Multilevel and multi-index sampling methods with applications

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First lecture: Multilevel Monte Carlo

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We will first recall, for a general audience, the use of Monte Carlo and Multi-level Monte Carlo methods in the context of Uncertainty Quantification. Then we will discuss the recently developed Adaptive Multilevel Monte Carlo (MLMC) Methods for (i) It Stochastic Differential Equations, (ii) Stochastic Reaction Networks modeled by Pure Jump Markov Processes and (iii) Partial Differential Equations with random inputs. In this context, the notion of adaptivity includes several aspects such as mesh refinements based on either a priori or a posteriori error estimates, the local choice of different time stepping methods and the selection of the total number of levels and the number of samples at different levels. Our Adaptive MLMC estimator uses a hierarchy of adaptively refined, non-uniform time discretizations, and, as such, it may be considered a generalization of the uniform discretization MLMC method introduced independently by M. Giles and S. Heinrich. In particular, we show that our adaptive MLMC algorithms are asymptotically accurate and have the correct complexity with an improved control of the multiplicative constant factor in the asymptotic analysis. In this context, we developed novel techniques for estimation of parameters needed in our MLMC algorithms, such as the variance of the difference between consecutive approximations. These techniques take particular care of the deepest levels, where for efficiency reasons only few realizations are available to produce essential estimates. Moreover, we show the asymptotic normality of the statistical error in the MLMC estimator, justifying in this way our error estimate that allows prescribing both the required accuracy and confidence level in the final result. We present several examples to illustrate the above results and the corresponding computational savings.

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