Optimized Schwarz algorithms for the time harmonic Maxwell equations discretized by a discontinuous Galerkin method

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This work is concerned with the numerical solution of the first order time-harmonic Maxwell equations on bounded domain and on heterogeneous media :

$$(i\omega\varepsilon(\mathbf{x}) + \sigma(\mathbf{x}))\mathbf{E}(\mathbf{x}) - \mathrm{rot}\,(\mathbf{H}(\mathbf{x})) = \mathbf{J}(\mathbf{x}) \quad \text{and} \quad i\omega\mu\mathbf{H}(\mathbf{x}) + \mathrm{rot}\,(\mathbf{E}(\mathbf{x})) = \mathbf{0},\ \mathbf{x} \in \Omega \subset \mathbb{R}^3 \qquad (1)$$

We are interested in discontinuous Galerkin (DG) methods based on a high-order polynomial interpolation and a centered or upwind approximation to estimate the numerical fluxes at interfaces between neighboring elements. The discontinuous nature of the approximation allows for a local definition of the interpolation order which is, in combination with a local refinement of the mesh, a central feature for obtaining a flexible and accuracte method. The discretization of the equations (1) by a DG method leads to the resolution of a sparse linear system which can be of very large dimension. The direct methods for solving these systems are too costly in CPU time and in memory space, we use a domain decomposition method of optimized Schwarz-type [2]. This method, which is well suited for parallel computation, consists in splitting the computational domain into several subdomains and in solving with an iterative procedure the problems of small size corresponding to each subdomain. The interface conditions between subdomains are chosen, by approximating a transparent operator, in order to obtain an optimal convergence factor. We apply these methods to the imaging of a subsurface. This imaging consists to determine the characteristics parameters ($\varepsilon(\mathbf{x})$, $\sigma(\mathbf{x})$) of the various media constituting a subsurface. To do this, we solve an inverse problem based in minimizing an objective function which is the sum of the squared differences between the simulated and measured data [3]. Figure 2 shows a simple example of the imaging where we compute the relative electric permittivity $\varepsilon(\mathbf{x})$ of a subsurface from synthetic data.

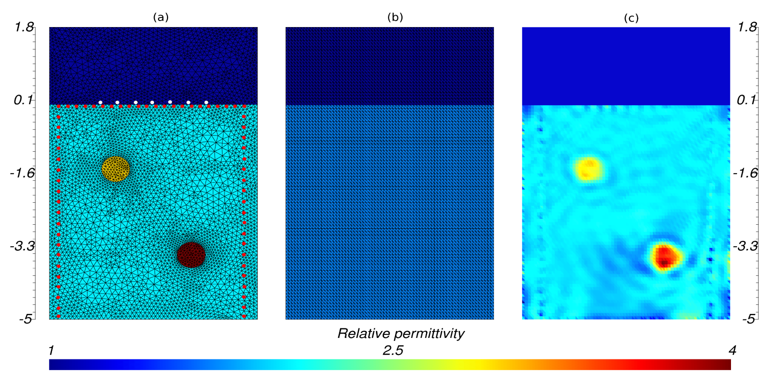

Figure 1: (a) represents the true model, the white bullets and red bullets are respectively the source positions and the recording positions; (b) and (c) respectively show the initial model and the result obtained after 200 conjugate gradient iterations in the inversion process.

Références

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