From Level Lines Shortening to Mean Curvature Image Motion

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Our purpose is to set forth a new image processing algorithm simulating a contrast invariant evolution of an image via the mean curvature motion. We show that *Curve Shortening and Mean Curvature Motion are equivalent*. We give an explicit method to construct a viscosity solution for the scalar mean curvature motion, provided we are given the curve shortening evolutions of the level lines of the initial data. Applications to *accurate computation of the curvature* are presented.

Background. The geometric information of an image relies on the topological boundaries of the connected components of the level sets, i.e. the level lines. This topographic map gives a complete representation of an image and has two advantages: it is independent of the gray scale and it is embedded in a tree structure. Monasse and Guichard developed in [3] a Fast Level Set Transform, which associates to an image the inclusion tree of connected components of its level sets.

Strategy. The Level Lines Shortening Algorithm. To achieve full invariance and to retrieve the proper curvatures of an image we suggest to evolve independently all its level curves by curve shortening and then to reconstruct, at each step, a new image whose level lines are the evolutions of the original level lines. Consider the bilinear interpolation u_0 of a digital image: for almost every level λ, the corresponding isolevel set is a finite union of disjoint Jordan curves. We can therefore apply, separately to each level line, the geometric curve shortening. By Grayson's theorem [2], the solutions exist, they are smooth and the curves disappear in finite time. Due to the inclusion principle, we can reconstruct, at each time t, a function $u(\cdot, t)$ for which the level lines are the evolutions, at time t, of the original level lines. We call this function the level lines shortening (LLS) evolution of u_0, at time t. We prove that the level lines shortening evolution of a function is a weak solution of the scalar mean curvature motion, in the setting of viscosity solutions theory [1].

Accurate curvature computation. The most local recognition methods involve salient points of a shape, which are the points where the curvature is zero - inflection points - and points of extreme curvature - the corners of the shape. Thanks to the geometric properties of the curve shortening evolution and to the subpixel implementation of the LLS algorithm, one is able to compute accurately these characteristic points.

Selected References

[1] L.C. Evans, J. Spruck, *Motion of level sets by mean curvature I*, J. Diff. Geom. 33 (1991), 635-681.

[2] M. Grayson, *The heat equation shrinks embedded plane curves to round points*, J. Diff. Geom. 26 (1987), 285-314.

[3] Pascal Monasse and Frédéric Guichard, *Fast computation of a contrast invariant image representation*, IEEE Trans. on Image Proc, 1998, vol. 9, 860-872.

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