Simulation based optimal control via deep learning

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Generally, the aim of solving an optimal decision problem, that is an optimal stopping or optimal control problem, is twofold. On the one hand, one aims at bounding its "true" value from below and above, and on the other hand one tries to find a "good" decision policy consistent with these bounds. In fact, a "good" (primal) decision policy yields a lower bound, and a "good" system of (dual) martingales yields an upper bound to the "true" value, respectively. Thus, naturally, solution methods for optimal decision problems can be classified in primal and dual approaches. For the standard optimal stopping problem, [1] succeeded to avoid the time consuming sub-simulations in the Andersen-Broadie algorithm by constructing the dual martingale via a discrete Clark-Ocone derivative of some approximation to the Snell-envelope, obtained by regression on a suitable set of basis functions. Later on in [3] a related regression method was developed that also avoids sub-simulations and, even more, does not require any input approximation to the solution of the problem (i.e. the Snell envelope). Particularly the later approach looked promising for generalization to quite general control problems. As a first non-trivial application, this method was successfully applied in the context of a hydro electricity storage model [2]. One of the main goals in this project proposal is a systematic numerical treatment of generic optimal decision problems in "real-life" applications by incorporating recent ideas of a relatively new concept of data analysis and prediction: *Deep Learning*. Typical "real-life" applications have a high dimensional nature and an effective numerical treatment of optimal stopping or control problems in this context, both from the primal and the dual side, is of prime importance and is considered a challenge from a mathematical point of view. In the recent work [2] a regression based framework was developed that, in principle, allows for simulating an upper biased bound and a lower biased bound to the solution to an optimal decision problem. As a main feature, the approach in [2] does not require nested simulation. However, there are fundamental problems that need to be tackled. In particular, in the dual approach the stochastic representation of the optimal solution requires in general a set of martingales with a cardinality equal to the typically high number of possible decisions. Further, recursive backward construction of a set of dual martingales in terms of a suitable set of basis functions involves, in principle, an optimization problem that is nonlinear at each step. In general, regression methods in stochastic optimal control heavily rely on the choice of the set of basis functions and in this respect backward construction of the set of dual martingales can be naturally combined with the Deep Learning idea. In particular, the solution from one layer may be incorporated in the set of basis functions for all higher layers. Summing up, the main goals in this proposal will be:

- Combination of regression methods with ideas of multilayer deep neural networks for solving stochastic optimal control problems
- Development of new generic methods for solving nonlinear stochastic optimization problems
- Development of a systematic way of estimating underlying price distribution processes
- Extension of the proposed methodology to optimal control problems for BSDEs and McKean-Vlasov nonlinear diffusions.

References

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