Measuring prediction uncertainty for approximate solutions to PDEs, and application to the Covid-19 pandemic

Olga Mula, Angelina Roche, and Robin J. Ryder

Centre de Recherche en Mathématiques de la Décision, Université Paris-Dauphine, Paris Sciences et Lettres

{mula,roche,ryder}@ceremade.dauphine.fr

CEMRACS, Summer 2021

The aim of this project is to extend the work by [BBMM21], by measuring the uncertainty in short-term predictions of a Partial Differential Equations model.

Sophisticated epidemiological models can include a large number of unknown parameters. This is true in particular of compartmental models such as extensions of the well-known Susceptible-Infectious-Recovered model, which can be represented by a system of PDEs with up to 20 scalar parameters. These models are of interest as they are grounded in expert knowledge, but fitting them to data is intractable either for computational reasons or because detailed enough data are unavailable.

[BBMM21] propose a model reduction technique, which projects the model with a large number of (constant in time) parameters onto a space with a small number of time-dependent parameters. A point prediction is then computed, based on a large number of simulated trajectories. We propose to adapt ideas from the Approximate Bayesian Computation literature [MPRR12] to leverage prediction uncertainty based on those same simulated trajectories. ABC has already been used for simpler PDE-based models in epidemiology by [MR19].

Specifically, we observe a marginal trajectory $y_{obs}(t)$ for $t \in [0, T]$; here, y_{obs} corresponds to the number of patients infected by and deceased of Covid-19 during the first lockdown in France. A set of parameters $\boldsymbol{p}$ leads to a trajectory $\tilde{y}(t; \boldsymbol{p})$ and we are aiming to minimize the quantity

$$\mathcal{J}(\boldsymbol{p}) = \int_0^T |\tilde{y}(t; \boldsymbol{p}) - y_{obs}(t)|^2 \, dt.$$

To solve this minimization problem approximately, [BBMM21] generate a large number of virtual scenarios, then restrict $\boldsymbol{p}$ to be a linear combination of a small number of carefully chosen parameter instances. The resulting estimate $\hat{\boldsymbol{p}} = \arg\min \mathcal{J}$ is shown to have interesting properties. Prediction is then easily available by extending the trajectory given by $\boldsymbol{p}$ to some interval $[T, T + \tau]$.

In this project, we shall consider instead an ensemble of estimates defined by

$$B_\epsilon = \{\boldsymbol{p} : \mathcal{J}(\boldsymbol{p}) \leq \mathcal{J}(\hat{\boldsymbol{p}}) + \epsilon\}$$

for some small value of ϵ. This leads naturally to an ensemble of predictions over the interval $[T, T + \tau]$. We shall explore various strategies to describe and sample from B_ϵ, and choose an appropriate value for ϵ.

References

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