Uncertainty quantification for the Vlasov equation

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The Vlasov equation models the evolution of a plasma in its self-consistent electro-magnetic fields. Simulations typically depend for instance on the intitial distribution and the guide field configuration. In this project, we are interested in quantifying how uncertainties in the configurations propagate over time. As a test problem, we will consider the 1d1v Vlasov–Poisson equation with an uncertainty in the initial value. More precisely, the width or position of a Maxwellian equilibrium could be uncertain. We will use a Karhunen–Loève expansion for the uncertain parameter and a discrete projection method to study the uncertainty following [6].

To sample the parameter space we will compare various methods including high-order quasi Monte Carlo [2], sparse grids, and possibly also generalized cross algorithms (cf. [1, 7]) and compare their efficiency based on the dimensionality of the parameter space. Our simulation of the Vlasov–Poisson equation will be based on solutions computed with the forward-backward Lagrangian method described in [5]. Compared to classical particle in cell techniques, FBL offers interesting improvements in accuracy but presents certain drawbacks regarding its practical implementation and we will explore ways to mitigate them.

After testing the discrete projection method with the various sampling techniques in the case of the 1d1v Vlasov–Poisson equation, we also plan to extend the study to more advanced 2d2v examples.

If time permits, we will apply the reduced modelling techniques developed in [4, 3] for the state estimation of a plasma by coupling Vlasov models with noisy measurement observations.

References

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