Mathematical and Statistical Analysis of Data-Model Coupling

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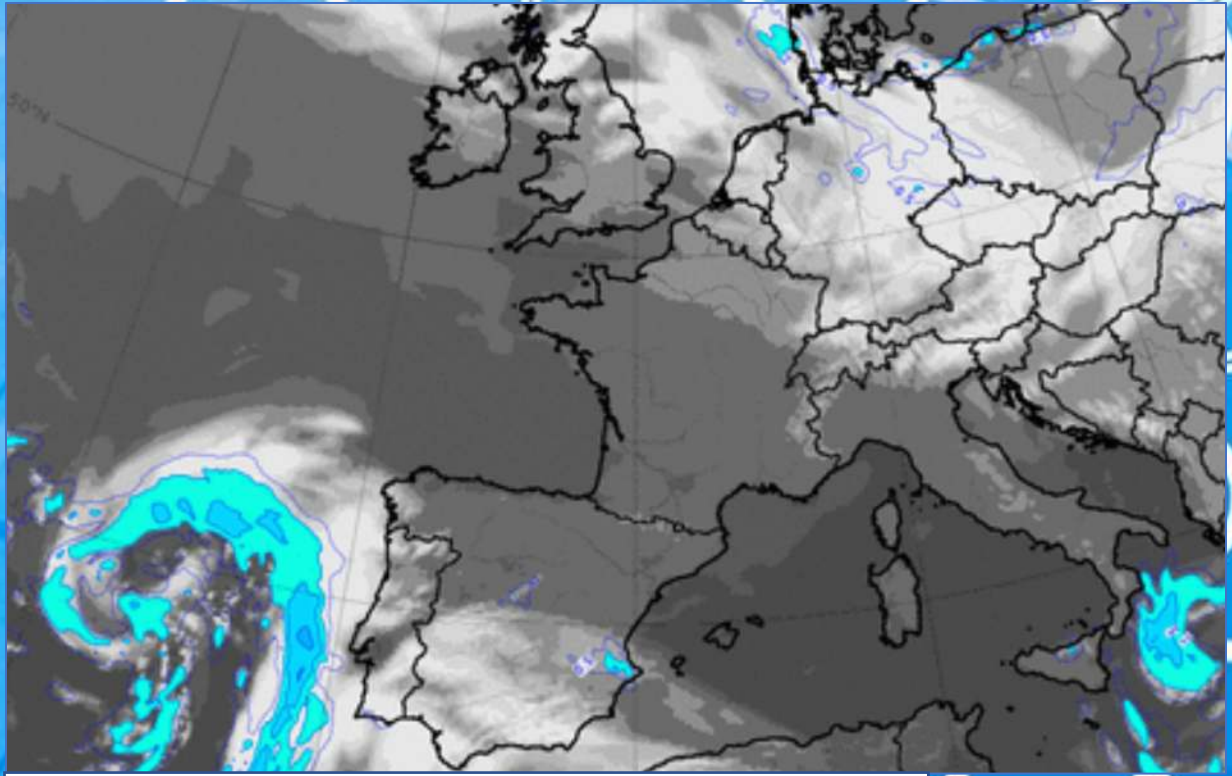
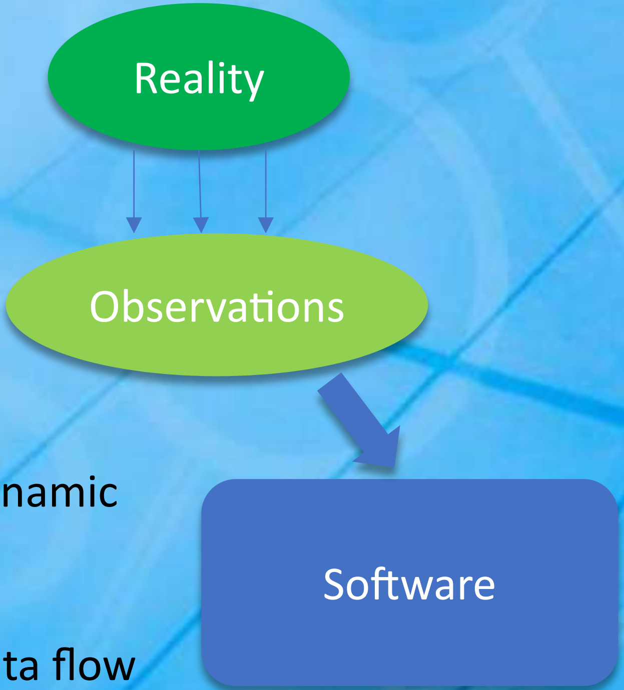
DATA ASSIMILATION

