Convection–diffusion equation in unbounded cylinders and related homogenization problems

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The first part of the talk focuses on the study of the behaviour at infinity of solutions to a second order elliptic equation with first order terms stated in a half-cylinder. The coefficients of the equation are assumed to be measurable and bounded; Neumann boundary condition is imposed on the lateral boundary of the cylinder, while on the base we assign the Dirichlet boundary condition. Under the assumption that the coefficients of the equation stabilize to a periodic regime exponentially, and the functions on the right-hand side decay sufficiently fast at infinity, we prove the existence and the uniqueness of a bounded solution and its stabilization to a constant at the exponential rate. Also we provide a necessary and sufficient condition for the uniqueness of a bounded solution.

The second part of the talk is devoted to the study of a convection-diffusion operator in an infinite cylinder being a union of two nonintersecting half-cylinders with a junction at the origin. The coefficients of the equation are supposed to be periodic in each of these cylinders, and the Neumann boundary condition is imposed on the lateral boundary of the cylinder. The existence of a bounded solution and its qualitative properties are discussed.

As an application of the obtained results we consider the homogenization problem of a stationary convection-diffusion equation in a thin cylinder being a union of two nonintersecting rods with a junction at the origin. It is assumed that each of these cylinders has a periodic microstructure, and that the microstructure period is of the same order as the cylinder diameter. Under some natural assumptions on the data we construct and justify the asymptotic expansion of a solution which consists of the interior expansion and the boundary layer correctors, arising both in the vicinity of the rod ends and the vicinity of the junction.