Optimal scaling of the transient phase of Metropolis Hastings algorithms

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We consider the Random Walk Metropolis algorithm on $\mathbf{R}^n$ with Gaussian proposals, and when the target probability measure is the n-fold product of a one dimensional law. It is well-known that, in the limit n tends to infinity, starting at equilibrium and for an appropriate scaling of the variance and of the timescale as a function of the dimension n, a diffusive limit is obtained for each component of the Markov chain. We generalize this result when the initial distribution is not the target probability measure. The obtained diffusive limit is the solution to a stochastic differential equation nonlinear in the sense of McKean. We prove convergence to equilibrium for this equation. We discuss practical counterparts in order to optimize the variance of the proposal distribution to accelerate convergence to equilibrium. Our analysis confirms the interest of the constant acceptance rate strategy (with acceptance rate between 1/4 and 1/3).