Modelling and simulation of a wave energy converter

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Overviews

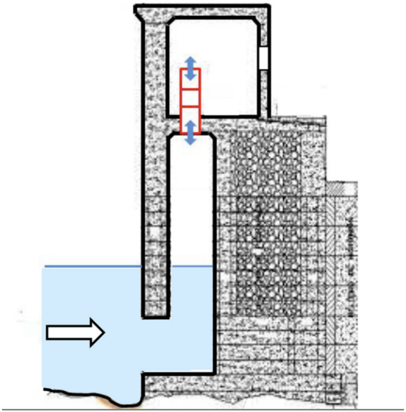
Introduction: Oscillating water column (OWC)

Closed chamber submerged with an opening below the free surface towards the incident wave

- Due to the waves motion, the water column acts as a piston compressing the air trapped inside the chamber.
- Pressurized air activates a turbine that is attached to the energy generator.

Some Advantages

- Easy maintenance
- There are no machine components in the water
- Efficient use of the marine space and is environment friendly



Taken from Falcao, Henriques, Renewable Energy, 2015.

Motivation: some experiences

Offshore OWC installed in Ireland, about 2008.

All these pictures are taken from Falcao, Henriques, Renewable Energy, 2015.

Onshore OWC installed in 1990 at Trivandrum, India.

Offshore OWC installed in Australia, about 2005.

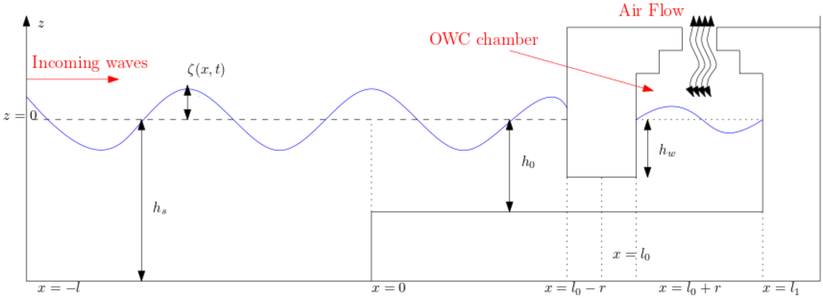


Figure: Configuration.

Notations

- ζ is the surface elevation around the rest state,
- h is the fluid height,
- q is the horizontal discharge,
- $\underline{P}$ is the surface pressure.

Wave energy converter: constrains

- **Exterior domain** $\mathcal{E}$,

$$\underline{P}_e = P_{\mathrm{atm}} \qquad \text{and} \qquad \zeta_e \text{ is unknown.}$$

- **Interior domain** $\mathcal{I}$,

$$\underline{P}_i \text{ is unknown} \qquad \text{and} \qquad \zeta_i = \zeta_w.$$

where $[\![f]\!]$ denotes the difference of f on the two side-walls of the solid, namely

$$[\![f]\!] = f(l_0 + r) - f(l_0 - r).$$

General Settings

The motion of the fluid is governed by the following nonlinear shallow water equations (NSW):

$$\begin{cases} \partial_t \zeta + \partial_x q = 0 \\ \partial_t q + \partial_x \left(\dfrac{q^2}{h} \right) + gh\partial_x \zeta = 0 \end{cases} \qquad x \in (-\infty, l_0 - r) \cup (l_0 + r, l_1)$$

The wave-structure interaction is described by the following two transmission conditions :

$$[\![q]\!] = 0,$$

$$\left[\!\!\left[\frac{q^2}{2h^2} + g\zeta \right]\!\!\right] = -\frac{2r}{h_w}\frac{dq_i}{dt}.$$

Initial conditions :

$$q(t = 0, x) = q^0(x); \qquad \zeta(t = 0, x) = \zeta^0(x).$$

Derivation of the Model

Step 1 : Reduce the problem

The motion of wave is described by the 1D shallow water equations :

In the exterior domain $\mathcal{E}$: $\begin{cases} \partial_t \zeta + \partial_x q = 0 \\ \partial_t q + \partial_x \left(\dfrac{q^2}{h} \right) + gh\partial_x \zeta = -\dfrac{1}{\rho} h \partial_x \underline{P}_{\text{atm}} = 0 \end{cases}$

In the interior domain $\mathcal{I}$: $\begin{cases} \partial_t \zeta_i + \partial_x q_i = 0 \\ \partial_t q_i + \partial_x \left(\dfrac{q_i^2}{h_i} \right) + gh_i\partial_x \zeta_i = -\dfrac{1}{\rho} h_i \partial_x \underline{P}_i \end{cases}$

