Isogeometric segmentation of boundary-represented solids*

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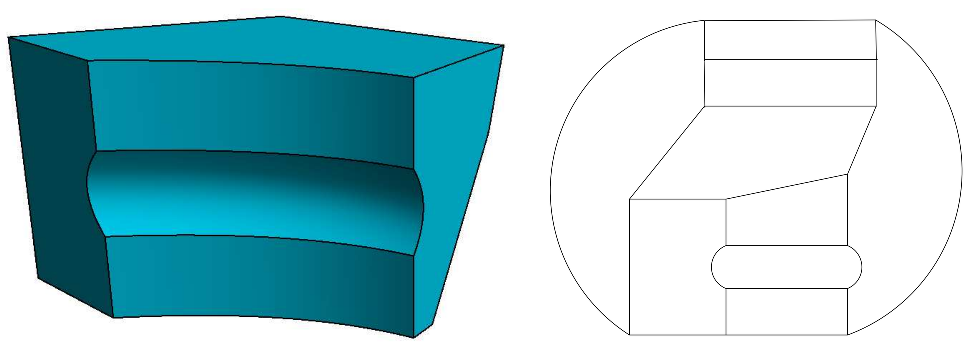
Introduction and preliminaries

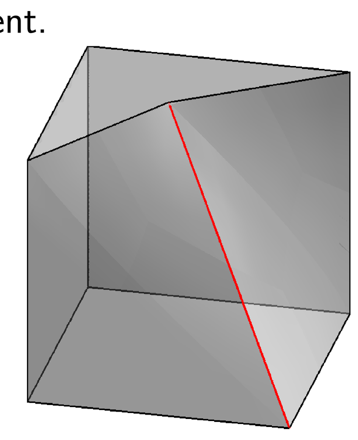
We address the challenge of segmenting a boundary-represented solid into a small number of topological hexahedra suitable for isogeometric analysis (IGA). IGA can be used to conduct simulations on a complex object if it is represented as a collection of volume parameterized pieces, e.g. trivariate NURBS volumes. However, in CAD systems it is typical to represent an object only by its boundary. The *isogeometric segmentation problem* is to convert a CAD model into an IGA suitable model by segmenting a boundary represented solid into a small number of (possibly distorted) topological hexahedra, allowing T-joints. The input for our algorithm is a boundary representation of a solid consisting of trimmed spline surfaces. Our approach [1, 2] focuses on the *edge graph* formed from the vertices, edges and faces of the solid.



A boundary-represented solid and a planar representation of its edge graph (see [1]).

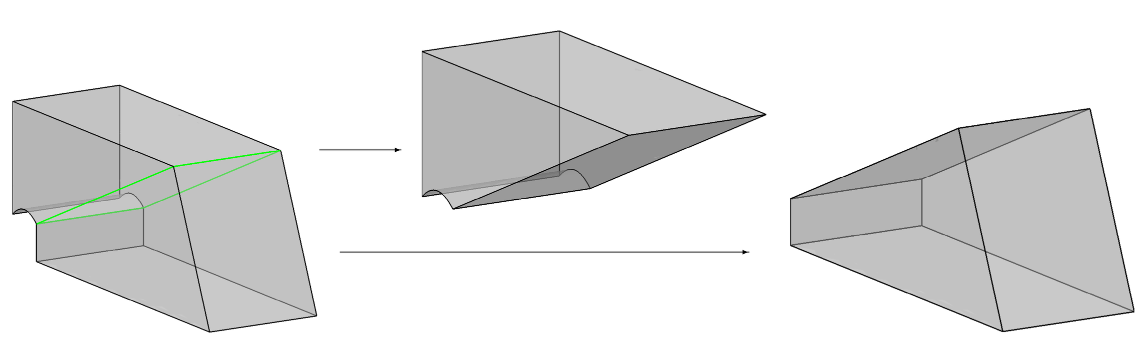
3-vertex-connectivity
The edge graph is *3-vertex-connected* if it is connected and has at least 4 vertices, and any 2 vertices can be deleted without disconnecting the graph. Steinitz's theorem says that a 3-vertex-connected planar graph can be realized as the edge graph of a convex polyhedron.

An edge $\mathbf{e}$ is *convex* if, for every point $\mathbf{u} \in \mathbf{e}$, the interior angle between the two incident faces to $\mathbf{e}$ at $\mathbf{u}$ is less than π. In order to produce topological hexahedra that can be parameterized in a nonsingular way, non-convex edges require special treatment.



A solid with a non-convex edge. The edge graph is isomorphic to that of a cube. However, the non-convex edge prevents C^1 volume parameterization by a cube. The segmentation algorithm eliminates non-convex edges as its first priority so that the resulting topological hexahedra have only convex edges.

