Convergence de schémas volumes finis pour les lois de conservation hyperboliques posées sur un espace-temps

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From the numerical analysis standpoint, it is challenging to analyze discretization schemes that are consistent with the geometry of a given manifold. Following earlier works by LeFloch and his collaborators, we consider here nonlinear hyperbolic conservation laws, posed on a differential (n + 1)-manifold with boundary referred to as a spacetime, and in which the "flux" is defined as a flux field of n-forms depending on a parameter (the unknown variable). We introduce a formulation of the initial and boundary value problem which is geometric in nature and is more natural than the vector field approach recently developed for Riemannian manifolds. Our main assumption is a global hyperbolicity condition, which provides a global time-orientation as is standard in Lorentzian geometry and general relativity. Assuming the existence of a foliation by compact slices, we establish the existence of a semi-group of entropy solutions. Given any two hypersurfaces with one lying in the future of the other, we establish a contraction-like property which compares two entropy solutions, in a (geometrically-natural) distance equivalent to the L1 distance. In the proofs, we rely on a new version of the finite volume method, which only requires the given n-volume form structure on the (n + 1)-manifold and involves the total flux across faces of the elements of the triangulations, only.

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