Large Time Behavior of Periodic Viscosity Solutions of Integro-differential Equations

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We are interested in the asymptotic behavior, as $t \to \infty$, of periodic viscosity solutions of integro-differential equations of the form

$$u_t - I[u] - Tr(a(x)D^2u) + b(x)|Du|^k = f(x)$$

where $I[u]$ is a Levy-Itô operator. We also address the problem to mixed integro-differential equations of the type

$$u_t + (-\Delta_{x_1})^{\beta}[u] - \Delta_{x_2}u + |Du|^k = f(x)$$

thus where fractional diffusion occurs in one direction and classical diffusion in the complementary one. It turns out that under some suitable assumptions the solution of the initial value problem behaves like $ct + v(x) + o(1)$ where v is a solution of the stationary ergodic problem corresponding to the ergodic constant c. In order to obtain this asymptotic behavior, we need to establish two key ingredients: the Lipschitz regularity of periodic solutions for (mixed) integro-differential equations and the strong maximum principle.

Références

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