A non-Newtonian rheology model applicable to complex flows

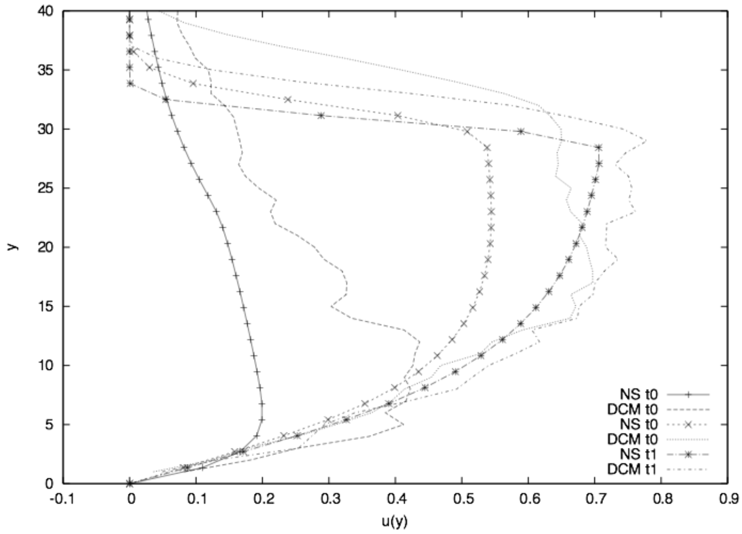
In a number of physical situations, Navier-Stokes equations can be used in flow modelling, even if the materials considered are not fluids in the intuitive sense of the term [4, 2]. This is the case, for example, for some granular flows (pyroclastic flows, avalanches) or for the lithosphere. The most complete models take into account, in one case, the detailed behaviour of the particles, and in the other case the complete mechanical and thermal behaviour of the system, in a three-dimensional context with a free surface. These systems are very complex to implement and very time consuming. In addition, some physical parameters are difficult to access.

In these contexts, it is interesting to develop simplified models, which are based both on scaling laws involving flow control parameters and on the rheology of the fluid, i.e. the modelling of its mechanical behaviour. This last point involves the use of non-Newtonian fluids, i.e. fluids where the relationship between stress and strain tensors is non-linear.

We will focus here on the particular case of a high viscosity contrast within a flow [3]. The figure shows numerical simulations of the temporal evolution of vertical velocity profiles during the collapse of a granular column. The dotted lines are obtained by a granular model, the markers by a Navier-Stokes calculation. It is not a question of working on the agreement between the two simulations, but of focusing on the Navier-Stokes profile, evacuating the almost horizontal upper part. The viscosity is read on the vertical variation of the velocity profile: near the ground (at the bottom of the figure), the profile is almost linear, which indicates a low viscosity.

The project consists of developing and testing a simplified model in two aspects, following a methodology proposed by Balmforth et al. [1] in a different physical context. The first is based on a thin film hypothesis, and requires the analysis of scaling factors in the equations. It allows by integration in the vertical direction to obtain averaged equations, and thus to gain a spatial dimension. Then, we try to implement a threshold rheology: the viscosity is constant in pieces, the change in value depending on the deformation of the fluid, one of the values being very small compared to the other. Initially, the work will be done in two spatial dimensions, with a final one-dimensional model. Dimension 3 will be study next.

Depending on the results obtained, the project may focus on analysis when the low viscosity tends towards 0, which can lead to a significant change in the model, or on the integration of this simplified model into a more general context, in geophysics for example.



References

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