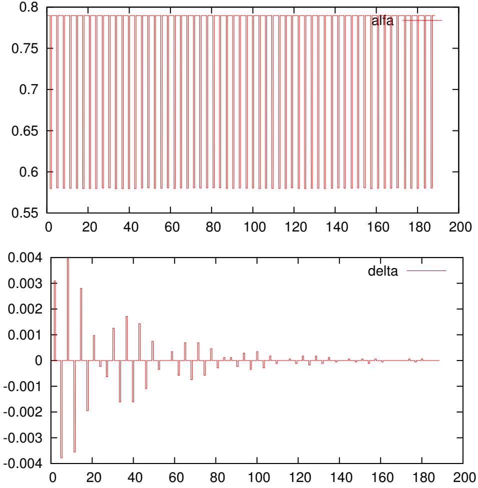


Saddle × Centre × Centre Projections



Results for the Geostorm Mission (RTBPS)

Variation of the sail orientation



2st Idea for finding $\alpha_{new}, \delta_{new}$

The computation of variational equations (of suitable order) w.r.t. α and δ gives explicit expressions for the effect of different orientations (close to the reference values $\alpha = \alpha_0, \delta = \delta_0$) trajectory.

$$\phi_t(x_0, \alpha_0 + h_a, \delta_0 + h_d) = \phi_t(x_0, \alpha_0, \delta_0) + \frac{\partial \phi}{\partial \alpha}(x_0, \alpha_0, \delta_0) \cdot h_a + \frac{\partial \phi}{\partial \delta}(x_0, \alpha_0, \delta_0) \cdot h_d,$$

With this we can impose conditions on the "future" of the orbit and find orientations that fulfil them (or show that the condition is unattainable).

- We will define the parameters ε_{max}, Dt_{min} and Dt_{max} that will vary for each mission application.
- We will find $\alpha_{new}, \delta_{new}$ and $dt \in [Dt_{min}, Dt_{max}]$ so that the trajectory is close to the fixed point.

Remarks

We use the variational equations up to first order. Hence, we have a linear map for the different final states.

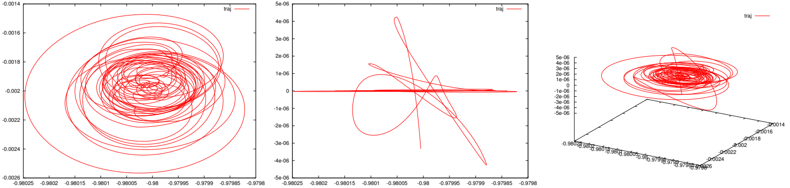
As before we want the final position to be close to the stable direction, keeping small the two centre projections.

One can think of different ways to solve this problem. We have seen that the best results are found if we:

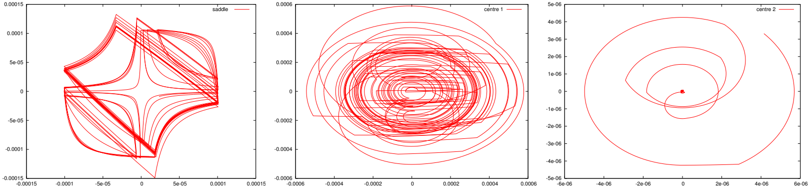
- For each $dt \in [Dt_{min}, Dt_{max}]$ we will find α_{new} and δ_{new} such that $s_1 = 0$ and (s_5, s_6) are minimum (i.e. we are close to stable direction and one of the centres is small).
- We finally choose the dt, α_{new} and δ_{new} that minimises the other centre projection (s_3, s_4).

