Multilayer approximation of the Navier-Stokes system
Analytical validation

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Due to computational issues associated with the free surface Navier-Stokes or Euler equations, the simulations of geophysical flows are often carried out with shallow water type models of reduced complexity. Thus the hydrostatic assumption for shallow water flows leads to non-linear hyperbolic systems of Saint-Venant type. For these vertically averaged models, efficient and robust numerical techniques [1, 2] are available and avoid to deal with moving meshes.

Here, we are interested in flows where the horizontal velocity can hardly be approximated – as in the Saint-Venant system – by a vertically constant velocity (large bottom friction, significant water depth, wind effects,...).

To model and simulate such complex flows, multilayer Saint-Venant models are often used but the proposed models do not allow mass exchanges between neighborhing layers and make a close relation to models for non-miscible fluids (see [3] and the references therein). Here, we derive another and simpler multilayer model where we prescribe the vertical discretization of the layers taking into account the (unknown) total height of water. In our approach [4, 5] the layer partition is merely a discretization artefact, and it is not physical. A critical distinguishing feature of our model is that it allows fluid circulation between layers. This changes dramatically the properties of the model and its ability to describe flow configurations that are crucial for the foreseen applications, such as recirculation zones.

The simulation of these flows requires stable, accurate, conservative schemes able to handle efficiently complex topographies, free surface deformations, and to capture robustly wet/dry fronts. The numerical solution of the multilayer system we propose is based on a kinetic interpretation of the model and uses a finite volume kinetic scheme with an extended hydrostatic reconstruction technique [1]. In contrast with Navier–Stokes solvers, our discretization technique allows easily satisfying properties such as conservation, positivity, well-balancing of source terms (cf. [1]), and handling robustly wet/dry interfaces over variable bottom topography.

The presentation is organized as follows. We start giving the cornerstones of the derivation of the multilayer model starting from the Navier-Stokes system. Then we propose a numerical scheme for the solution of the obtained model. A special emphasis is placed on the analytical validations and the confrontation with experimental data.

Références

[1] F. Bouchut. *Nonlinear stability of finite volume methods for hyperbolic conservation laws and well-balanced schemes for sources.* Birkhäuser, 2004.

[2] R.J. LeVeque. *Numerical methods for conservation laws.* Birkhauser, 1999.

[3] M.J. Castro, J.A. García-Rodríguez, J.M. González-Vida, J. Macías, C. Parés, and M.E. Vázquez-Cendón. *Numerical simulation of two-layer shallow water flows through channels with irregular geometry. J. Comput. Phys.*, 195(1):202–235, 2004.

[4] E. Audusse, M.-O. Bristeau, B. Perthame, and J. Sainte-Marie. *A multilayer Saint-Venant system with mass exchanges for Shallow Water flows. Derivation and numerical validation. ESAIM: M2AN*, 45:169–200, 2011.

[5] E. Audusse, M.-O. Bristeau, M. Pelanti, and J. Sainte-Marie. *Approximation of the hydrostatic Navier-Stokes system for density stratified flows by a multilayer model. Kinetic interpretation and numerical validation. Accepted for publication in J. Comp. Phys., doi: 10.1016/j.jcp.2011.01.042*, 2011.

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