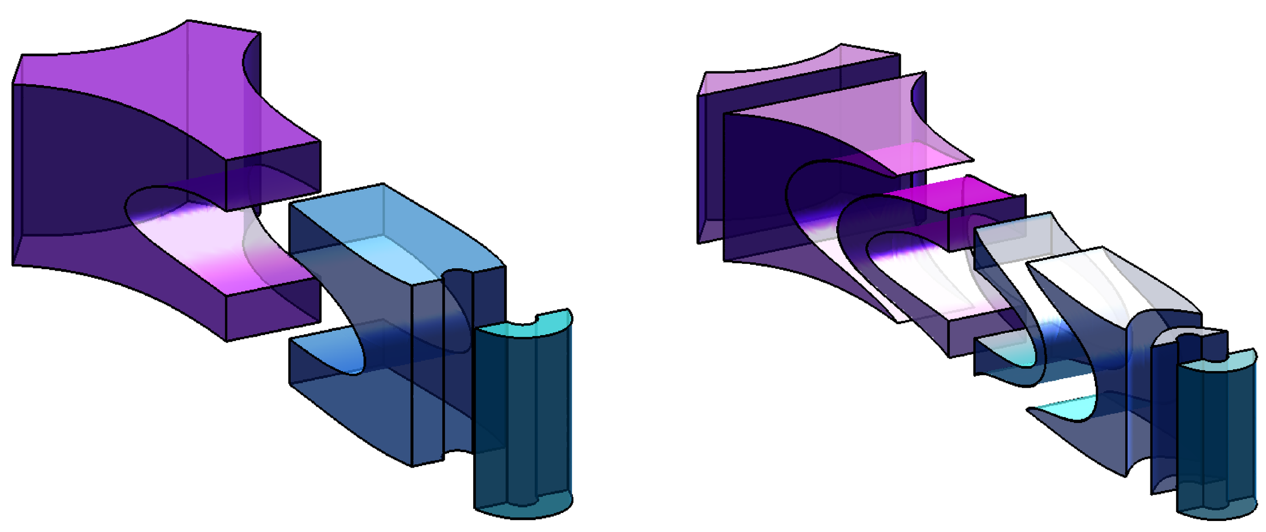
Summary of the algorithm



A model is segmented into topological hexahedra which can now be parameterized for IGA-based simulation.

Our approach to the isogeometric analysis problem is the following procedure.

- First, decompose a solid into contractible solid pieces, then create a sufficiently connected edge graph for each piece (not discussed in this poster).
- Search for a *cutting loop* which segments the edge graph into two simpler graphs.
- Use the cutting loop to construct a *cutting surface* which segments the solid.
- Repeat until all the pieces are reduced to specific base solids which have predefined segmentations into topological hexahedra.
- Create volume parameterizations of the hexahedra (not discussed).

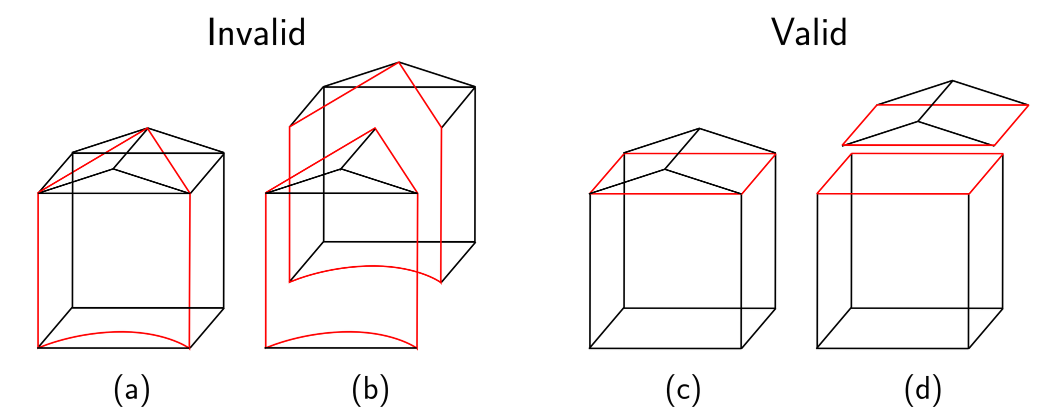


Left: a boundary-represented model, preprocessed into contractible pieces by cutting along planes. Right: the algorithm segments the contractible pieces into topological hexahedra.

