A Posteriori Error Estimation for the DDFV Discretization of Nonlinear Elliptic Equations in Two Dimensions

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Let Ω be an open bounded polygonal subset of $\mathbb{R}^2$, $\Gamma = \partial\Omega$, f be a given function from Ω to $\mathbb{R}$ and H a given function. We consider an approximate solution of the following nonlinear equation:

$$\begin{cases} -\mathrm{div}(H(\hat{u})\nabla\hat{u})(x) & = f(x), \ x \in \Omega \\ \hat{u}(x) & = 0, \quad x \in \Gamma \end{cases}$$

where the functions H and f satisfy several conditions in order that this equation a has unique solution, we refer [4], [1].
Our purpose is to derive an a posteriori error estimation for this nonlinear diffusion equation. We use the fixed point method to linearize the nonlinear equation and discretize linear equation by DDFV scheme (see [5]) on trianglar meshes. From the discrete solution, we reconstruct an approximate function like in [3] and we use [6] to obtain an estimation of the difference between the exact solution and this approximate function, this estimation includes 2 terms: discretization and linearization estimators. Hence the iterative linearization can be stopped whenever the linearization estimator drops below the fraction of the discretization estimator. This lead to computional savings, see more in [2]. Numerical tests are performed with several functions H and a singular solution.

Références

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