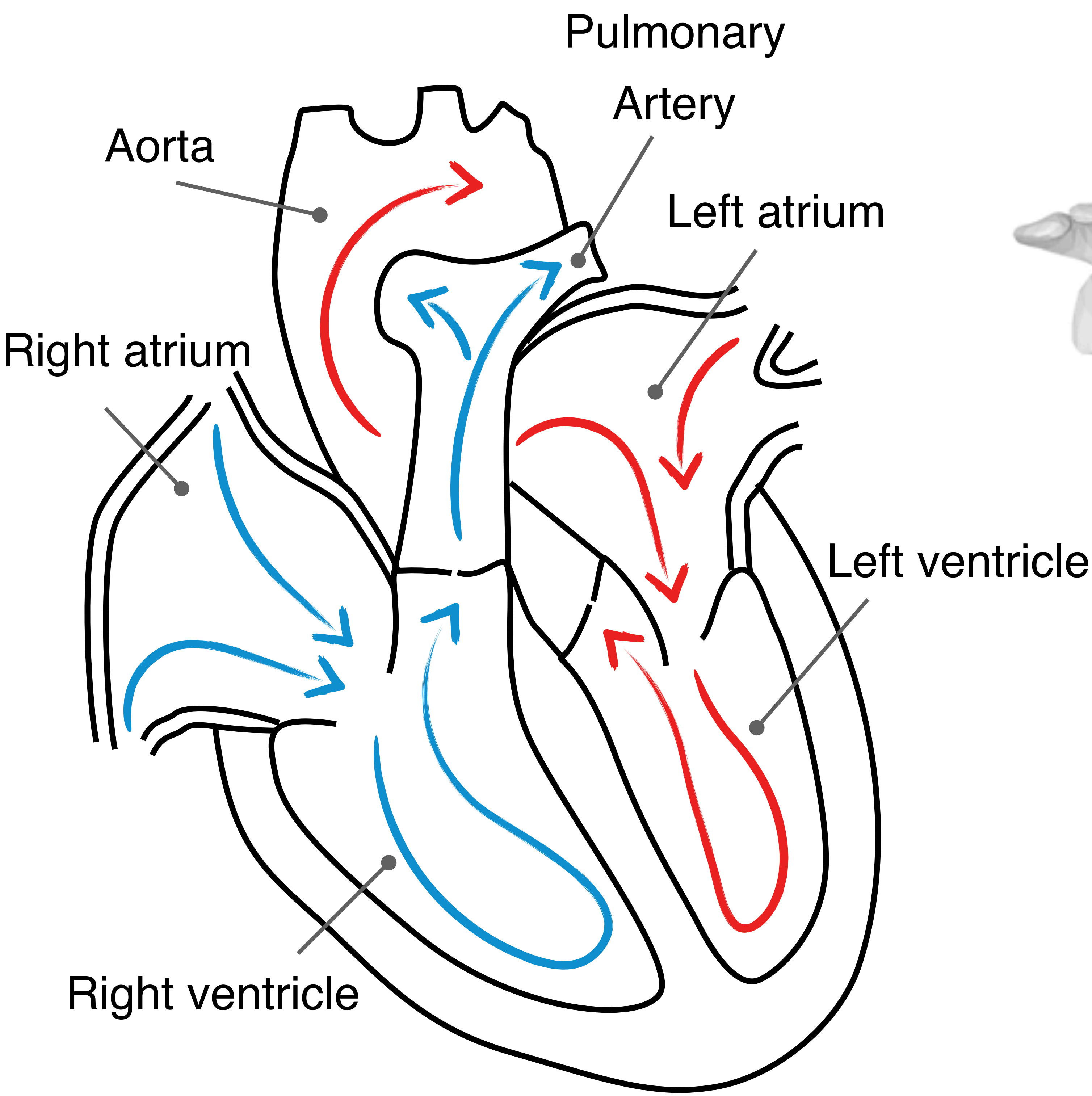
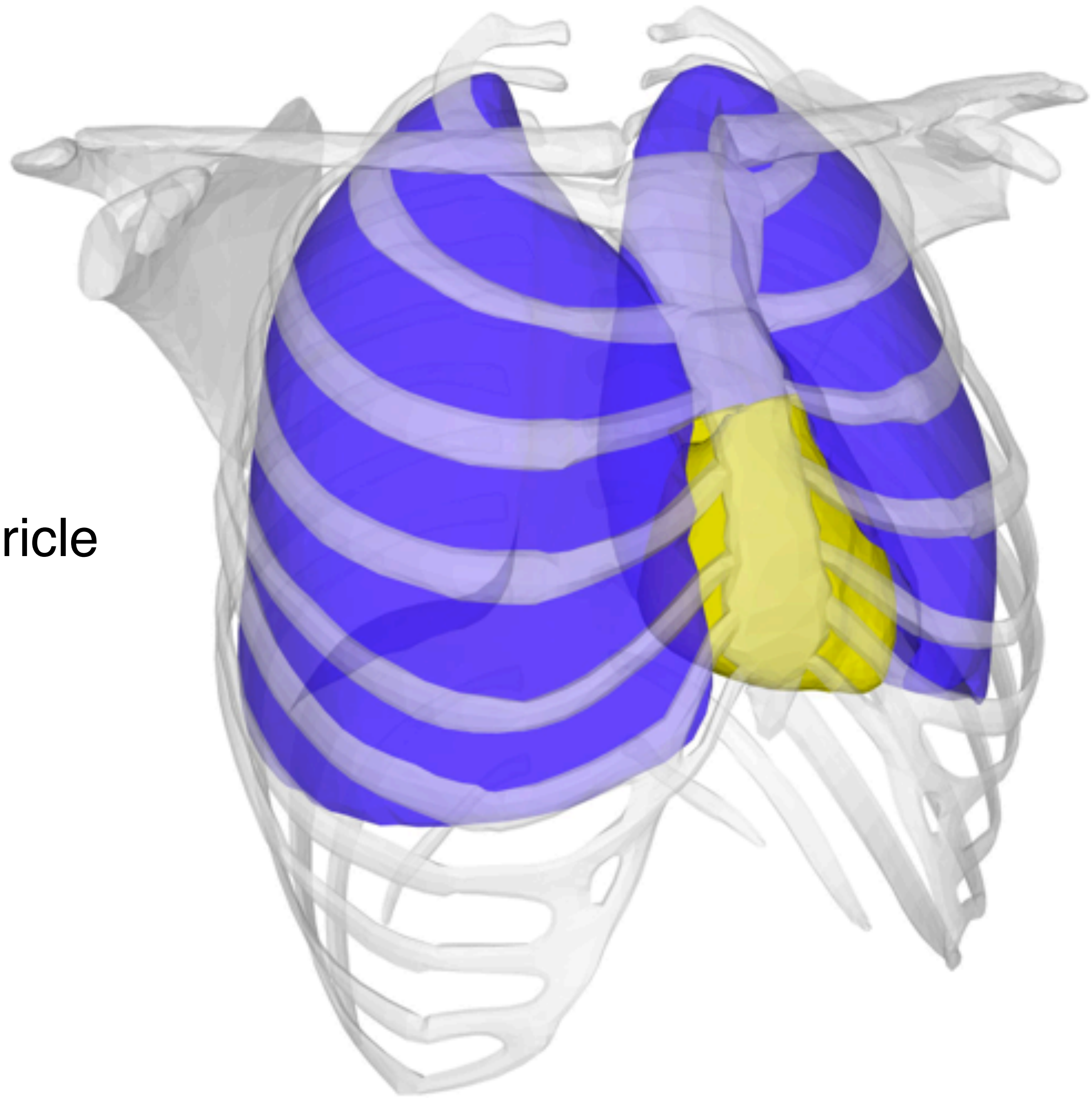
MODELING AND DATA ASSIMILATION IN CARDIAC ELECTROPHYSIOLOGY