Cutting loops and the main result

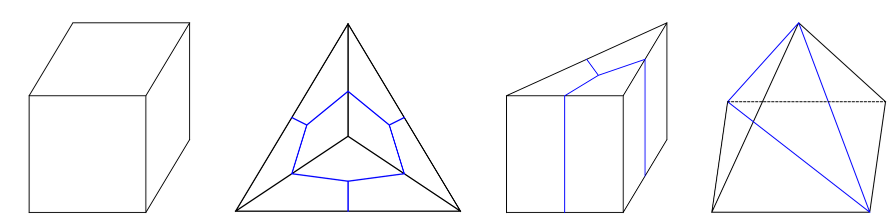
Cutting loops
Assume the solid is contractible and the edge graph is 3-vertex-connected.
A *cutting loop* is a cycle consisting of
- edges of the edge graph;
- *auxiliary edges* which can be created between any two vertices on the same face.

A cutting loop is *valid* if it can be used as the boundary of a surface which cuts the solid into two pieces whose edge graphs are 3-vertex-connected.



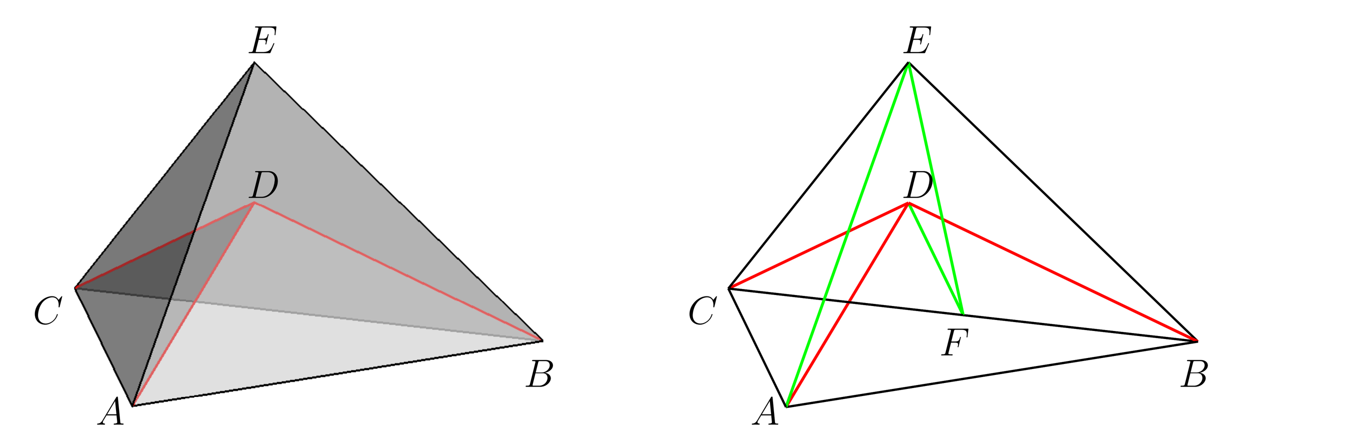
The cutting loop in (a) is not valid, as it results in two solids with graphs shown in (b), one of which is not 3-vertex-connected. The cutting loop in (c) is valid and results in solids with edge graphs shown in (d).

Base solids
We repeatedly use cutting loops to segment a solid into *base solids*: a collection of simple solids with predefined segmentations into topological hexahedra.



Base solids: topological hexahedra, tetrahedra, prisms and pyramids (see [1]).

For solids with only convex edges, validity of a cutting loop is essentially a graph-theoretic condition. When non-convex edges exist, additional geometric criteria arise. These can prevent the existence of a valid cutting loop that uses only the vertices of the solid's edge graph. This problem can be handled by inserting additional *auxiliary vertices* along the edges of the graph.