Results for the Geostorm Mission (RTBPS)

XY and XZ and XYZ Projections

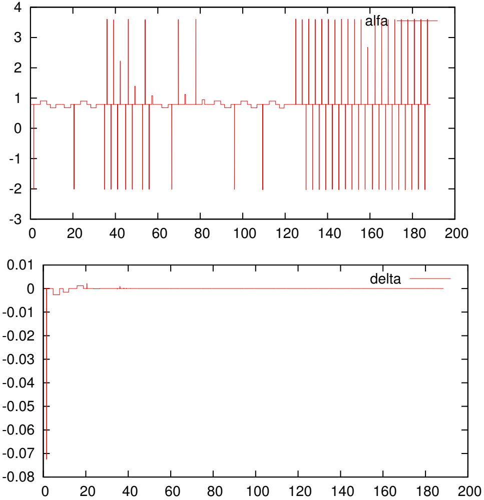


Saddle × Centre × Centre Projections



Results for the Geostorm Mission (RTBPS)

Variation of the sail orientation



Results

We have applied these station keeping strategy to different mission scenarios. We show the results for the Geostorm Warning Mission.

For each mission:

- We have done a Monte Carlo simulation taking a 1000 random initial conditions.
- For each simulation we have applied the station keeping strategy for 30 years.
- We have tested the robustness of our strategy including random errors on the position and velocity determination, as well as on the orientation of the sail at each manoeuvre.

Note : All the simulations have been done using the full set of equations, we only use the linear dynamics to decide the change on the sail orientation.

Results for the Geostorm Mission (RTBPS)

Algorithm Used: **Fixed Point Algorithm**

EType	% Success	Δt (days)	$\Delta\alpha$ (deg)	$\Delta\delta$ (deg)
E0	100.0 %	158.32 - 38.90	0.211 - 0.209	4.884e-03 - 5.040e-06
P0	100.0 %	158.44 - 38.86	0.211 - 0.209	4.879e-03 - 5.041e-06
V1	100.0 %	166.28 - 38.41	0.212 - 0.207	6.399e-03 - 2.461e-05
V2	100.0 %	233.42 - 37.01	0.219 - 0.199	2.324e-02 - 1.003e-04
V3	100.0 %	363.40 - 35.38	0.228 - 0.189	5.588e-02 - 2.179e-04
V4	79.0 %	370.85 - 28.23	0.283 - 0.101	2.123e-01 - 6.749e-04

EType stands for the kind of errors considered in each simulation: E0 = No errors, P0 = Errors on Position and Velocity only, V1, V2, V3, V4 = Errors on Position, Velocity and Sail Orientation, where V1 $= 0.001^\circ$, V2 $= 0.005^\circ$, V3 $= 0.01^\circ$, V4 $= 0.05^\circ$

Results for the Geostorm Mission (RTBPS)

Algorithm Used: **Variational Equation Algorithm**

EType	% Success	Δt (days)	$\Delta\alpha$ (deg)	$\Delta\delta$ (deg)
E0	100.0 %	317.88 - 2.32	2.82 - 0.109	0.160 - 0.000
P0	100.0 %	317.88 - 2.32	2.82 - 0.109	0.160 - 0.000
V1	100.0 %	321.57 - 2.32	2.82 - 0.110	0.163 - 3.78e-05
V2	100.0 %	346.67 - 2.32	2.86 - 0.104	0.292 - 1.72e-04
V3	100.0 %	361.96 - 2.32	4.09 - 0.098	0.557 - 2.38e-04
V4	100.0 %	334.56 - 2.32	4.47 - 0.041	2.598 - 4.54e-04

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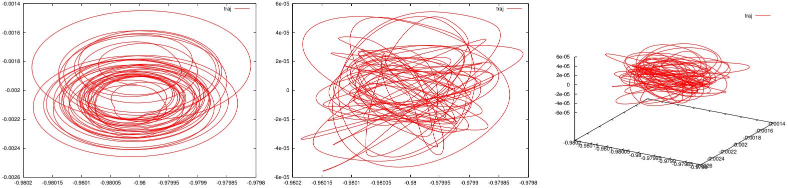
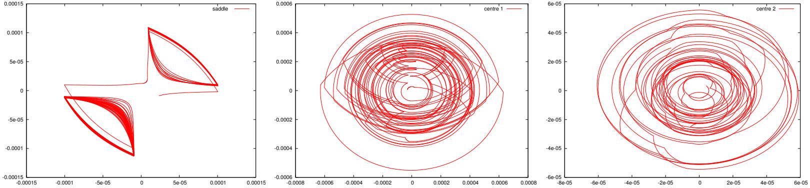
Results for the Geostorm Mission (RTBPS)