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Coupling conditions :

$$q(t, l_0 \pm r) = q_i(t, l_0 \pm r)$$
$$\partial_t \zeta_i = 0 \rightsquigarrow q_i(t,x) = q_i(t)$$

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$\rightsquigarrow$ first transmission condition :

$$[\![q]\!] = 0$$

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Remark :

- Free surface, constrained pressure in the exterior domain : ζ, $\underline{P}_{\text{atm}}$
- Constrained surface, free pressure in the interior domain : $\zeta_w = \zeta_i$, $\underline{P}_i$

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- Free surface, constrained pressure in the exterior domain : ζ, $\underline{P}_{\text{atm}}$
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Goal : find the evolution equation for q_i !

Step 2: Derive the transmission condition

$$\begin{cases} \partial_t \zeta + \partial_x q = 0 \\ \partial_t q + \partial_x \left(\frac{q^2}{h} \right) + gh\partial_x \zeta = -\frac{1}{\rho} h \partial_x \underline{P} \end{cases}$$

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- Local energy conservation in the exterior region.

$$\partial_t \mathfrak{e}_{ext} + \partial_x \mathfrak{f}_{ext} = 0.$$

with

$$\mathfrak{e}_{ext} = \frac{q^2}{2h} + g\frac{\zeta^2}{2} \quad \text{and} \quad \mathfrak{f}_{ext} = \frac{q^3}{2h^2} + g\zeta q,$$

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- Total energy : $$E_{\text{tot}} = \int_{\mathcal{E}} \mathfrak{e}_{ext} + \int_{\mathcal{I}} \frac{\rho}{2} \left(\frac{q_i^2}{h_w} + g\zeta_w^2 \right)$$

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$$0 = [\![\mathfrak{f}_{ext}]\!] + \frac{2r\rho}{h_w} q_i \frac{d}{dt} q_i$$

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Wave-structure interaction

The original problem can be reduced to a transmission problem :

$$\begin{cases} \partial_t \zeta + \partial_x q = 0 \\ \partial_t q + \partial_x \left(\dfrac{q^2}{h} \right) + gh\partial_x \zeta = 0 \end{cases} \qquad x \in \mathcal{E} \tag{1}$$

with transmission conditions provided at the contact points $x = l_0 \pm r$:

$$[\![q]\!] = 0, \tag{2}$$

$$\left[\!\!\left[\frac{q^2}{2h^2} + g\zeta \right]\!\!\right] = -\frac{2r\rho}{h_w}\frac{dq_i}{dt} = -\alpha\frac{dq_i}{dt}. \tag{3}$$

Discretization

Riemann invariants

For a pair of system of hyperbolic conservation laws

$$\begin{bmatrix} \zeta \\ q \end{bmatrix}_t + \begin{bmatrix} q \\ \frac{q^2}{h} + \frac{g}{2} h^2 \end{bmatrix}_x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

it is known that a pair of Riemann invariants exist so that the system can be rewritten as

$$\partial_t R + \lambda_+(\zeta, q)\partial_x R = 0; \quad \partial_t L - \lambda_-(\zeta, q)\partial_x L = 0$$

where (R, L) are the Riemann invariants and λ_+ and $-\lambda_-$ are the two eigenvalues

$$R = 2(\sqrt{gh} - \sqrt{gh_0}) + \frac{q}{h}, \quad L = 2(\sqrt{gh} - \sqrt{gh_0}) - \frac{q}{h};$$

$$\lambda_+(U) = \sqrt{gh} + \frac{q}{h}, \quad -\lambda_-(U) = -\sqrt{gh} + \frac{q}{h}$$

Discretization of the Model

Let us first rewrite the shallow water equations in a more compact form by introducing $U = (\zeta, q)^T$:

$$\partial_t U + \partial_x(F(U)) = 0, \tag{4}$$

with

$$F(U) = (q, \frac{1}{2}g(h^2 - h_0^2) + \frac{q^2}{h})^T,$$

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Then the Lax-Friedrichs scheme for solving the above partial differential equation is given by:

$$\frac{U_i^{n+1} - \frac{1}{2}(U_{i+1}^n + U_{i-1}^n)}{\Delta t} + \frac{F(U_{i+1}^n) - F(U_{i-1}^n)}{2\,\Delta x} = 0$$

which implies

$$U_i^{n+1} = \frac{1}{2}(U_{i+1}^n + U_{i-1}^n) - \frac{\Delta t}{2\,\Delta x}(F(U_{i+1}^n) - F(U_{i-1}^n))$$

