Symmetric Discontinuous Galerkin Formulation For Maxwell's equations

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In this paper, a new symmetric discontinuous Galerkin formulation for the time-dependent Maxwell's equations with superconductive boundary has been proposed. Its hp analysis is carried out and error estimates that are optimal in the meshsize h and slightly suboptimal in the approximation degree p are obtained. Some numerical results are given to confirm the convergence rates as a function of the meshsize.

In [3], the discontinuous Galerkin method with solutions that are exactly divergence-free inside each element, is developped for numerically solving the Maxwell equations.
Here, we consider a symmetric interior penalty discontinuous Galerkin method to approximate in space an initial boundary value problem derived from Maxwell's equations in "stable medium" with supraconductive boundary.

$$\frac{\partial^2 u}{\partial t^2} + c^2 \nabla \times (\nabla \times u) = f, \;\; \nabla \cdot u = 0 \quad \text{in} \;\; \Omega \times I; \tag{1}$$

$$n \times u(x,t) = 0 \quad \text{on} \;\; \partial\Omega \times I, \quad u(x,0) = u_0(x), \;\; \frac{\partial u}{\partial t}(x,0) = u_1(x) \quad \text{on} \;\; \Omega. \tag{2}$$

Here Ω is a convex polyhedron, $I = [0, t^*] \subset \mathbb{R}$, u_0 and u_1 are in $H_0(\nabla\times, \Omega) \cap H(\nabla\cdot, \Omega)$ and f is defined on $\Omega \times I$. In this paper we used the same notations for spaces as in [1].
In order to derive a weak formulation of (1)-(2), we note that formulas (1)-(2) in [1] implies for any u with $\nabla \times u \in H(\nabla\times, \Omega)$

$$c^2(\nabla \times (\nabla \times u), v) = c^2(\nabla \times u, \nabla \times v) + a(u,v)$$

where we have denoted by

$$a(u,v) = c^2 < n \times (\nabla \times u), v > -c^2 \textstyle\sum_{e \subset F_h^I} < [v]_T, \{\nabla \times u\} >_e .$$

Now, we introduce the penalty term via the form

$$J^\sigma(u,v) = \sum_{e \subset F_h^I} < \sigma[u]_N, [v]_N >_e + \sum_{e \subset F_h} < \sigma[u]_T, [v]_T >_e \qquad u,v \in H^1(\nabla\times, \Pi_h)^3$$

where $\sigma := \kappa p^2/h$ is a stabilization parameter and κ is a constant supposed ≥ 1. We also define
$A(u,v) = c^2(\nabla \times u, \nabla \times v) + a(v,u) - a(u,v) + J(u,v)$ where $J(u,v) = (\nabla.u, \nabla.v)$
So our formulation is $B(u,v) = A(u,v) + J^\sigma(u,v)$.

Références

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