Fixed Point Algorithm. Error type V3

XY and XZ and XYZ Projections



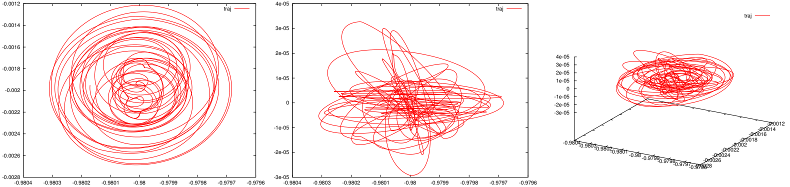
Saddle × Centre × Centre Projections



Results for the Geostorm Mission (RTBPS)

Variational Equations Algorithm. Error type V3

XY and XZ and XYZ Projections



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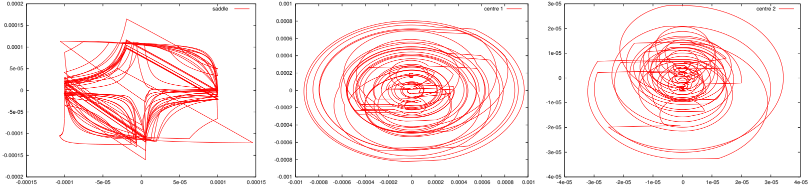
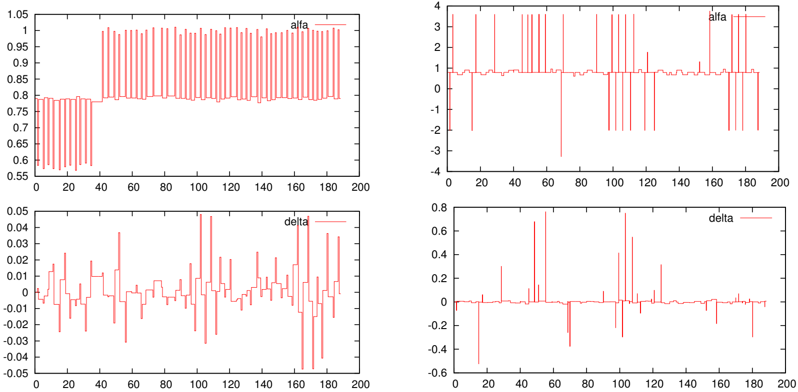


Results for the Geostorm Mission (RTBPS)

Variation of the sail orientation



Left: ALgorithim Fixed Point, Right: Algorthim Variational Equations

Including more realism to the dynamical model

There are several ways to include more realism to the dynamical model. For example,

- taking a more realistic model for the Solar Sail by including the force produced by the absorption of the photons, the reflectivity properties of the sail material, ...
- taking a more realistic model for the gravitational perturbations by including the eccentricity in the Earth - Sun system. Or the gravitational attraction of other bodies, i.e. the Moon, Jupiter, ...

We have started by considering the eccentricity in the Earth - Sun system and studied the robustness of our strategies. So we take the Elliptic Restricted Three Body Problem with a Solar sail as a model.

In the ERTBP + Solar Sail

The fixed points that existed in the RTBP + Solar sail no longer exist in this model. They have been replaced by periodic orbits of same period as the Earth's orbit around the Sun.

We can apply the same ideas to remain close to one of these periodic orbits and fulfil the mission requirements of the Geostorm mission or the Polar Observer.

Notice that:

- for each sail orientation (α, δ) we have an unstable periodic orbit replacing the fixed point.
- taking the Floquet modes we have a periodic reference system that will give us a good description of the local dynamics around these periodic orbits.
- we can apply the same ideas as before considering this appropriate reference system.