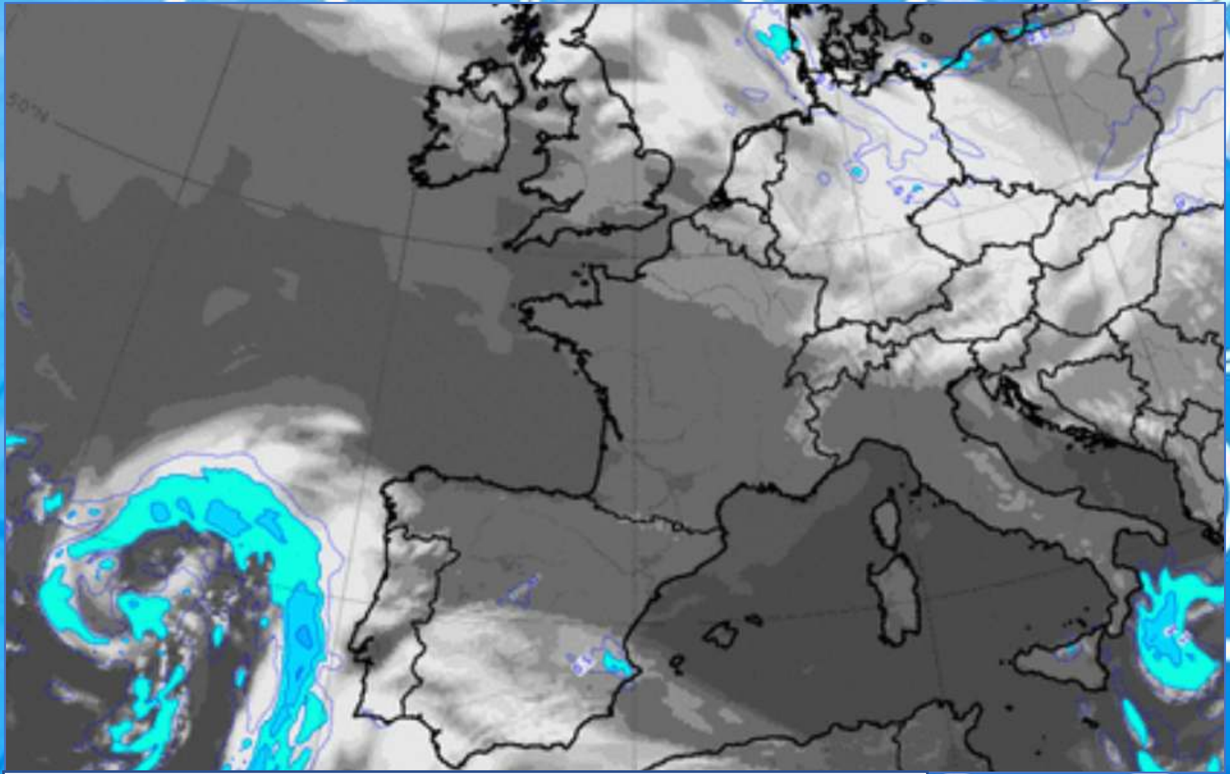


Satellite picture and expected rain (Arpege model) © Météo-France

DATA ASSIMILATION



- **Simulation** of the dynamic of the reality