Collaborators: Dominique Chapelle, Jean-Frédéric Gerbeau,
Philippe Moireau, Sébastien Impériale, Elisa Schenone

Cardiac electrophysiology - Introduction



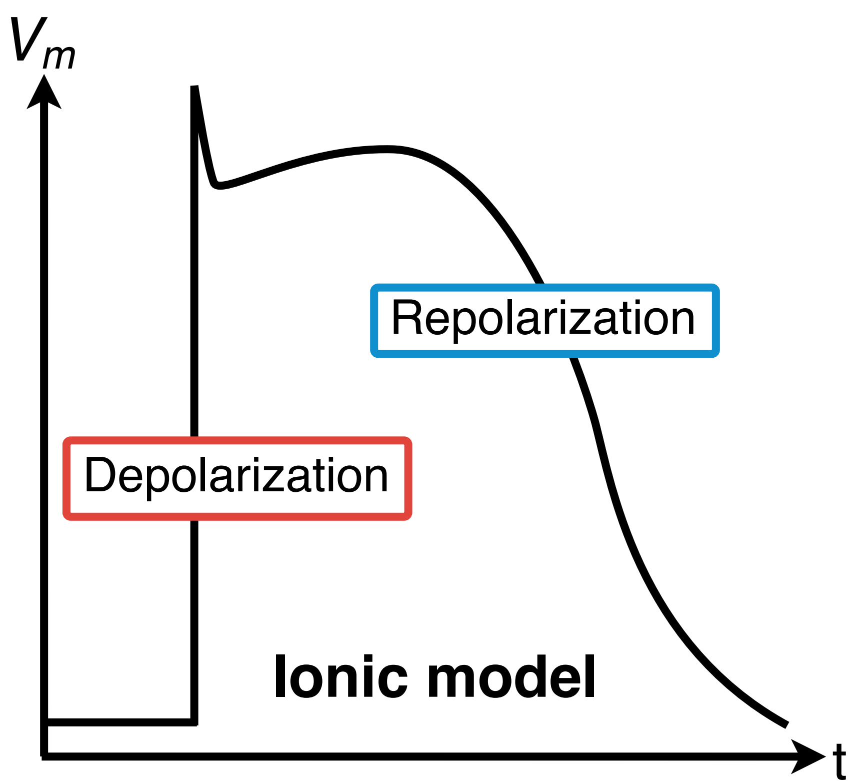
Blood circulation in the heart

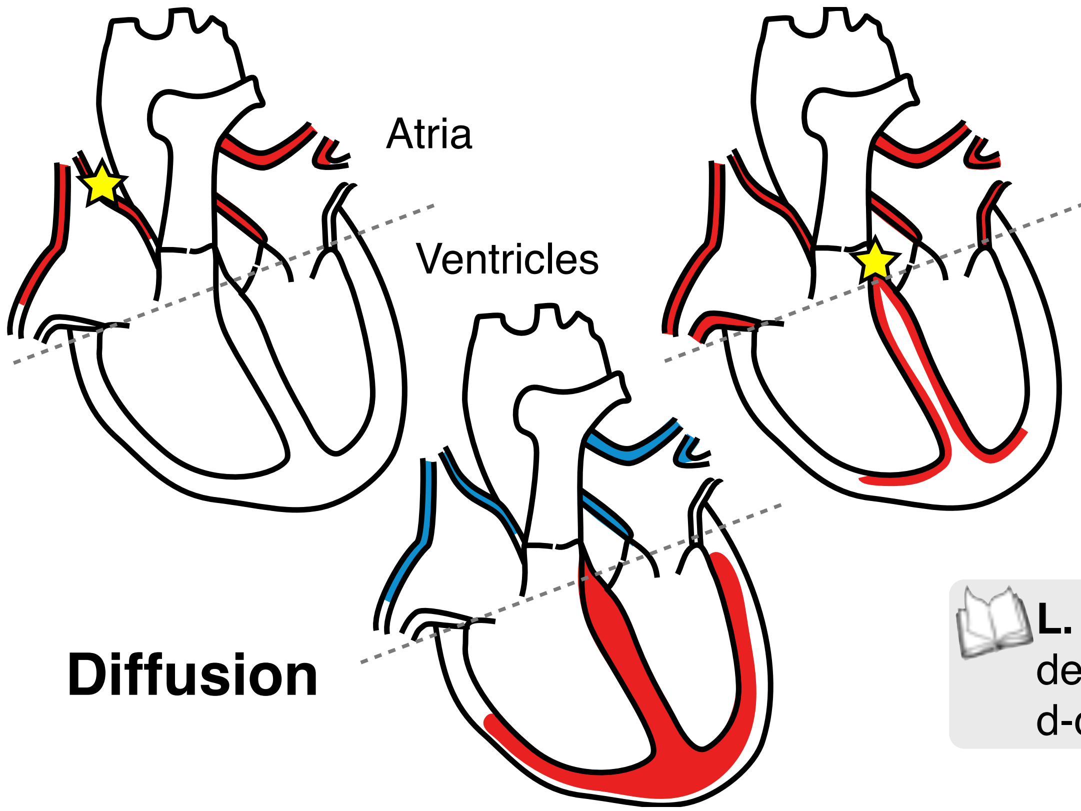


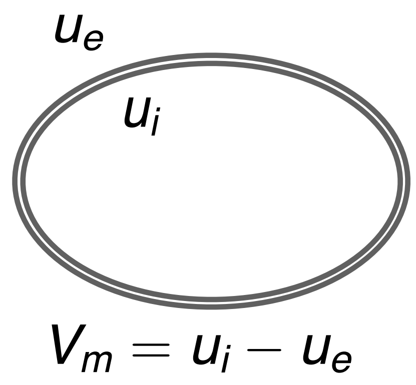
Electrical activity = origin of the mechanical activity

Bidomain model

- Bidomain model a classical electrophysiological model







L. Tung. A bi-domain model for describing ischemic myocardial d-c potentials. 1978.

- Non linear reaction-diffusion system coupled with one (or more) ODE(s)

$$\begin{cases} A_m\Big(C_m \dfrac{\partial V_m}{\partial t} + I_{ion}(V_m, \cdots)\Big) - \mathrm{div}(\vec{\vec{\sigma}}_i \cdot \vec{\nabla} V_m) = \mathrm{div}(\vec{\vec{\sigma}}_i \cdot \vec{\nabla} u_e) + I_{app}, & \text{in } \mathcal{B} \times (0, T), \\ \mathrm{div}\Big((\vec{\vec{\sigma}}_i + \vec{\vec{\sigma}}_e) \cdot \vec{\nabla} u_e\Big) = -\mathrm{div}(\vec{\vec{\sigma}}_i \cdot \vec{\nabla} V_m), & \text{in } \mathcal{B} \times (0, T) \end{cases}$$

- Existence and uniqueness under assumptions.

P. Colli-Franzone. G. Savaré. Degenerate evolution systems modeling the cardiac electric field at micro and macroscopic level. 2009.

- Monodomain model (simplified version)

