Collective motion of biofilms

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Collective organisms in nature are capable of a wide range of motility patterns to find food, reproduce or avoid predation. Remarkably, multicellular systems can also display collective movement to achieve similar goals. *Myxococcus xanthus*, a soil predatory bacterium, assembles multicellular biofilms to collectively predate on other microorganisms. During the hunting phase, the bacterial biofilm is composed with isolated bacteria and largers clusters of various sizes. In her PhD thesis [4] , S. Rombouts conducted experiments and analysis about various collective behaviours occurring during predation. One of the main outcome of her study is the evidence of synergistic effects when various motility apparatus are expressed within the cell population, with the consequence of increasing the rate of predation.

The goal of the project is to explore the fascinating biological issues raised in [4] from a mathematical point of view.

In this work, we will mainly focus on the interplay between isolated cells and clusters to drive the propagation of *Myxococcus xanthus* when it penetrates the prey population. We describe the spatial dynamics of the predator population in a one dimensional space ($x \in \mathbb{R}$). We propose to distinguish between two types of bacteria: isolated bacteria with density $\rho_1(t,x)$ and clusters of two bacteria, say, with density $\rho_2(t,x)$. The two mechanisms we take into account to model the experiments are the growth of isolated bacteria with rate $\alpha > 0$, diffusion of the two species with respective rates θ_1 and θ_2, fragmentation of clusters into single cells occuring with rate τ_2 and coagulation of two single cells into one cluster with rate τ_1. The system is written as

$$\begin{cases} \dfrac{\partial \rho_1}{\partial t} - \theta_1 \Delta \rho_1 = -\tau_1 \rho_1^2 + 2\tau_2 \rho_2 + \alpha \rho_1, \\ \dfrac{\partial \rho_2}{\partial t} - \theta_2 \Delta \rho_2 = \dfrac{\tau_1}{2} \rho_1^2 - \tau_2 \rho_2. \end{cases} \tag{1}$$

Following [4], the diffusion of bacterial clusters is higher than the diffusion of single cells, i.e. we assume

$$\theta_2 > \theta_1. \tag{2}$$

Well-posedness for such reaction-diffusion systems have been widely studied (see for instance [3]).

In the case $\tau_2 = 0$, the system is decoupled and the first equation is identified as the Fisher-KPP equation [2] for which is is known that travelling waves propagate with the minimal speed $2\sqrt{\theta_1 \alpha}$.

The open problem that we want to investigate in this project is how the spreading of clusters of size 2 affect this result. Does system (1) exhibit travelling waves? If so, how the speed of propagation is impacted by the presence of clusters? The questions are motivated by the observation of travelling waves whose velocity has not yet been identified experimentally.

1. First step will be to conduct numerical experiments to explore the asymptotic behaviour of (1)

2. Second step will be to extend (1) to a large number of clusters (see, for instance, [1]), and to address modelling questions

3. Third (and related) step will consist in drawing some conclusions in comparison with the experimental data from [4].

4. In parallel, any conclusions derived from the numerical exploration may be subject to heuristical and theoretical investigation.

References

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