- **Correction** by the data flow



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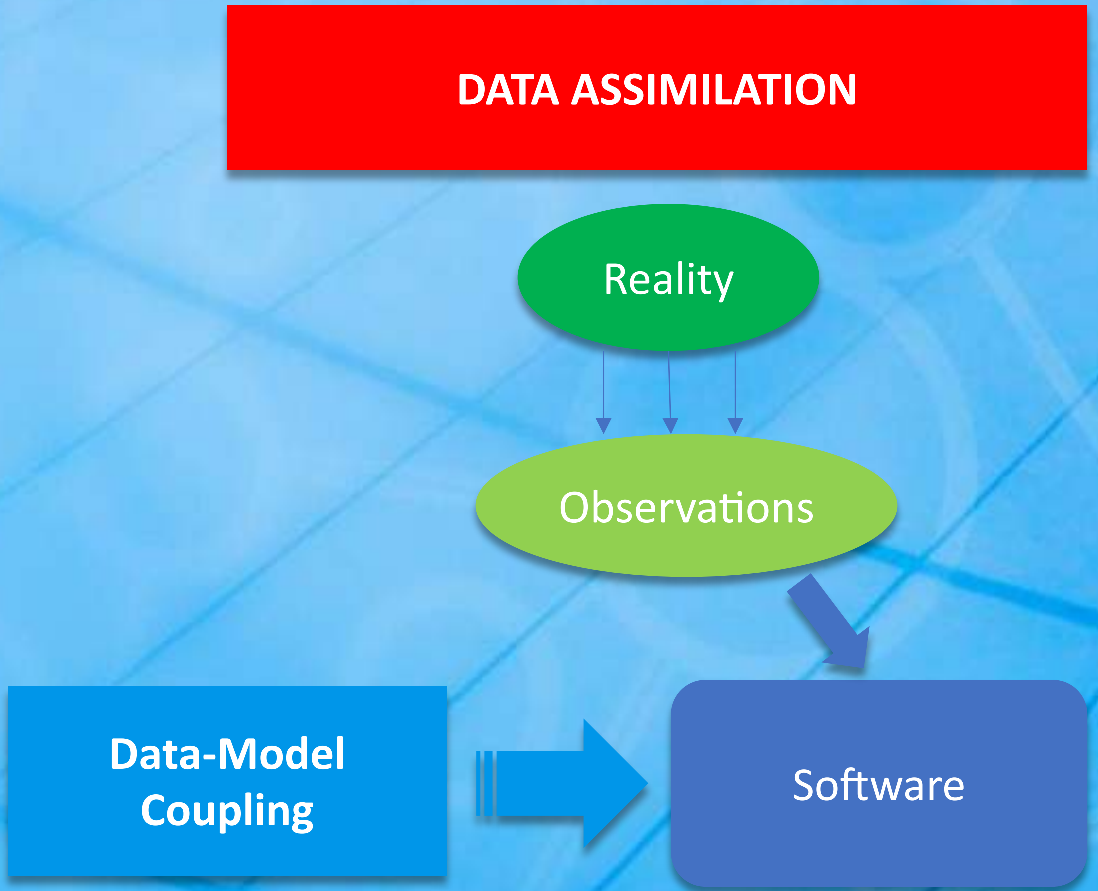
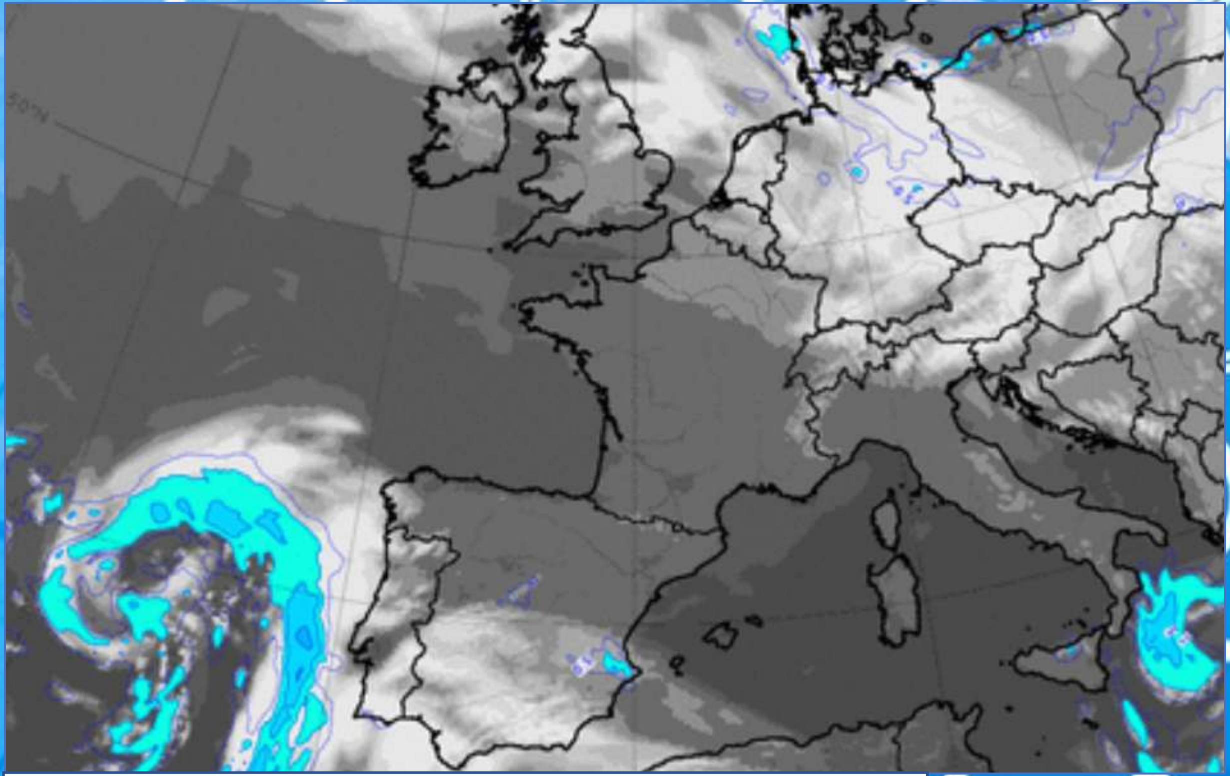


