Diffusion and Cascading Behavior in Random Networks

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(1) Diffusion Model
inspired from game theory
and statistical physics.

(2) Results
from a mathematical analysis.

(3) Adding Clustering
Joint work with Emilie Coupechoux
(0) Context

Crossing the Chasm
(Moore 1991)
(1) Diffusion Model

(2) Results

(3) Adding Clustering
(1) Coordination game...

- Both receive payoff $q$.
- Both receive payoff $1-q > q$.
- Both receive nothing.
(1)...on a network.

• Everybody start with ICQ.

• Total payoff = sum of the payoffs with each neighbor.

• A seed of nodes switches to (Blume 95, Morris 00)
(1) Threshold Model

- State of agent $i$ is represented by

\[ X_i = \begin{cases} 
0 & \text{if } \text{icq} \\
1 & \text{if } \text{talk} 
\end{cases} \]

- Switch from $\text{icq}$ to $\text{talk}$ if:

\[ \sum_{j \sim i} X_j \geq qd_i \]
(1) Model for the network?

$p = 0.04$

$p = 0.05$

$p = 0.08$

Statistical physics: bootstrap percolation.
(1) Model for the network?
(1) Random Graphs

• Random graphs with given degree sequence introduced by (Molloy and Reed, 95).

• Examples:
  – Erdős-Rényi graphs, $G(\eta, \lambda/n)$.
  – Graphs with power law degree distribution.

• We are interested in large population asymptotics.

• Average degree is $\lambda$.

• No clustering: $C=0$. 
(1) Diffusion Model

\[ q = \text{relative threshold} \]
\[ \lambda = \text{average degree} \]

(2) Results

(3) Adding Clustering
(1) Diffusion Model

\[ q = \text{relative threshold} \]
\[ \lambda = \text{average degree} \]

(2) Results

(3) Adding Clustering
(2) Contagion (Morris 00)

- Does there exist a finite groupe of players such that their action under best response dynamics spreads contagiously everywhere?
- Contagion threshold: $q_c = \text{largest } q \text{ for which contagious dynamics are possible.}$
- Example: interaction on the line

\[ q_c = \frac{1}{2} \]
(2) Another example: d-regular trees

\[ q_c = \frac{1}{d} \]
(2) Some experiments

Seed = one node, $\lambda=3$ and $q=0.24$ (source: the Technoverse blog)
(2) Some experiments

Seed = one node, $\lambda = 3$ and $1/q > 4$

(source: the Technoaverse blog)
(2) Some experiments

Seed = one node, $\lambda=3$ and $q=0.24$ (or $1/q>4$)
(source: the Technoverse blog)
(2) Contagion threshold

In accordance with (Watts 02)
(2) A new Phase Transition
(2) Pivotal players

- Giant component of players requiring only one neighbor to switch: $\text{deg} < 1/q$.

Tipping point: Diffusion like standard epidemic

Chasm: Pivotal players = Early adopters

Mean degree
(2) q above contagion threshold

- New parameter: size of the seed as a fraction of the total population $0 < \alpha < 1$.
- Monotone dynamic $\rightarrow$ only one final state.
(2) Minimal size of the seed, \( q > 1/4 \)

Tipping point: Connectivity helps

Mean degree

Chasm: Connectivity hurts
Connectivity helps the diffusion.

Size of the seed

Size of the contagion

(2) $q > 1/4$, low connectivity
Connectivity inhibits the global cascade, but once it occurs, it facilitates its diffusion.
(2) Equilibria for $q < q_c$

- Trivial equilibria: all A / all B
- Initial seed applies best-response, hence can switches back. If the dynamic converges, it is an equilibrium.

- Robustness of all A equilibrium?
- Initial seed = 2 pivotal neighbors
  
  -> pivotal equilibrium
(2) Strength of Equilibria for $q < q_c$

Mean number of trials to switch from all $A$ to pivotal equilibrium

In Contrast with (Montanari, Saberi 10)
Their results for $q \approx 1/2$
(2) Coexistence for $q < q_c$

Size giant component

Connected Players A

Players B

Coexistence
(1) Diffusion Model

(2) Results

(3) Adding Clustering

joint work with Emilie Coupechoux
(3) Simple model with tunable clustering

- **Clustering coefficient:**
  \[ C = \frac{3 \text{ number of triangles}}{\text{number of connected triples}} \]

- Adding cliques *(Trapman 07)*
(3) Pivotal players are the same!
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(3) Contagion threshold with clustering

Clustering inhibits contagion

Clustering helps contagion

Global cascades

No cascade
(3) Low connectivity: clustering hurts contagion
(3) High connectivity: clustering helps contagion

Contagion threshold
(3) Intermediate regime: non-monotone effect of clustering

Contagion threshold

Mean Degree = 3.19
(3) Effect of clustering on the cascade size

Fraction of pivotal players and size of the cascade
(3) Another model

Separate communities (Trapman 07)  Overlapping communities (Newman 03)
(3) Local Structure
(3) Diffusion with overlapping communities
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Conclusion

• Simple tractable model:
  – Threshold rule
  – Random network: heterogeneity of population
  – Tunable degree/clustering

• 1 notion: Pivotal Players and 2 regimes:
  – Low connectivity: tipping point / clustering hurts
  – High connectivity: chasm / clustering helps activation

• More results in the papers:
  – Heterogeneity of thresholds, active/inactive links, rigorous proof.
Merci!

Available at http://www.di.ens.fr/~lelarge