On an inverse Cauchy problem arising in tokamaks

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In this talk we investigate an inverse Cauchy problem which consists in the identification of the plasma boundary inside a tokamak only through the knowledge of magnetic measurements at the outer boundary of the device.

It is well-known that Maxwell's equations enable to describe the spatial evolution of the magnetic poloidal flux u inside the tokamak, more precisely in the vacuum located between the outer boundary and the limiter (mechanical part built so that the plasma remains entirely contained within this latter). Due to the axisymmetry of the device, the equation under study may be formulated as a linear homogeneous second order elliptic equation in an 2D-annular domain Ω :

$$\nabla.(\sigma\nabla u) = 0 \text{ in } \Omega \subset \mathbb{R}^2 \text{ with } \sigma \in W^{1,\infty}(\Omega, \mathbb{R}) \text{ and } 0 < c \leq \sigma < C.$$

In fact, identify the plasma boundary remains to find level lines of u from overdetermined Cauchy data on the outer boundary. Such an issue is known to be ill-posed and has generated several technics of regularization, for instance Tikhonov, iterative, level set, integral equations, quasi-reversibility or conformal mappings methods. We choose to look at this problem under the point of view of complex analysis. Indeed, following an idea of [1], the equation for u may be viewed as a compatibility one for the conjugate Beltrami equation

$$\overline{\partial} f = \nu \overline{\partial f} \text{ in } \Omega \text{ with } \nu = \frac{1-\sigma}{1+\sigma} \in W^{1,\infty}(\Omega, \mathbb{R}) \text{ and } |\nu| < \kappa < 1.$$

Looking for solutions to this last equation for non smooth ($L^p, p > 1$) Dirichlet condition leads to the introduction of particular classes of functions that can be considered as generalized analytic functions [2]. Properties of those latter allow to reformulate our inverse problem as a best approximation one under constraint admitting an explicite unique solution [3, 4].

To compute this explicit solution in the case of L^2 boundary data, the magnetic poloidal flux and its normal derivative are first expand on complete families of solutions, namely the toroidal harmonics which fit to the topology of the device. Afterwards the approximation problem under constraint is solved by an algorithm iterating on the geometry of the plasma boundary. We present some recent numerical simulations obtained from magnetic data measured on the outer boundary of the tokamak *Tore Supra* built at CEA Cadarache (France).

Références

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