- Parameterized PDEs
- Database



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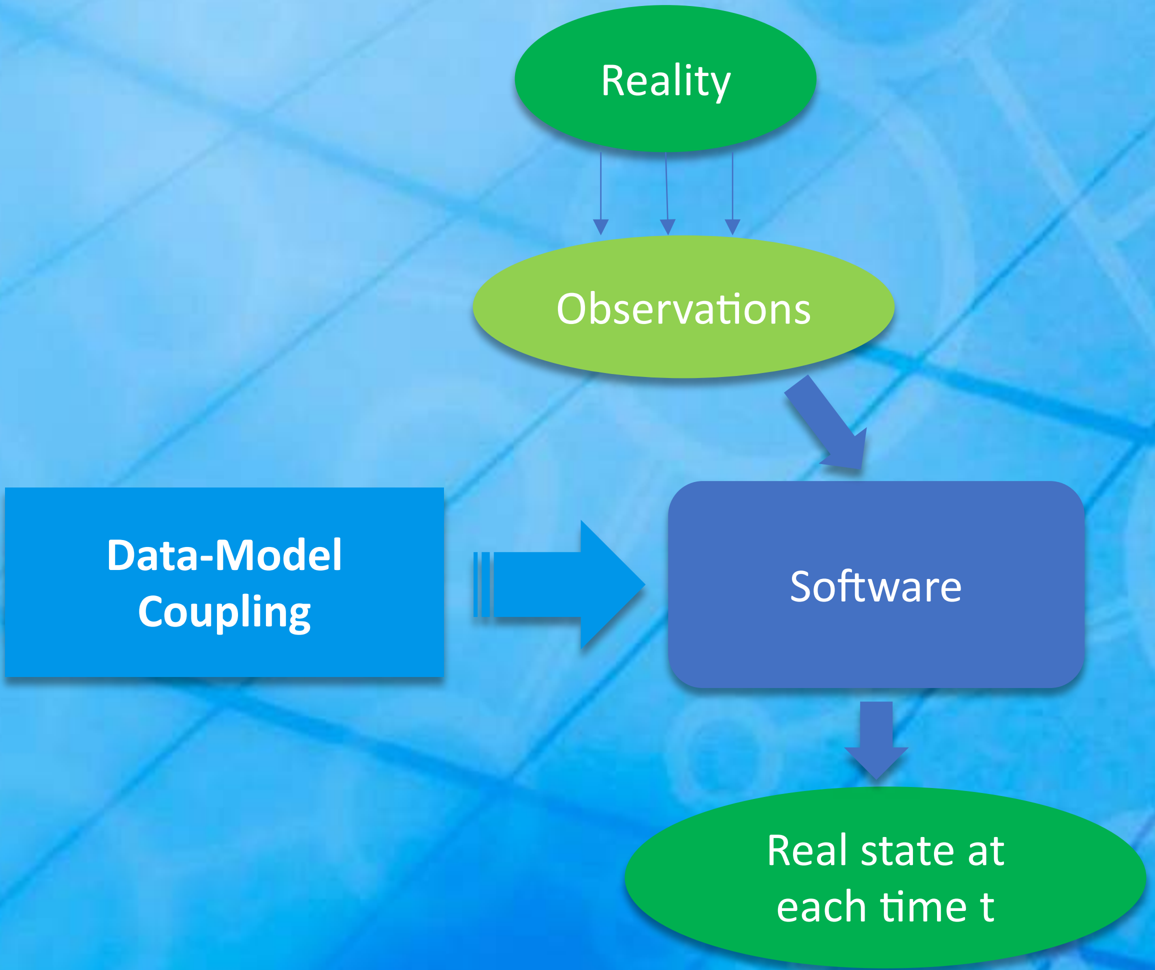
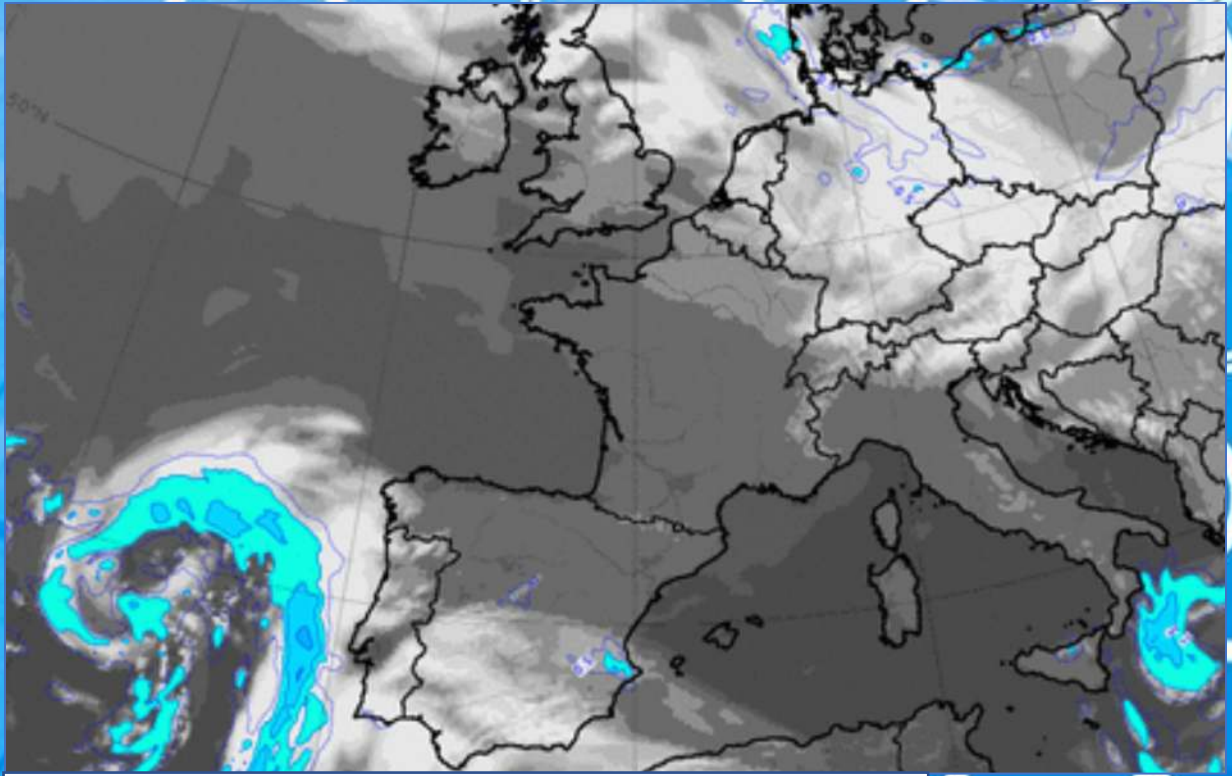
DATA ASSIMILATION





