State estimation methods involving physical model corrections. Application to neutronics.

1 Goal of the project

In the field of inverse problems, one often combines available observations coming from sensors with physical PDE models in order to provide an approximation of the state of the physical system under consideration or certain outputs of interest. In general, even the best existing PDE models available to describe many physical systems present a model error, which is difficult and costly to estimate. This model error can however be large, and have a disastrous impact on the final quality of the reconstruction if not taken into account. In this projet, we propose to develop data-driven methods to learn how to introduce corrections/improvements to the reconstructions when the physical model is not perfect.

As a support for our tests, we propose to explore an example from the field of neutronics, which describes the behavior of neutrons in reactor cores. This type of system can be described via two models: the first, most accurate, is the so-called multigroup transport equation. The second model is a diffusion equation, less accurate by easier to implement and faster to compute. Assuming that the true system is given by the accurate transport model, the goal is to do state estimation using the diffusion model and incorporating data-driven corrections accounting for the fact that the model is inexact.

2 Organization details

- **Supervisors:** François Madiot (CEA), Olga Mula (Paris Dauphine, Inria), Tommaso Taddei (Inria)
- **Number of students:** 2

3 Transport and diffusion models for neutronics

An accurate model to describe the behavior of neutrons is the multigroup transport equation, which expresses the balance between the generation and disappearance of neutrons [1].

Let R be the domain of the core reactor. We consider the stationary case where the angular flux $\psi := (\psi^g)_{g\in[1,G]}$ depends on six variables, namely the position $r \in R$, the direction $\omega \in \mathbb{S}_2$ where $\mathbb{S}_2$ is the unit sphere and $g \in [1, G]$ the energy group.

The multigroup neutron transport equation writes

$$\text{Find } (\psi, \lambda) \text{ such that for all } g \in [1, G],$$
$$\begin{cases} L^g\psi^g(r,\omega) = H^g\psi(r,\omega) + \lambda F^g\psi(r,\omega) \quad \text{in } R \times \mathbb{S}_2, \\ \psi = 0 \quad \text{on } \{r \in \partial R, n(r)\cdot\omega < 0\}, \end{cases} \tag{1}$$

where

$$L^g\psi^g(r,\omega) = (\omega\cdot\nabla + \sigma_t^g(r))\psi^g(r,\omega) \text{ is the advection operator,}$$

$$H_g\psi(r,\omega) = \int_{\mathbb{S}_2}\sum_{g'=1}^{G}\sigma_s^{g'\to g}(r,\omega'\cdot\omega)\psi^{g'}(r,\omega')d\omega' \text{ is the scattering operator,}$$

$$F_g\psi(r,\omega) = \frac{\chi^g(r)}{4\pi}\sum_{g'=1}^{G}(\nu\sigma_f)^{g'}(r)\phi^{g'}(r) \text{ is the fission operator,}$$

$$\phi^g(r) = \int_{\mathbb{S}_2}\psi^g(r,\omega)d\omega \text{ is the scalar flux.}$$

In the listed terms, $\sigma_t^g(r)$ denotes the total cross-section and $\sigma_s^{g'\to g}(r,\omega'\cdot\omega)$ is the scattering cross-section from energy group g' and direction ω' to energy group g and direction ω, $\sigma_f^g(r)$ is the fission cross-section, $\nu^g(r)$ is the average number of neutrons emitted per fission and $\chi^g(r)$ is the spectrum.

The transport model is often approximated at the reactor core scale by a diffusion model. The multigroup diffusion model writes

$$\text{Find } (\phi,\lambda) \text{ such that for all } g\in[1,G],$$
$$\begin{cases} -\operatorname{div}(D^g(r)\nabla\phi^g(r)) + \sigma_t^g(r)\phi^g(r) = H_0^g\phi(r) + \lambda F^g\phi(r) & \text{in } R,\\ D^g(r)\nabla\phi^g(r)\cdot n + \frac{1}{2}\phi^g(r) = 0 & \text{on } \partial R, \end{cases} \tag{2}$$

where

$$D^g(r) \text{ is the diffusion coefficient,}$$

$$H_0^g\phi(r) = \sum_{g'=1}^{G}\sigma_{s,0}^{g'\to g}(r)\phi^{g'}(r) \text{ is the scattering operator,}$$

$$F^g\phi(r) = \frac{\chi^g(r)}{4\pi}\sum_{g'=1}^{G}(\nu\sigma_f)^{g'}(r)\phi^{g'}(r) \text{ is the fission operator,}$$

and $\sigma_{s,0}^{g'\to g}$ is the isotropic scattering cross-section from energy group g' to energy group g.

References

[1] James J Duderstadt. *Nuclear reactor analysis.* Wiley, 1976.