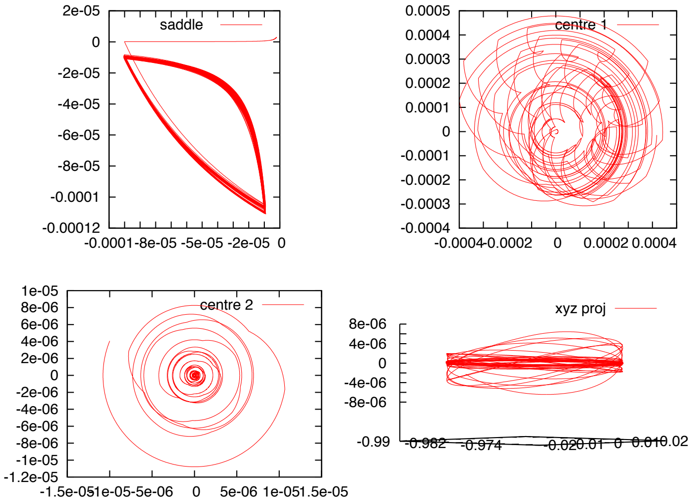
Results for the Geostorm Mission (ERTBPS)

Algorithm Used: **Fixed Point Algorithm**

EType	% Success	Δt (days)	$\Delta\alpha$ (deg)	$\Delta\delta$ (deg)
E0	100.0 %	155.65 - 92.46	0.174 - 0.168	3.650e-03 - 4.087e-06
P0	100.0 %	170.83 - 92.61	0.177 - 0.168	4.093e-03 - 2.268e-05
V1	100.0 %	173.56 - 92.70	0.177 - 0.167	5.762e-03 - 2.256e-05
V2	100.0 %	262.21 - 95.06	0.183 - 0.160	2.478e-02 - 9.557e-05
V3	100.0 %	362.44 - 99.53	0.190 - 0.149	5.313e-02 - 1.928e-04
V4	29.0 %	351.75 - 87.30	0.242 - 0.700	2.101e-01 - 6.972e-04

EType stands for the kind of errors considered in each simulation: E0 = No errors, P0 = Errors on Position and Velocity only, V1, V2, V3, V4 = Errors on Position, Velocity and Sail Orientation, where V1 = 0.001°, V2 = 0.005°, V3 = 0.01°, V4 = 0.05°

Results for the Geostorm Mission (ERTBPS)



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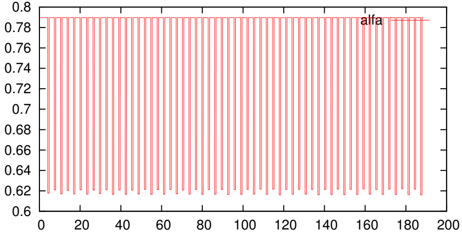
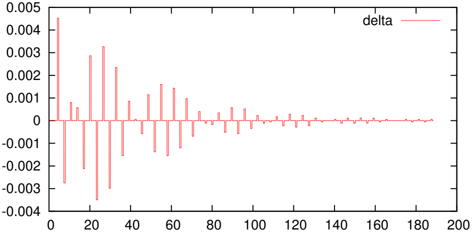




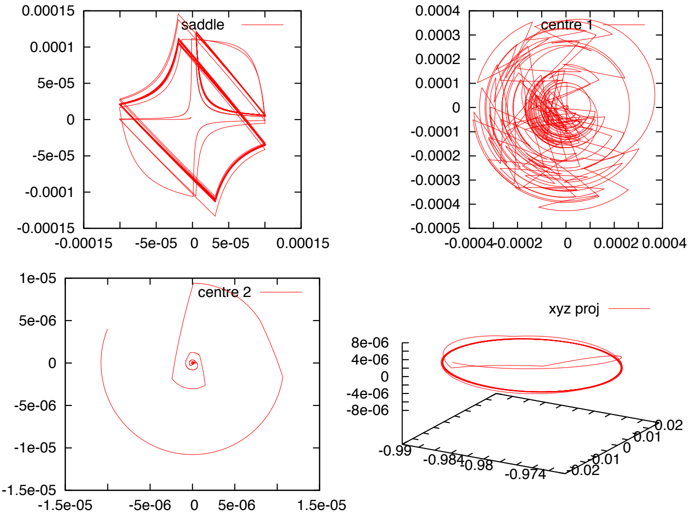
Results for the Geostorm Mission (ERTBPS)