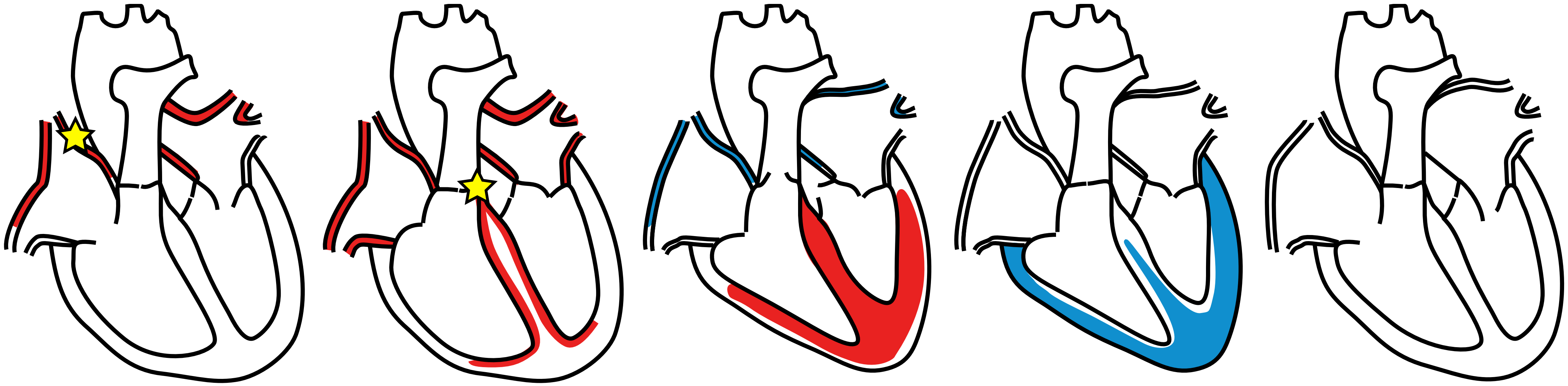
$$A_m\Big(C_m \partial_t V_m + I_{ion}\Big) + \underline{\nabla} \cdot \Big(\underline{\underline{\sigma}} \cdot \underline{\nabla} V_m\Big) = I_{app}$$

$$\text{with } \underline{\underline{\sigma}} = \Big(\underline{\underline{\sigma}}_i + \underline{\underline{\sigma}}_e\Big)^{-1} \underline{\underline{\sigma}}_i \underline{\underline{\sigma}}_e$$

Some issues …

- I. Forward problem - Modelling
  - Effects of the mechanical deformations
    Methods: Homogenization, Mixture theory

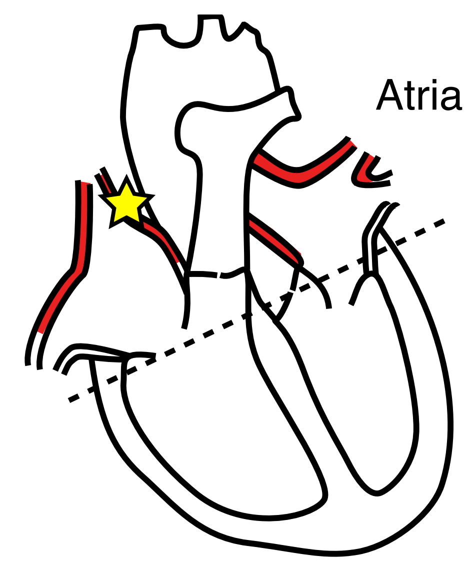
From cell models to 3D model



For the sake of simplicity, in what follows we will neglect these effects.

  - Specific difficulties of the atria (very thin) - Bidomain surface model
    Method: Asymptotic analysis inspired from shell theory

