CEMRACS 2017 Project
Network of interacting neurons with random synaptic weights

Supervised by Etienne Tanré (Inria Sophia-Antipolis, team ToSCA), François Delarue (LJAD, Nice), Julien Chevallier (Univ. Cergy-Pontoise).

Aim

The goal of this research project is to study and simulate large networks of interacting neurons with several forms of synaptic weights, like

1. Models with random synaptic weights
2. Models with singular synaptic weights.

In all cases, we are interested in the macroscopic behavior of the model, which includes short and long-run analysis.
Several extensions may be addressed during the project, say for instance models with a spatial structure or models with different sources of noise.

Applicants with a strong background in one of the following field will be strongly appreciated:

1. Stochastic analysis and numerical probabilities;
2. Numerical methods and programming;
3. Biology, neurosciences and computational neurosciences.

Introduction

Mathematical models of interacting neurons have known a surge of interest for the past ten years, see for instance the list of references below. Generally speaking, models in hand are based on the following twofold structure:

- First, the state of a neuron is characterized by its membrane potential. The dynamics of the neuron potential may be described by several forms of stochastic processes, which include diffusion or point processes, see for instance [7, 6] or [5] and [10].

- Second, the model exhibits some specific times as a remarkable feature: A fast variation of the membrane potential is observable from time to time (biologically, a large number of ion channels are open simultaneously). We say that the neuron spikes (or emits an action potential) at these times. Then, the effect of the spike is transmitted to the other neurons connected with the spiking neuron. The intensity of the interaction is given by the synaptic weights.

As explained below, several models have been addressed in the literature for the synaptic weights.

Mean-field effect

From the modeling point of view, there are several issues. The first one is to exhibit a tractable but relevant form of interaction between the neurons. Generally speaking, the huge number of neurons involved in many areas of the brain makes the global system difficult to study and simulate. However, in some cases, one may provide a quite explicit description of the whole behavior of the network as the number of neurons tends to the infinity. In this regard, the most popular framework is the so-called "mean field regime", in which neurons are assumed to interact with the others through their collective state. The "mean field effect" is a mathematical paradigm that permits to describe the asymptomatic behavior quite simply.

Choice of the synaptic weights

Another issue is that there is no precise knowledge of the nature of the connections in the brain and that it is very difficult to appreciate it and to measure it accurately. Several strategies are conceivable. The first one is to choose some deterministic rules, which is precisely what has been done in [6] and [7]; therein, the interaction is somewhat singular and may exhibit, as a noticeable feature, blow-ups –i.e. times at which the system features a massive proportion of spikes–. Another classical approach is to choose the synaptic weights as the realizations of random variables. In this regard, it is of a peculiar interest to understand which global quantities should be fixed so that the model makes sense from the biological point of view. For instance, one may play with the proportion of active connections or one may require a balance between excitatory and inhibitory synapses.

Goal of the project

The goal is to study the effect of the model for the synaptic weights onto the mean field limit. For instance, one may play with the intensity of the interaction or with the law of the synaptic weights when they are chosen randomly. In this regard, we are especially interested in the persistence of well-known macroscopic behaviors such that the spontaneous oscillations, the stability of the activity, etc. In case when the synaptic weights are drawn at random, we are especially interested in obtaining quenched results, i.e. results that are satisfied for any realization of a given law, and not only annealed results i.e. results that are satisfied for averages over all the realizations of the synaptic weights.

Several extensions will be conceivable :

1) the influence of the nature of the noise (diffusive noise versus point processes) is a natural question;

2) the correlation structure of the noise is another interesting question;

3) models with delay may be addressed;

4) extension to models featuring a spatial structure is certainly meaningful from the biological point of view.

References

[1] M. Bachar, J. Batzel, and S. Ditlevsen, editors. *Stochastic biomathematical models*, volume 2058 of *Lecture Notes in Mathematics*. Springer, Heidelberg, 2013. With applications to neuronal modeling, Mathematical Biosciences Subseries.

[2] B. Barbour, N. Brunel, V. Hakim, and J.-P. Nadal. What can we learn from synaptic weight distributions? *Trends Neurosci*, 30(12):622–629, 2007.

[3] P. C. Bressloff. Stochastic neural field theory and the system-size expansion. *SIAM J. Appl. Math.*, 70(5):1488–1521, 2009/10.

[4] M. a. J. Cáceres, J. A. Carrillo, and B. Perthame. Analysis of nonlinear noisy integrate & fire neuron models: blow-up and steady states. *J. Math. Neurosci.*, 1:Art. 7, 33, 2011.

[5] J. Chevallier. Mean-field limit of generalized hawkes processes, 2015.

[6] F. Delarue, J. Inglis, S. Rubenthaler, and E. Tanré. Global solvability of a networked integrate-and-fire model of McKean-Vlasov type. *Ann. Appl. Probab.*, 25(4):2096–2133, 2015.

[7] F. Delarue, J. Inglis, S. Rubenthaler, and E. Tanré. Particle systems with a singular mean-field self-excitation. Application to neuronal networks. *Stochastic Process. Appl.*, 125(6):2451–2492, 2015.

[8] O. Faugeras and J. MacLaurin. A large deviation principle and an expression of the rate function for a discrete stationary Gaussian process. *Entropy*, 16(12):6722–6738, 2014.

[9] O. Faugeras and J. MacLaurin. A representation of the relative entropy with respect to a diffusion process in terms of its infinitesimal generator. *Entropy*, 16(12):6705–6721, 2014.

[10] N. Fournier and E. Löcherbach. On a toy model of interacting neurons. *Ann. Inst. Henri Poincaré Probab. Stat.*, 52(4):1844–1876, 2016.

[11] W. Gerstner, W. M. Kistler, R. Naud, and L. Paninski. *Neuronal Dynamics: From Single Neurons To Networks And Models Of Cognition.* Cambridge University Press, 2014.

[12] E. Luçon. Quenched large deviations for interacting diffusions in random media. *Journal of Statistical Physics*, 166(6):1405–1440, jan 2017.

[13] B. Perthame and D. Salort. On a voltage-conductance kinetic system for integrate & fire neural networks. *Kinet. Relat. Models*, 6(4):841–864, 2013.

[14] T. Schwalger, M. Deger, and W. Gerstner. Towards a theory of cortical columns: From spiking neurons to interacting neural populations of finite size, 2016.