Discretization of entry condition

Entry condition at $x = -l$:

- Surface elevation ζ is given by $\zeta(t^n, x = -l) = f(t^n)$;
- Horizontal discharge q can be derived by Left Riemann invariant L :

$$q = h(2(\sqrt{gh} - \sqrt{gh_0}) - L)$$

After discretization, we have

$$q^n|_{x=-l} = (h_0 + f(t^n))(2(\sqrt{g(h_0 + f(t^n))} - \sqrt{gh_0}) - \overbrace{L^n|_{x=-l}}^{?}).$$

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By using characteristic equation of L, we have

$$\frac{L_0^n - L_0^{n-1}}{\delta_t} - \lambda_- \frac{L_1^{n-1} - L_0^{n-1}}{\delta_x} = 0.$$

Thus, $L^n|_{x=-l}$ can be determined by

$$L_0^n = (1 - \lambda_- \frac{\delta_t}{\delta_x}) L_0^{n-1} + \lambda_- \frac{\delta_t}{\delta_x} L_1^{n-1}.$$

Discretization of discontinuous topography

Coupling conditions near the discontinuous topography at $x = 0$:

- Continuity of the surface elevation ζ : $\zeta^l|_{x=0} = \zeta^r|_{x=0}$;
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Using Riemann invariants, we find two expressions of q describing $q^l|_{x=0}$ and $q^r|_{x=0}$, respectively,

$$\begin{cases} q^l|_{x=0} = (h_s + \zeta^l|_{x=0})\Big(R^l|_{x=0} - 2(\sqrt{g(h_s + \zeta^l|_{x=0})} - \sqrt{g h_s})\Big) \\ q^r|_{x=0} = (h_0 + \zeta^r|_{x=0})\Big(2(\sqrt{g(h_0 + \zeta^r|_{x=0})} - \sqrt{g h_0}) - L^r|_{x=0}\Big) \end{cases} \quad (5)$$

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Here, $R^l|_{x=0}$ and $L^r|_{x=0}$ can be determined by their characteristic equations :

$$(R^l)_0^n = \left(1 - \lambda_+^l \frac{\delta_t}{\delta_x}\right)(R^l)_0^{n-1} + \lambda_+^l \frac{\delta_t}{\delta_x}(R^l)_{-1}^{n-1},$$

$$(L^r)_0^n = \left(1 - \lambda_-^r \frac{\delta_t}{\delta_x}\right)(L^r)_0^{n-1} + \lambda_-^r \frac{\delta_t}{\delta_x}(L^r)_{+1}^{n-1}$$

Discretization on the sector of the fixed submerged object

Transmission conditions in the interior domain $\mathcal{I} = (l_0 - r, l_0 + r)$ **:**

- $[\![q]\!] = 0 \quad \rightsquigarrow \quad q^l|_{l_0-r} = q_i = q^r|_{l_0+r}.$

Using Riemann invariants, we find

$$\begin{cases} q^l|_{l_0-r} = (h_0 + \zeta^l|_{l_0-r})\Big(R^l|_{l_0-r} - 2\big(\sqrt{g(h_0 + \zeta^l|_{l_0-r})} - \sqrt{gh_0}\big)\Big); \\ q^r|_{l_0+r} = (h_0 + \zeta^r|_{l_0+r})\Big(2\big(\sqrt{g(h_0 + \zeta^r|_{l_0+r})} - \sqrt{gh_0}\big) - L^r|_{l_0+r}\Big). \end{cases} \tag{6}$$

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Here, $R^l|_{l_0-r}$ and $L^r|_{l_0-r}$ can be determined by their characteristic equations as before.