Reduced model



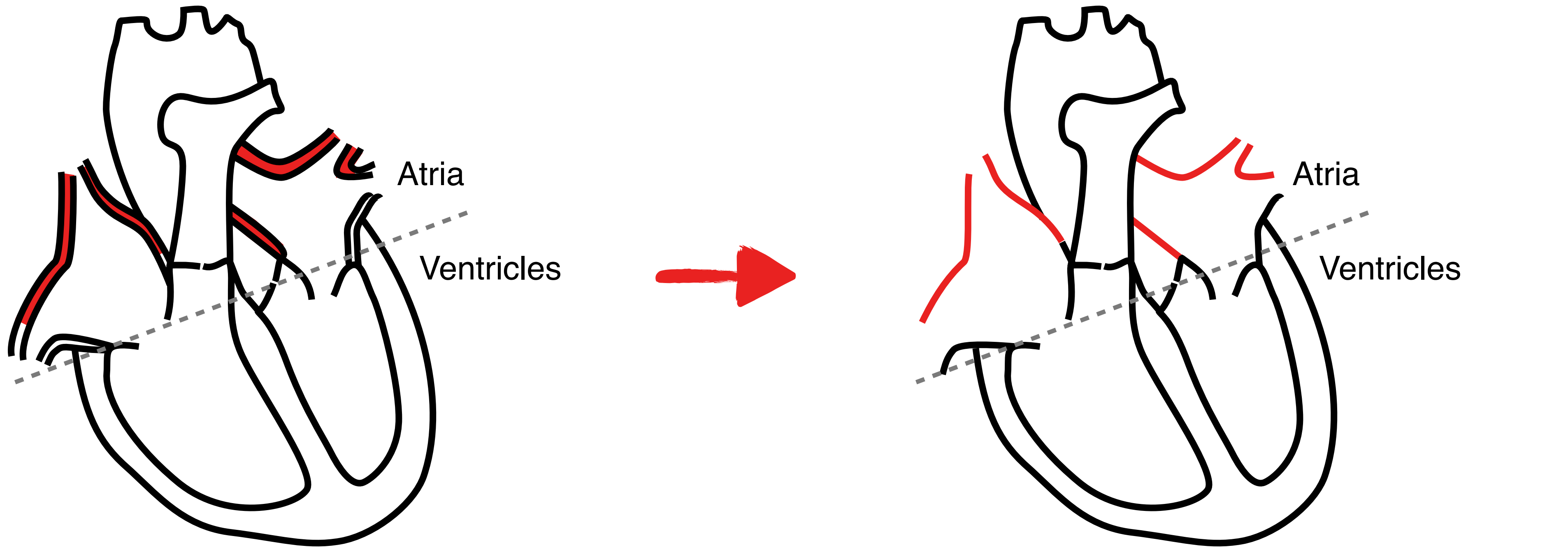
- II. Data assimilation - Patient specific model
  - Estimation with data in the form of level sets of the electrical potential (isochrones)

Parametric estimation: particle models

Bidomain surface model

- Specific difficulties of the atria
  - Very thin
  - Only surface apparent in medical imaging

- Objectives
  - Derive a surface electrophysiology model using **asymptotic analysis** (shell theory)
  - Drastically decrease the computing time



Bidomain surface model

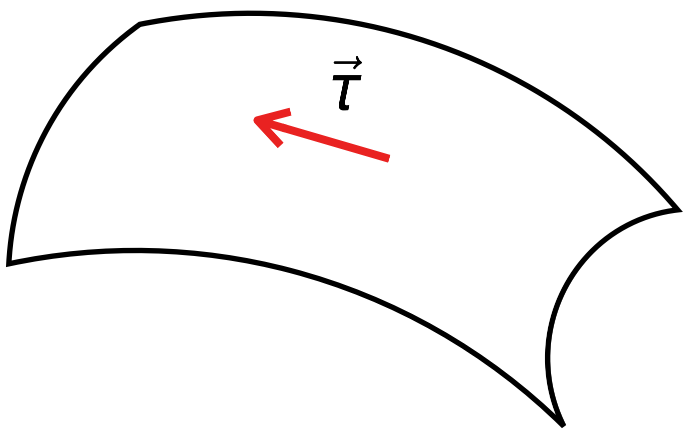
- Bidomain model

$$\begin{cases} A_m\Big(C_m\dfrac{\partial V_m}{\partial t}+I_{ion}(V_m,\cdots)\Big)-\mathrm{div}\big(\vec{\vec{\sigma}}_i\cdot\vec{\nabla}V_m\big) & = & \mathrm{div}\big(\vec{\vec{\sigma}}_i\cdot\vec{\nabla}u_e\big)+I_{app}, & \text{in } \mathcal{B}\times(0,T), \\ \mathrm{div}\Big(\big(\vec{\vec{\sigma}}_i+\vec{\vec{\sigma}}_e\big)\cdot\vec{\nabla}u_e\Big) & = & -\mathrm{div}\big(\vec{\vec{\sigma}}_i\cdot\vec{\nabla}V_m\big), & \text{in } \mathcal{B}\times(0,T), \end{cases}$$

  - Main difficulty: the anisotropy resulting from the preferred conduction direction along the muscle fibers - which may rapidly vary across the thickness.

