Multilayer approximation of the Navier-Stokes system. Variable density and coupling

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Mots-clés : Navier-Stokes equations, Free surface stratified flows, Multilayer system, Kinetic scheme, Droop model, Raceway, Hydrodynamics and biology coupling.

The water density stratification due to the heterogeneous vertical distribution of temperature, salinity,... leads to important effects in geophysical flows. Although the Navier-Stokes equations give a complete description of hydrodynamics, they are computationnaly demanding. Thus, the shallow water assumption is often invoked when considering shallow depth flows and an averaged description is often used.
However for complex flows, we can neither restrict ourselves to a model that assumes uniformity along the vertical dimension nor with a superposition of shallow water models that would prevent exchanges of fluid between the layers. Therefore we derive a multilayer Saint-Venant model that allows mass exchanges, with variable density [4, 5]. This derivation is obtained by a finite element Galerkin type discretization of the Navier-Stokes sytem along the vertical direction.
Several applications involving complex flows and their couplings are provided. First of all we concentrate on algae growth in aquaculture for biofuel production. In this context, the coupling between hydrodynamics and biology has mostly been studied – with simple hydrodynamics models – in photobioreactors [2] that are narrow closed tubes in which a turbulent flow transports the algae. We are rather interested in situations where the algae culture takes place in small circular ponds called raceways, used for the intensive outside culture of algae. As far as we know, such studies have only been carried out in simplified situations [3] that are not fully adapted to the problem we address. We propose to use our variable density multilayer system for the hydrodynamics part, while we derive another multilayer system for the biological part, starting from an improved Droop model [1] that includes light effects on the algae growth. For this problem coupling hydrodynamics and biology, 2d and 3d simulations illustrate our work and pave the way for further optimization of the aquaculture.
Second of all, we present simulations of the haline stratification in the Gibraltar strait. The results are confronted with experimental data.
Finally, we investigate couplings between flows and structures and give numerical results dealing with erosion problems.

Références

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