Algorithm Used: **Variational Equation Algorithm**

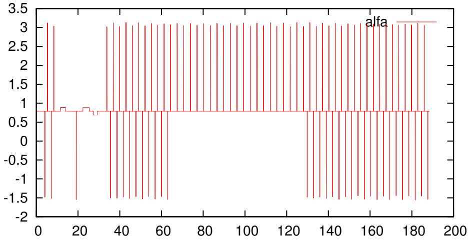
EType	% Success	Δt (days)	$\Delta\alpha$ (deg)	$\Delta\delta$ (deg)
E0	100.0 %	280.56 - 64.07	2.35 - 0.213	0.12 - 0.00
P0	100.0 %	338.21 - 67.29	2.37 - 0.102	0.12 - 1.13e-05
V1	100.0 %	323.99 - 66.60	2.38 - 0.096	0.13 - 5.09e-05
V2	100.0 %	369.22 - 69.12	2.62 - 0.085	0.28 - 1.54e-04
V3	100.0 %	355.27 - 69.72	4.00 - 0.077	0.55 - 2.88e-04
V4	100.0 %	321.73 - 61.39	4.71 - 0.025	2.34 - 5.90e-04

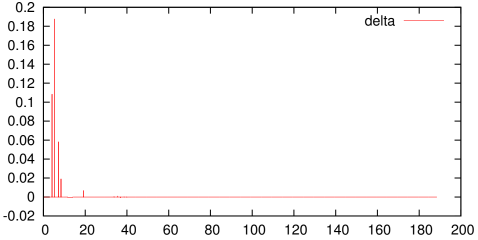
EType stands for the kind of errors considered in each simulation: E0 = No errors, P0 = Errors on Position and Velocity only, V1, V2, V3, V4 = Errors on Position, Velocity and Sail Orientation, where V1 = 0.001°, V2 = 0.005°, V3 = 0.01°, V4 = 0.05°

Results for the Geostorm Mission (ERTBPS)



Results for the Geostorm Mission (ERTBPS)





References

[1] Farrés, A. & Jorba, À.: *Dynamical system approach for the station keeping of a solar sail.* The Journal of Astronautical Science, 58:2, pp. 199-230, 2008.

[2] Farrés, A. & Jorba, À.: *Solar sail surfing along families of equilibrium points.* Acta Astronautica, Vol. 63, Issues 1-4, pp. 249-257, 2008.

[3] Farrés, A.: *Contribution to the Dynamics of a Solar Sail in the Earth-Sun System* PHD Thesis, University of Barcelona, October 2009.

[4] Farrés, A. & Jorba, À.: *Sailing Between The Earth and Sun.* Proc. of the Second International Symposium on Solar Sailing. New York, USA, 20-22 July 2010. Ed: R.Ya. Kezerashvili, pp. 177-182, 2010.

[5] Farrés, A. & Jorba, À.: *On the station keeping of a Solar sail in the Elliptic Sun-Earth system.* Advances in Space Research. To appear in 2011.

Conclusions & Future Work

- We have described the linear dynamics around an unstable equilibrium point and how it varies when the sail orientation changes.
- We have designed two station keeping strategies using this information and applied them to a particular mission.
- We have extended these strategies to a more complex model, and we are now working to include the effect of the other planets.
- We have discussed the robustness of these algorithms when different sources of errors are included. We have seen that the controllability of the sail is strictly related to the nature of the neighbourhood of the equilibrium point.
- Notice that these strategies do not require previous planning as the decisions are taken depending on the sails position at each time.

Merci pour votre attention