Uniform bounds for positive random functionals with application to density estimation

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Mots-clés :
Résumé : The talk consists of three parts. In the first one we presents upper functions for very general stochastic objects namely for positive random functionals. The corresponding results are used for deriving the uniform bounds for gaussian random fields and for the empirical processes. This part is ended by the discussion on the relation of the obtained abstract probabilistic results to the well-known phenomena arising in minimax and minimax adaptive estimation. In the second part we consider some special random processes such that kernel density estimation process and convoluting kernel density estimation process. Both of them are the special cases of empirical processes. Using the results obtained in the first part of the talk we prove non-asymptotical versions of the law of iterated logarithm and the law of logarithm and compare them with existing asymptotical results. Moreover, we establish also some moments inequalities for the supremum norm of the both mentioned above processes. These results are the crucial tool for the considerations done in the third part of the talk devoted to statistical problems. We study the estimation of a probability density on R^d and consider the risk described by supremum norm. We propose very general selection rule from the family of kernel estimators. The main ingredient of our construction are majorants which are the upper functions for the processes considered in the second part. For the selected estimator we prove so-called sup-norm oracle inequality. Being established, an oracle inequality is the informative tool for deriving minimax adaptive results. We use our sup-norm oracle inequality in order to prove that the selected estimator is adaptive over the scale of anisotropic Hölder classes.

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