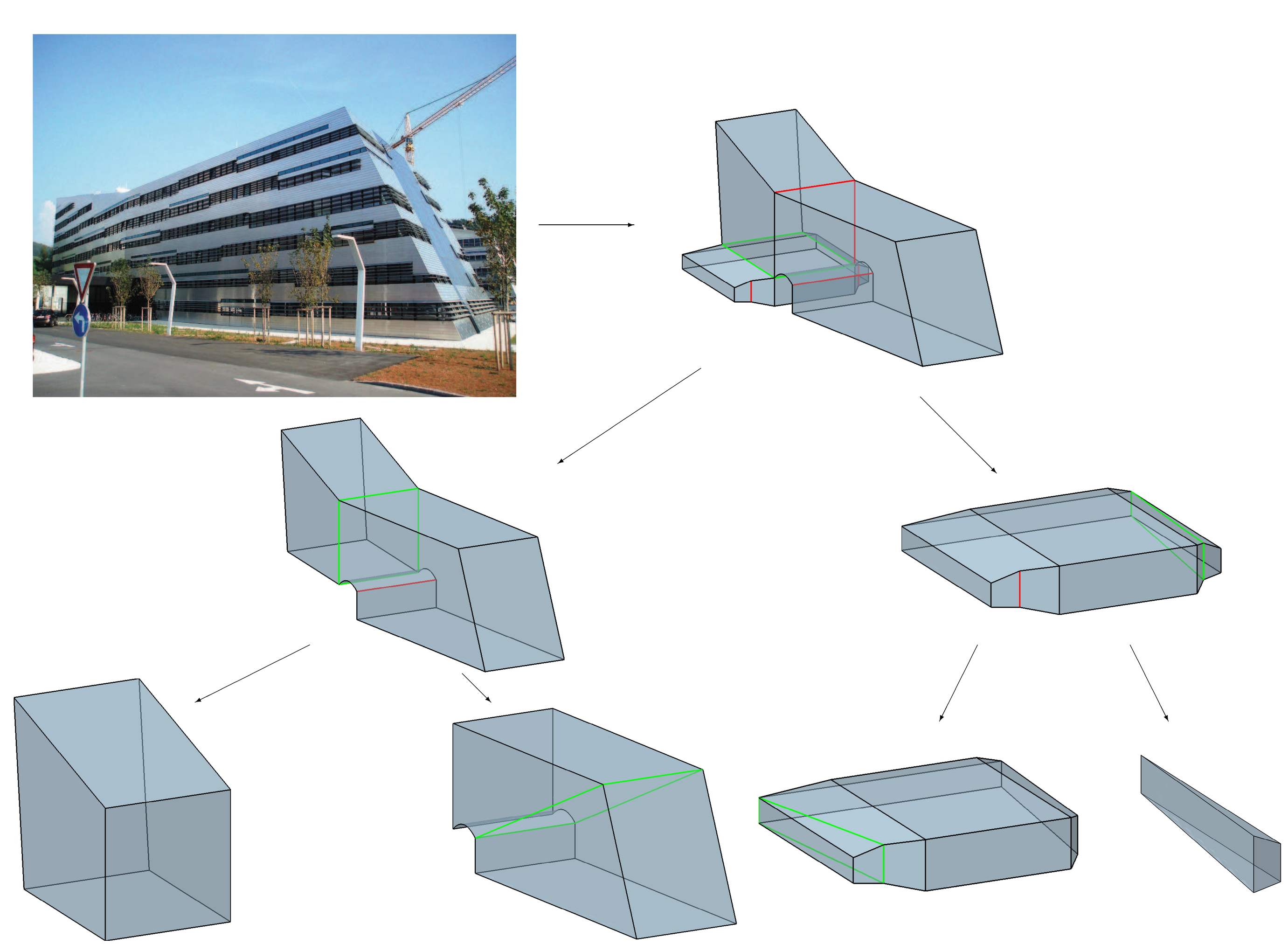


Left: a solid with non-convex edges AD, BD and CD. There is no valid cutting loop which cuts any of the non-convex edges and only uses existing vertices. Right: an auxiliary vertex has been added to the solid, and now the edge AD can be cut (see [2]).

Main Result
Assume a solid is contractible and has a 3-vertex-connected edge graph. Using cutting loops, possibly with auxiliary vertices, it can be segmented into base solids with only convex edges.

Typically, many valid cutting loops exist. The algorithm chooses one using a cost function which is a combination of combinatorial (based on the edge graph) and geometric criteria. These include the number of edges in the cutting loop and in the new faces resulting from adding auxiliary edges and a measurement of the planarity of the cutting loop.

Additional example



Several steps of the algorithm applied to a simplified building model (see [2]).

Geometric problems

The implementation of a robust construction of auxiliary edges is an ongoing development. This amounts to producing a spline curve between given points in a (planar) trimmed domain. The curve must not intersect itself or the boundary of the trimmed domain. This is achieved by optimizing a penalty function that tends to infinity as the curve tends towards intersecting itself or the boundary. Similar optimization-based approaches could be applied for the construction of a cutting surface from a given cutting loop; the higher dimension increases the computational difficulty. Another ongoing geometric development is the construction of spline-based volume parameterizations of the resulting topological hexahedra.

References

[1] B. Jüttler, M. Kapl, D.-M. Nguyen, Q. Pan, and M. Pauley. Isogeometric segmentation. Part I: Decomposing contractible solids without non-convex edges. Submitted.

[2] D.-M. Nguyen, M. Pauley, and B. Jüttler. Isogeometric segmentation. Part II: On the segmentability of contractible solids with nonconvex edges. *to appear in Graphical Models*, 2014.

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