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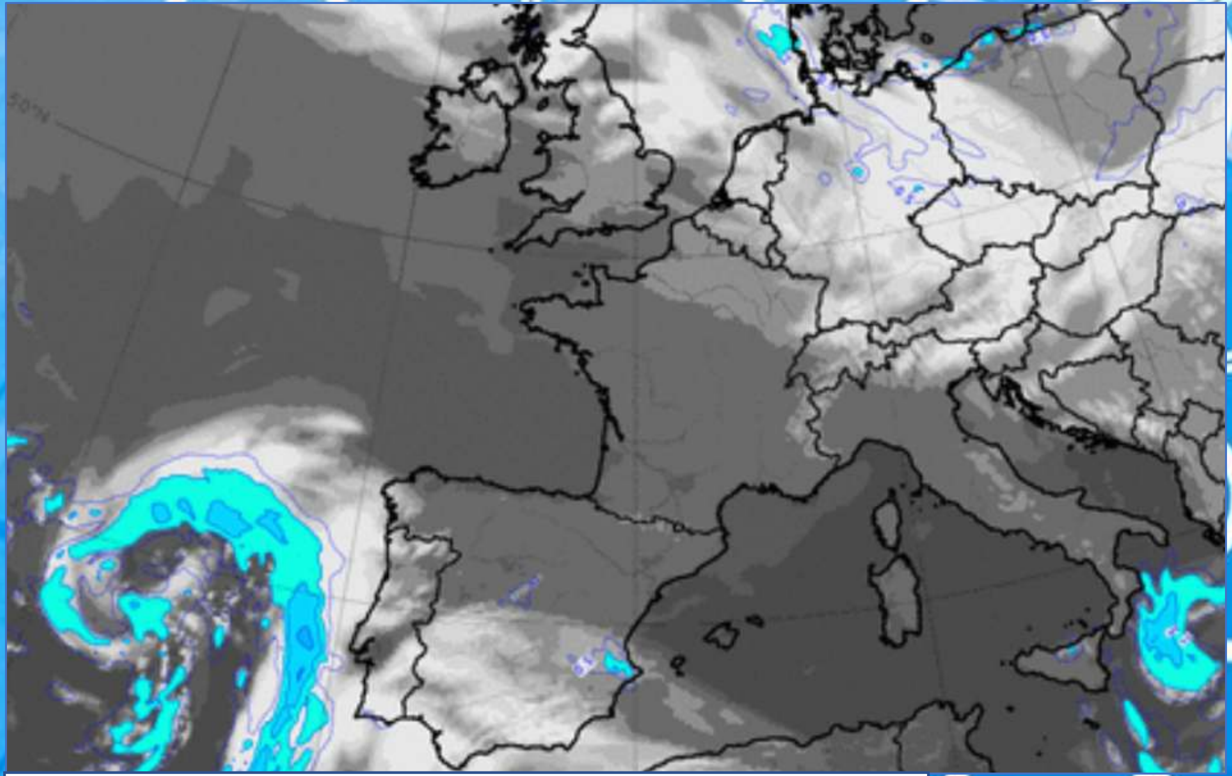
Other definition of Data Assimilation :

$$\begin{cases} x(t) = M[x(t-1)] + \eta(t) \\ y(t) = H[x(t)] + \varepsilon(t) \end{cases}$$

