Domain decomposition method
combined with a linear multigrid algorithm

L. Fournier, S. Lanteri, Y. Maday

In order to gain efficiency in parallel linear multigrid methods different strategy have been studied since the 80's. Here we propose to keep the usual multiplicative multigrid algorithm combined with a parallel smoother.

The natural parallelism introduced by the use of domain decomposition method is very attractive in our context. Smoothing properties of a family of additive Schwarz method is the main aspect of this project. We can find in the literature different kinds of additive Schwarz methods. In addition to the classical continuous formulation, there also exist several algebric forms of the method. One of the most recent, called the Restricted Additive Schwarz method [CS97] allows a significant decrease of communication with a better convergence rate. These algebric methods are usualy used as preconditioners of a Krylov method, but we are here interested in its smoothing property (this kind of analysis can be found in [Wes91]).

A first study with a continuous model equation has shown a wrong treatment of the correction in the overlapping region. This observation seems to be an indicator of the better comportment of the RAS method. The problem is apparently due to the algebric formulation and the use of simple prolongation operators. In order to complete our understanding of the method, we have reformulated the study on the Poisson equation done by Hackbusch [Hac85]. This has allowed us to obtain the eigenvalues of the method and more precisely for the algebric formulations.

This work should be extend in next months to the advection diffusion model equation. But in order to have a good behavior with the multigrid method, we are going to estimate the effect of an inexact local solver (this one should only be a smoother). This study should be concluded with numerical experiments.

References

[CS97] Cai X. C. et Sarkis M. – A restricted additive Schwarz preconditioner for general sparse linear systems, *Tech Report CU-CS-843-97, Dept. of Comp. Sci., Univ. of Colorado at Boulder*, 1997. (SIAM J. Numer. Anal., submitted)

[Hac85] Hackbusch W. – *multi-grid methods and applications.* – Springer Verlag, 1985, *Springer series in Computationl Mathematics*, volume 4.

[Wes91] Wesseling P. – *An introduction to multigrid methods.* – John Wiley and sons, 1991.