

Local polynomial approximation of drivers and branching process based resolution of BSDE

We propose to investigate a new type of numerical schemes for (reflected) Backward Stochastic Differential Equations based on branching processes, that has been first proposed by [2]. It is based on the Feynman-Kac representation of the KPP equation of Skorokhod [11], Watanabe [13] and McKean [9], recently further explored by Rasulov, Raimov and Mascagni [10] and [6, 7, 8], which provides a pure forward representation of BSDEs with polynomial drivers in terms of a family of branching processes. The main idea of [2] is to make these polynomials local, so as to be able to approximate any Lipschitz driver. This is obtained by using a Picard type iteration, but which has the main advantage of not requiring a precise initial prior.

The first aim of this project will be prove the convergence of the method in the case where the driver depends on the Z component of the BSDE and the ghost approach is used to avoid explosion of the variance, see [12]. Secondly, the group will study how this method can be adapted to approximate reflected BSDEs. Numerical experiments will be done on examples coming from the industry. Various variance reduction techniques will also be considered.

References

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