- The transmission condition near the object

$$\left[\!\!\left[\frac{q^2}{2h^2} + g\zeta\right]\!\!\right] = -\alpha\frac{dq_i}{dt}$$

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$$\rightsquigarrow \frac{(q^r|_{l_0+r})^2}{2(h_0+\zeta^r|_{l_0+r})^2} + g\zeta^r|_{l_0+r} - \frac{(q^l|_{l_0-r})^2}{2(h_0+\zeta^l|_{l_0-r})^2} - g\zeta^l|_{l_0-r} = -\alpha\frac{dq_i}{dt}$$

Discretization on the wall

There is a physical boundary at $x = l_0 + l$ given by the wall and the corresponding boundary condition is :

$$\bar{v}(t, l_0 + l) = 0$$

which implies $q(t, l_0 + l) = 0$ at the wall, so that

$$0 = h(R - 2(\sqrt{gh} - \sqrt{gh_0})) \rightsquigarrow \zeta = \frac{1}{g}(\frac{R}{2} + \sqrt{gh_0})^2 - h_0, \text{ at } x = l_0 + l$$

After discretization, we have

$$\zeta(t^n, l_0 + l) = \frac{1}{g}(\frac{R(t^n, l_0 + l)}{2} + \sqrt{gh_0})^2 - h_0$$

Numerical Results

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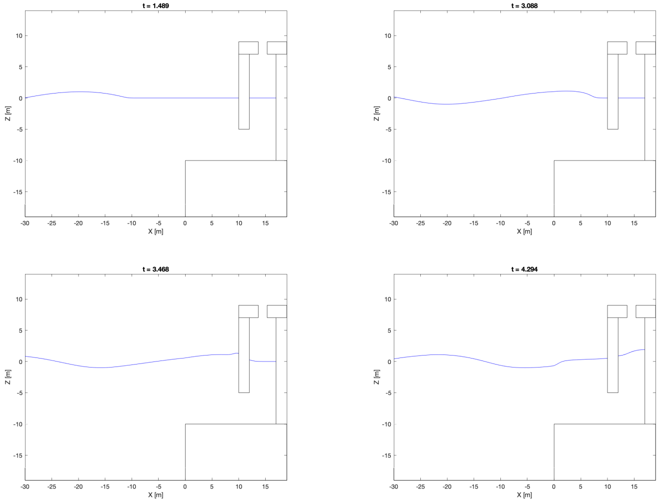

Figure: Amplitude = 1 and period = 3.

Numerical Results: Differences

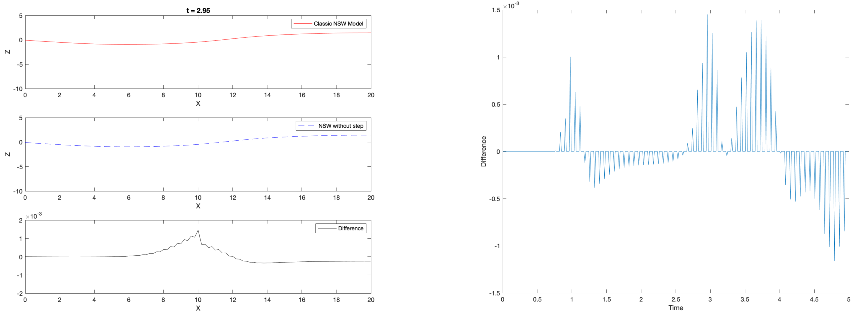

Figure: Amplitude = 1 and period = 3.

Absorbed power of the OWC-WEC device

- The incident wave power P_{inc} is defined as the product of the incoming wave energy and the group velocity c_g :

$$P_{inc} = Ec_g$$

with

$$E_{inc} = \frac{1}{2}\rho g L A^2 \quad \text{and} \quad c_g = \sqrt{gh_0}$$

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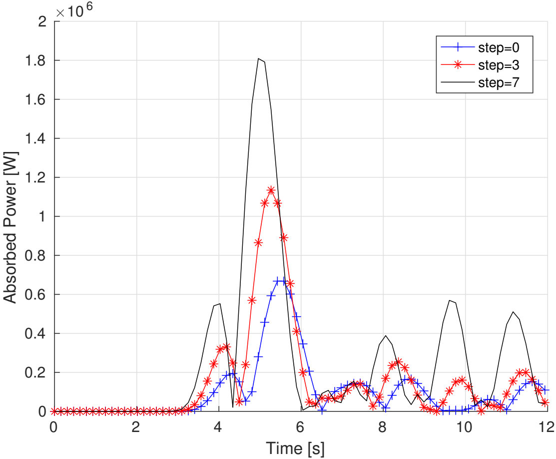
- The absorbed power is defined as

$$P_a = \frac{1}{T}\int_0^T \Delta P Q dt$$

where ΔP is the instantaneous differential pressure between the chamber and the exterior domain, and Q the airflow rate through the turbine, which simply can be presented by

$$bl \, \frac{d\zeta_{average}}{dt}.$$

Absorbed power



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Thanks for your attention!