- **t** the discrete time
- **x** the real state
- **y** the associated observation
- **M** the model
- **H** the observation operator
- **η** the model uncertainty
- **ε** the observation error



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State of the art:

- ***H. Flourent, E. Frénod & V. Sincholle*** *(preprint). An Innovating Statistical Learning Tool Based on Partial Differential Equations, Intending Livestock Data Assimilation.*

→ Simulation and prediction methods of biological data

- ***R. Lguensat, P. Tandeo, P. Ailliot, M. Pulido & R. Fablet*** *(2017). The analog data assimilation. Monthly Weather Review, 145(10), 4093-4107.*

→ Data assimilation methods, based on the nearest neighbors

Food for thought:

- Use of the **maximum likelihood** for learning.
- Modeling with **non linear PDEs**.
- **Deep learning** : construction of an initial state for a deep Neural Network from PDEs.

Fields of application:

- **A.I**
- **Meteorology**
- **Smart Farming**
- **Industry**
- **Bank**
- **SMEs**

Thank you for your attention！

Model based on PDEs for biological data

Annex 1

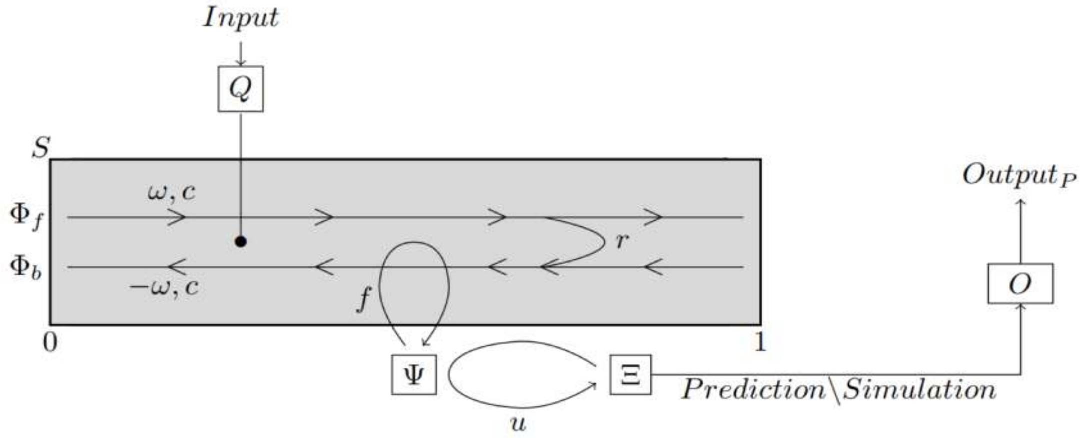
$$\frac{\partial\{\Phi_f(\mathtt{d})\}}{\partial t}(t,\mathbf{x}) + \omega_{\mathtt{d}}\frac{\partial\{\Phi_f(\mathtt{d})\}}{\partial \mathbf{x}}(t,\mathbf{x}) - \frac{\partial\left[c_{\mathtt{d}}\chi\frac{\partial\left[\{\Phi_f(\mathtt{d})\}+\{\Phi_b(\mathtt{d})\}\right]}{\partial\mathbf{x}}\right]}{\partial\mathbf{x}}(t,\mathbf{x}) = \frac{1}{2}\{Q(\mathtt{d})\}(t,\mathbf{x}) - f_{\mathtt{d}}\{F(\mathtt{d})\}(\mathbf{x})\{\Phi_f(\mathtt{d})\}(t,\mathbf{x}) - r_{\mathtt{d}}\{\Phi_f(\mathtt{d})\}(t,\mathbf{x}) \quad (1)$$

$$\begin{cases} \mathbf{x(t) = M[x(t\text{-}1)] + \eta(t)} \\ \mathbf{y(t) = H[x(t)] + \varepsilon(t)} \end{cases}$$

$$\frac{\partial\{\Phi_b(\mathtt{d})\}}{\partial t}(t,\mathbf{x}) - \omega_{\mathtt{d}}\frac{\partial\{\Phi_b(\mathtt{d})\}}{\partial \mathbf{x}}(t,\mathbf{x}) - \frac{\partial\left[c_{\mathtt{d}}\chi\frac{\partial\left[\{\Phi_f(\mathtt{d})\}+\{\Phi_b(\mathtt{d})\}\right]}{\partial\mathbf{x}}\right]}{\partial\mathbf{x}}(t,\mathbf{x}) = \frac{1}{2}\{Q(\mathtt{d})\}(t,\mathbf{x}) - f_{\mathtt{d}}\{F(\mathtt{d})\}(\mathbf{x})\{\Phi_b(\mathtt{d})\}(t,\mathbf{x}) + r_{\mathtt{d}}\{\Phi_f(\mathtt{d})\}(t,\mathbf{x}) \quad (2)$$

