Least Squares Residual Minimization for Friedrichs Model and Cauchy Riemann equation

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Given a set of partial differential equations and a triangulation of the domain of interest, we can define residuals by integrating the equations over each element. Using piecewise linear interpolation functions, the solution is approximated by minimizing the residuals not only with respect to the nodal values but also with respect to the nodal coordinates. Applying this scheme to the Friedrichs Model, we only obtain a small increase of the error in comparison to the error of the solution on a fixed grid. This is due to the fact that the solution of the minimizing problem is not unique. Hence, we have to include an additional quantity into the functional to be minimized in order to impose some extra condition on the solution. For the Friedrichs Model the equal distribution of the difference of the Hermite interpolation and Lagrange interpolation polynomials turns out to be the right term.

The goal of the project was to generalize this idea for the 2D Cauchy Riemann equations. Minimizing only the residuals with respect to the nodal values and coordinates results in a promising mesh movement, meaning the nodes are moving towards the singularities but the error decreases only slightly. In order to be able to include standard error estimates to be minimized, we showed that in the case of the Cauchy Riemann equations the Least Squares method is equivalent to the application of the finite element method to two Laplace equations, one with Dirichlet boundary conditions and one with Neumann boundary conditions. But to achieve a good performance in minimizing the functional, we need dimensional consistency which turned out to be a problem for the Cauchy Riemann equation.