$$\vec{\vec{\sigma}}_{i,e}=\sigma^t_{i,e}\vec{\vec{I}}+(\sigma^l_{i,e}-\sigma^t_{i,e})\vec{\tau}\otimes\boxed{\vec{\tau}}$$

unit vector parallel to the local fiber direction



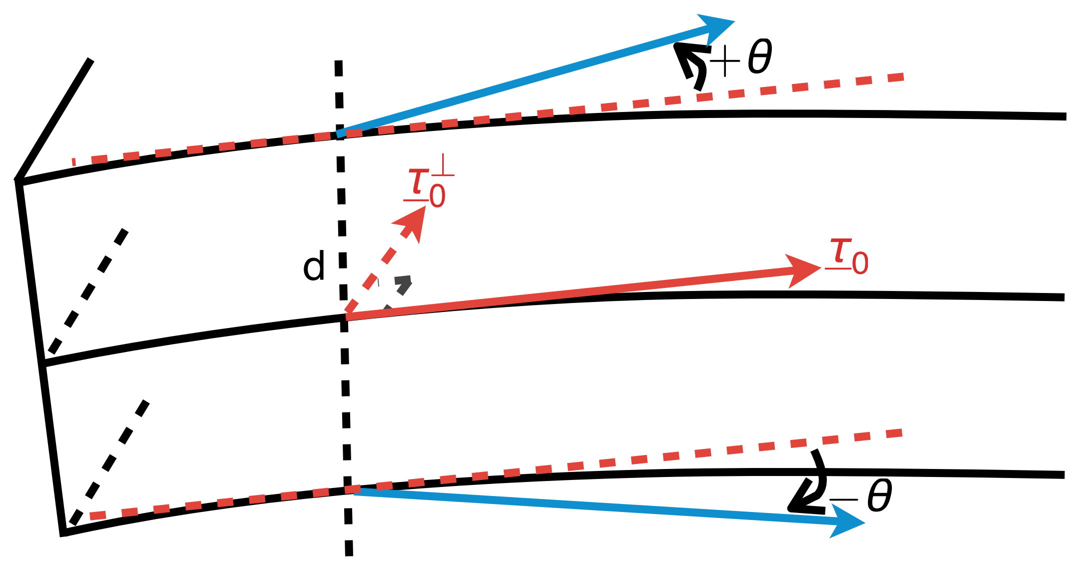
Asymptotic analysis: diffusion problem

Bidomain surface model

- Asymptotic analysis (inspired from the shell theory)
- Diffusion problem
  - Weak form

$$\mathcal{V}^{3D} = H^1(\mathcal{B}) \cap (\mathcal{BC}), \text{ seeking } u \in \mathcal{V}^{3D} \text{ such that}$$
$$\int_{\mathcal{B}} (\vec{\vec{\sigma}} \cdot \vec{\nabla} u) \cdot \vec{\nabla} v \, dV = \int_{\mathcal{B}} f v \, dV, \quad \forall u \in \mathcal{V}^{3D}.$$

  - Diffusion tensors



$$\vec{\vec{\sigma}} = \sigma_m \vec{\vec{I}} + (\sigma_M - \sigma_m) \vec{\tau} \otimes \vec{\tau},$$

$$\vec{\tau} = \underline{\tau}_0(\xi^1, \xi^2) \cos\left(\frac{2\theta\xi^3}{d}\right) + \underline{\tau}_0^{\perp}(\xi^1, \xi^2) \sin\left(\frac{2\theta\xi^3}{d}\right)$$

Bidomain surface model

- Study the problem in the subspace $\mathcal{V} \subset \mathcal{V}^{3D}$: $\forall \epsilon, u^\epsilon = u_0^\epsilon + \xi^3 u_1^\epsilon + (\xi^3)^2 u_2^\epsilon$

$$\int_\Omega (\vec{\vec{\sigma}} \cdot \vec{\nabla}(u_0+\xi^3 u_1+(\xi^3)^2 u_2)) \cdot \vec{\nabla}(v_0+\xi^3 v_1+(\xi^3)^2 v_2) dV = \int_\Omega f(v_0+\xi^3 v_1+(\xi^3)^2 v_2) dV.$$

$\longrightarrow$ Existence and uniqueness of a solution $(u_0^\epsilon, u_1^\epsilon, u_2^\epsilon)$, for all $\epsilon = \frac{d}{\mathrm{diam}(\mathcal{S})}$.