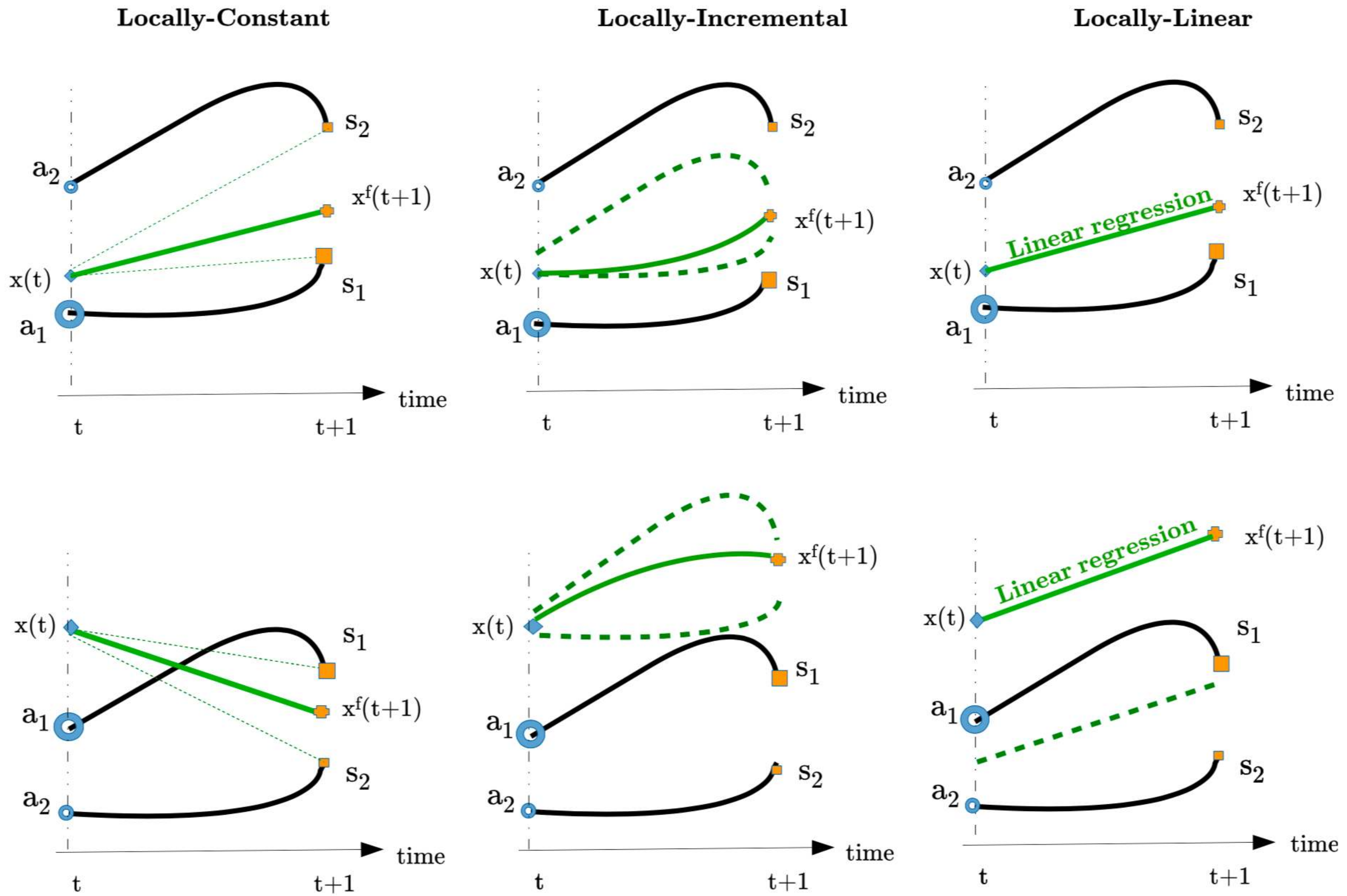
$$\frac{\partial\{\Psi(\mathtt{d})\}}{\partial t}(t,\mathbf{x}) = f_{\mathtt{d}}\{F(\mathtt{d})\}(\mathbf{x})\left[\{\Phi_b(\mathtt{d})\}(t,\mathbf{x}) + \{\Phi_f(\mathtt{d})\}(t,\mathbf{x})\right] - u_{\mathtt{d}}\{\Psi(\mathtt{d})\}(t,\mathbf{x}) \quad (3)$$

