Two-Aircraft Optimal Control Problem. The in-flight noise reduction

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The aim of this talk is to present and solve a mathematical model of a two-aircraft optimal control problem reducing the noise on the ground during the approach.
The mathematical formulation of this problem is given by:

$$\begin{cases} \min_{u\in\mathbf{U}} J_{12}(y,u) & = \int_{t10}^{t_{1f}} g_1(y_1(t),u_1(t))dt + \int_{t20}^{t_{1f}} g_{12}(y_1(t),u_1(t),y_2(t),u_2(t))dt + \\ & \int_{t20}^{t_{2f}} g_2(y_2(t),u_2(t))dt \\ \dot{y}(t) & = f(u(t),y(t)), \forall t \in [t_{10},t_{2f}], t_{10}=0, y(0)=y_0, u(0)=u_0 \\ k_{1i}(y_i(t),u_i(t)) & \leq 0, \\ k_{2i}(y_i(t),u_i(t)) & \geq 0, i=1,2. \end{cases} \tag{1}$$

where J_{12} is the two-aircraft noise on ground, g_1, g_2 are respectively the first and the second aircraft noise level and g_{12} shows the aircraft coupling noise. $y(t) := (y_1(t), y_2(t))$ is state variable and $u(t) := (u_1(t), u_2(t))$ is the control variable.
The equation on $\dot{y}(t)$ summarizes the dynamic of the system. $k_{1i}(y_i(t),u_i(t))$, $k_{2i}(y_i(t),u_i(t))$ are mixed constraints on u and y; they correspond to the aerodynamical and flight limitations in (3D). (1) is a non-convex optimal control governed by ordinary non-linear differential equations [1]. To solve this problem, we use a direct method and a Runge-Kutta 'RK4' discretization schema. We chose this discretization schema because it is a sufficiently high orde and it does-not require computation of the partial derivatives of f. We apply the Nonlinear Interior point Trust Region Optimization solver 'KNITRO' [2]. We will present a large set of numerical experiments. The obtained results give feasible trajectories with a significant noise reduction.
The optimal trajectories provide optimal flight characteristics and coincide for a large portion of the flight as soon as the one runways is considered when the continuous descent is initiated. Further methods are needed to extend the developped problem to air traffic control while a specified airport is considered.

Références

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