3D Scale-space for Point Sets

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Scale spaces invented by Witkin and Koenderink ([2]) are efficient ways of processing 2D images. The processing task is performed after several filtering steps and the result is back propagated to the finest scale. A filtering step consists in applying the heat equation to the image pixels. We propose to extend this notion to the 3D case: the 3D equivalent to the heat equation is the mean curvature motion ([1]). Our input data are points acquired on a real object surface by a laser scanner. Most 3D processing methods start by building a mesh and then compute point curvatures by using the connectivity information. But in case of irregular sampling, this connectivity information can lead to irrelevant processing. On the contrary, our methods succeed in processing point samples previously to the estimation of any topological or differential structure and therefore preserve mathematical consistency. The novelties are that our new method implements the mean curvature motion on an irregularly sampled raw data set point previously to any triangulation. A consistency theorem is also proven for the new scheme. Finally, this scheme permits a backward tracking of the point evolution to the original raw data set. In consequence, it allows to reliably find the topology of a raw data point set and to triangulate it, thus preserving texture and fine details

Mathematical results. To approximate the mean curvature motion, we define a simple projection operator: each point is projected onto its local regression plane. Then we can show that this operator approximates one step of the mean curvature motion $\frac{\partial p}{\partial t} = H\vec{n}$ (where $\vec{n}$ is the point normal and H its curvature). This operator also yields a direct algorithm to compute the mean curvature. This scale space defines a framework which allows us to deal with most 3D point clouds problems.

Application to the mesh reconstruction problem. We use the scale space framework to build a mesh from a set of point samples with the special constraint that vertices must be a subset of the original points. Instead of directly meshing the samples which would lead to singularities or smoothing (depending on the method used), we iterate our projection filter and then use a simple triangulation method. Since the point set is smoothed, the triangulation will be singularity free. We then back project the result to the finest scale and get a texture preserving mesh. Results can be seen on figure 1.

Figure 1: Initial raw point set; Point set curvature; Picture of the real object; Mesh obtained at coarse scale; Mesh back projected to a finer scale

A comparison between details of the back projected mesh and of the mesh we obtained by a direct meshing of the samples, reveals that the back projected mesh preserves well the textures and small object asperities.

References

[1] Fred Almgren, Jean E. Taylor, and Lihe Wang. Curvature-driven flows: a variational approach. *SIAM J. Control Optim.*, 31(2):387–438, 1993.

[2] Andrew P. Witkin. Scale-space filtering. In *8th Int. Joint Conf. Artificial Intelligence*, volume 2, pages 1019–1022, Karlsruhe, August 1983.

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