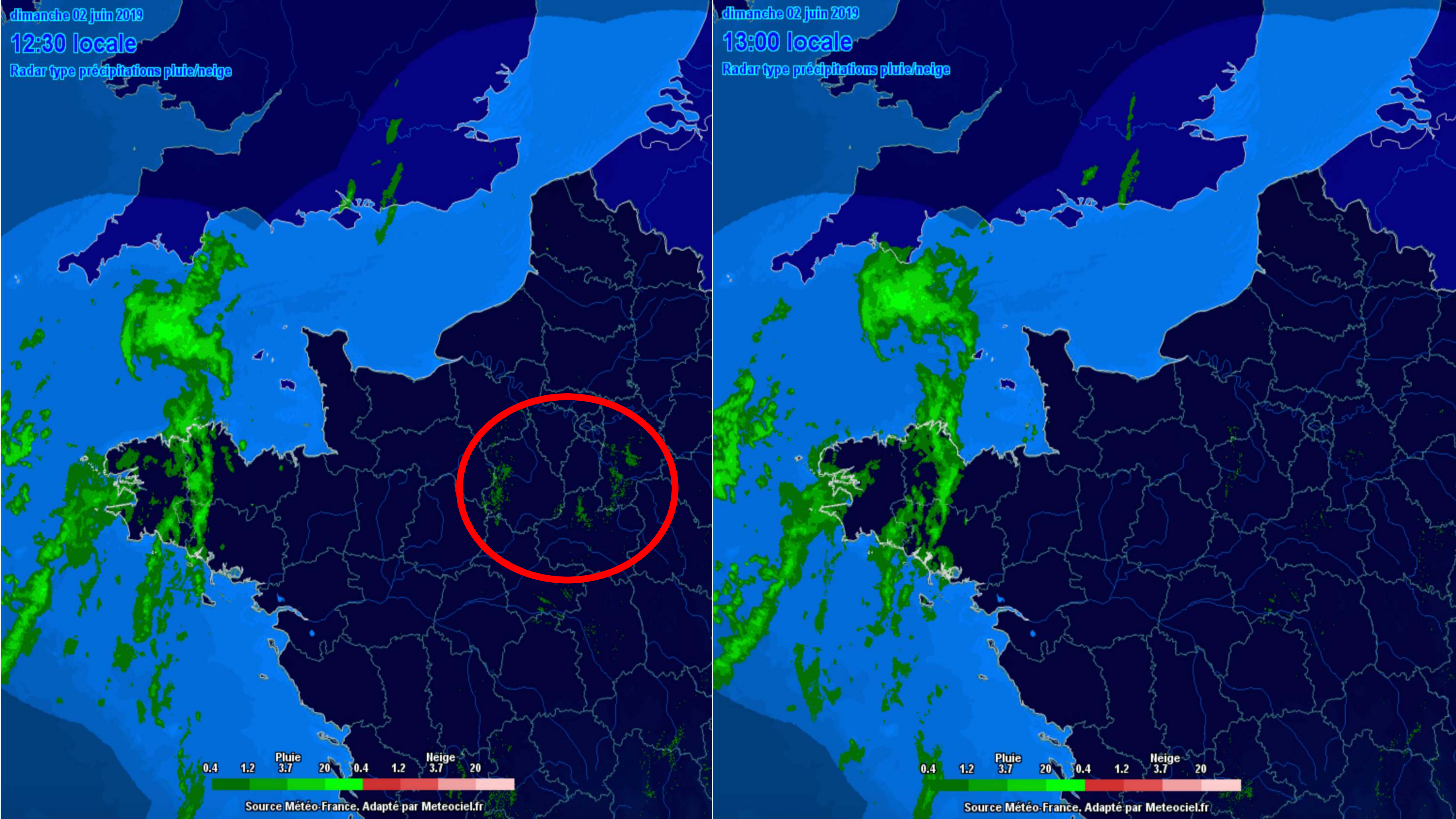
<u>Mathematical model:</u>



Example of prediction methods by the nearest neighbors

Annex 2

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12:30 locale
Radar type précipitations pluie/neige
Pluie
0.4
1.2
3.7
20
Neige
0.4
1.2
3.7
20
Source Météo-France. Adapté par Meteociel.fr
dimanche 02 juin 2019
13:00 locale
Radar type précipitations pluie/neige
Pluie
0.4
1.2
3.7
20
Neige
0.4
1.2
3.7
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Source Météo-France. Adapté par Meteociel.fr