

Multilevel and multi-index sampling methods with applications

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Second lecture: Multi-index Monte Carlo and Multi-index Stochastic Collocation

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We describe and analyze the Multi-Index Monte Carlo (MIMC) and the Multi-Index Stochastic Collocation (MISC) method for computing statistics of the solution of a PDE with random data. MIMC is both a stochastic version of the combination technique introduced by Zenger, Griebel and collaborators and an extension of the Multilevel Monte Carlo (MLMC) method first described by Heinrich and Giles. Instead of using first-order differences as in MLMC, MIMC uses mixed differences to reduce the variance of the hierarchical differences dramatically. These mixed differences yield new and improved complexity results, which are natural generalizations of Giles's MLMC analysis, and which increase the domain of problem parameters for which we achieve the optimal convergence. On the same vein, MISC is a deterministic combination technique based on mixed differences of spatial approximations and quadratures over the space of random data. Provided enough mixed regularity, MISC can achieve better complexity than MIMC. Moreover, we show that, in the optimal case, the convergence rate of MISC is only dictated by the convergence of the deterministic solver applied to a one-dimensional spatial problem. We propose optimization procedures to select the most effective mixed differences to include in MIMC and MISC. Such optimization is a crucial step that allows us to make MIMC and MISC computationally efficient. We show the effectiveness of MIMC and MISC in some computational tests using the mimclib open source library, including PDEs with random coefficients and Stochastic Interacting Particle Systems.

Finally, we will briefly discuss the use of Markovian projection for the approximation of prices in the context of American basket options.

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