Adaptive Multiscale Schemes for Hyperbolic Conservation Laws

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During the last decade, finite volume schemes have been proven to be very robust and reliable approximation schemes for conservation laws as has been verified by numerous applications. However, the construction of *robust and efficient* schemes is not yet finished. More advanced approaches are based on the *heterogenity* of the flow field, i.e., several physical effects occuring on different spatial and temporal scales have to be resolved appropriately. In order to capture these effects by a numerical scheme, different discretization lengthes are required where the highest resolution is determined by the smallest scale. Thus, storage capacities or computational time might be exceeded, especially for multi–dimensional flow problems. Therefore, algorithms have to be developed where the computational effort and memory capacities are proportional to the complexity of the underlying problem. This means that the finest resolution has to be chosen only in regions where small–scale effects are present while elsewhere a much coarser discretization is sufficient. In view of realistic simulations of multi–dimensional flow problems around complex geometries uniform discretizations are no longer adequate as they exceed presently accessible resources of modern computer systems. Therefore, discretizations have to be adaptive instead of uniform, in order to use efficiently the available resources.

In this project, we have been developing and investigating a new concept how to incorporate multiscale techniques into already existing finite volume schemes in order to achieve an adaptive scheme. Opposite to recent work in this field based on *hybrid* techniques (Harten 1995, Sjögreen 1995, Bihari/Harten 1997 and Gottschlich–Müller/Müller 1996–98), we now have improved Harten's original idea by incorporating *adaptivity*. To this end, we use *local* multiscale decompositions which correspond to *adaptive grids*. First test calculations have shown that the complexity of the resulting algorithm corresponds to the number of *significant* multiscale coefficients, i. e., those coefficients which are greater than a threshold value. In our applications, this number turns out to be tremendously smaller than the number of *all* coefficients with respect to the finest discretization level.

Our future work is devoted to time adaptivity. Presently, the temporal discretization length is significantly restricted by the CFL number because of a very small number of significant multiscale coefficients of the finest level. This constraint becomes even more serious if more levels are involved. In order to overcome this obstruction, we want to combine our concept with a *local time–stepping* strategy. Additionally, we will investigate the adaptive concept in the context of *implicit* finite volume schemes.