Sensitivity analysis: a sparse grid approach

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The goal of this project is to design methods that allow to compute sensitivities (a.k.a. Greeks) for some structured products in a high dimensional setting. Our main motivation is the management of risks on a balance sheet. This requires the construction of hedging portfolios which leads itself to the (re-)computation of various greeks (delta and gamma with respect to several underlying assets, various durations for rho and rho convexity,...), as necessary tools to evaluate the risk and the possible mitigation effect of the addition of new positions.

The typical method to solve this question uses a simulation approach that rapidly becomes intractable as the number of inputs (factors, parameters) increases. Alternatives to reduce the time complexity issue, such as Algorithmic Differentiation [6], frequently imply the need to (pre-)compute the sensitivities on the whole space (or at least the support of the various risk scenarii), leading to memory management issues. However, when some regularity is known a priori on the function that has to be pre-computed, one can try to use a sparse grid representation of the function to avoid (at least partially) this problem.

Sparse grids are now a quite well-known tool [1] and have been applied already in finance [5] mainly in relation with PDE methods. In this project we will study a different approach that relies on a probabilistic representation of the price and sensitivities of the structured product, deduced respectively from the classical Feynman-Kac formula and the Malliavin calculus approach as introduced in [3].

Applying these probabilistic representations for numerical calculations leads obviously to the computation of expectations and then to the question of the discretisation of the various probability laws involved. We will study mainly cubature methods (possibly on the Wiener space) for this part [8, 7].

Our starting point will be to compute the Delta and Gamma on sparse grids using the aforementioned probabilistic approach adapting the backward algorithm introduced in [2]. Once this is achieved, we will investigate the following points both numerically and theoretically

- How to parallelise the method? [4]
- How to take into account irregularities in the value functions?
- How to compute other greeks?

References

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