Parallel computing to study the Davey-Stewartson II equation in small dispersion limit.

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The Davey Stewartson systems read

$$\begin{aligned} i\epsilon u_t + \epsilon^2 u_{xx} - \alpha\epsilon^2 u_{yy} + 2\rho\left(\phi + |u|^2\right) u &= 0 \\ \phi_{xx} + \alpha\phi_{yy} + 2\,|u|^2_{xx} &= 0 \end{aligned} \tag{1}$$

where α, $\rho = \pm 1$, $\epsilon \ll 1$ is a small dispersion parameter, and ϕ represents the velocity potencial of the fluid. The case $\rho = -1$ corresponds to the focusing case and $\rho = 1$ to the defocusing case.
These systems appear in many physical applications, for example in the study of nonlinear water waves [1] and in plasma physics [2], [3].
To give an asymptotic description of solutions of these systems in small dispersion limit, (i.e. when ϵ tends to 0), where a zone of rapid oscillations is expected to appear; we have to use high resolution computation.
We developped a parallel FORTRAN 90 code, which computes numerical solutions of DS II system by using successively, a semi-discretisation in spatial coordinates in Fourier space, and then a time-splitting sheme of order 4.
This code is expected to allow us to study completely, as much as possible, the semi-classical DS II limit. We study in particular the the apparition or not of rapid oscillations zone in the solution before probable blow up. Actually, there doesn't exist analytical theory or even predictions about solutions of (1), so numerical simulations become the only way to investigate this problem.

Références

[1] A. Davey and K. Stewartson, *On three-dimensional packets of surface waves.* Proc. R. Soc. Lond. A, 338:101-110 (1974).

[2] K. Nishinari, K. Abe, and J. Satsuma. *A new type of soliton behaviour in a two dimensionnal plasma system.* J. Phys. Soc. Japan, 62:20212029, 1993.

[3] K. Nishinari, K. Abe, and J. Satsuma. *Multidimensional behavior of an electrostaic ion wave in a magnetized plasma.* Phys. Plasmas 1, 8:25592565, 1994.

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