- Limit of $(u_0^\epsilon, u_1^\epsilon, u_2^\epsilon)$ when ϵ tends to 0 ?
  - First step: Identify a limit problem

$\forall v_0 \in H^1(\mathcal{S}) \cap (\mathcal{BC}')$, $\exists! u_0^l \in H^1(\mathcal{S}) \cap (\mathcal{BC}')$ such that

$$\sigma_m \int_\omega \underline{\nabla} u_0^l \cdot \underline{\nabla} v_0 \, dS + (\sigma_M - \sigma_m) \int_\omega \Big((I_0(\theta)\, \underline{\tau}_0 \otimes \underline{\tau}_0 + J_0(\theta) \underline{\tau}_0^\perp \otimes \underline{\tau}_0^\perp) \cdot \underline{\nabla} u_0^l \Big) \cdot \underline{\nabla} v_0 dS \; = \int_\omega f_0 \, v_0 dS,$$

where $I_0(\theta) = \dfrac{1}{2} + \dfrac{\sin(2\theta)}{2\theta}$ and $J_0(\theta) = 1 - I_0(\theta)$ (mean direction $\theta = 0$).

  - Second step: Convergence theorems …
    - $(u_0^\epsilon, u_1^\epsilon)_\epsilon$ *converges strongly to* $(u_0^l, 0)$ *in* $H^1(\mathcal{S}) \times L^2(\mathcal{S})$ *when* ϵ *tends to* 0,
    - ϵu_1^ϵ *and* $\epsilon^2 u_2^\epsilon$ *converge strongly to* 0 *in* $H^1(\mathcal{S})$ *when* ϵ *tends to* 0.

D.Chapelle, A.Collin and J.-F.Gerbeau. A surface-based electrophysiology model relying on asymptotic analysis and motivated by cardiac atria modeling. M3AS, 2012.

Bidomain surface model

- Presentation of the proposed model

  (by applying the results of the derivation of the diffusion terms)

$$
\begin{cases}
A_m \int_\omega \left(C_m \dfrac{\partial V_m}{\partial t} + I_{ion} \right) \phi \, dS + C_m \int_\omega \left(\underline{\underline{\sigma}}_i \cdot (\underline{\nabla} V_m + \underline{\nabla} u_e) \right) \cdot \underline{\nabla} \phi \, dS & = \int_\omega I_{app} \phi \, dS, \\
\int_\omega \left((\underline{\underline{\sigma}}_i + \underline{\underline{\sigma}}_e) \cdot \underline{\nabla} u_e \right) \cdot \underline{\nabla} \psi \, dS + \int_\omega \left(\underline{\underline{\sigma}}_i \cdot \underline{\nabla} V_m \right) \cdot \underline{\nabla} \psi \, dS & = 0,
\end{cases}
$$

with

- $\underline{\underline{\sigma}}_{i,e} = \sigma^t_{i,e} \underline{\underline{I}} + (\sigma^l_{i,e} - \sigma^t_{i,e}) \big(I_0(\theta) \underline{\tau}_0 \otimes \underline{\tau}_0 + J_0(\theta) \underline{\tau}_0^\perp \otimes \underline{\tau}_0^\perp \big),$
- $I_0(\theta) = \dfrac{1}{2} + \dfrac{1}{4\theta} \sin(2\theta)$ and $J_0(\theta) = \dfrac{1}{2} - \dfrac{1}{4\theta} \sin(2\theta) = 1 - I_0(\theta).$

Note: mean direction $\theta = 0$

Finite Elements for Life Sciences and Engineering

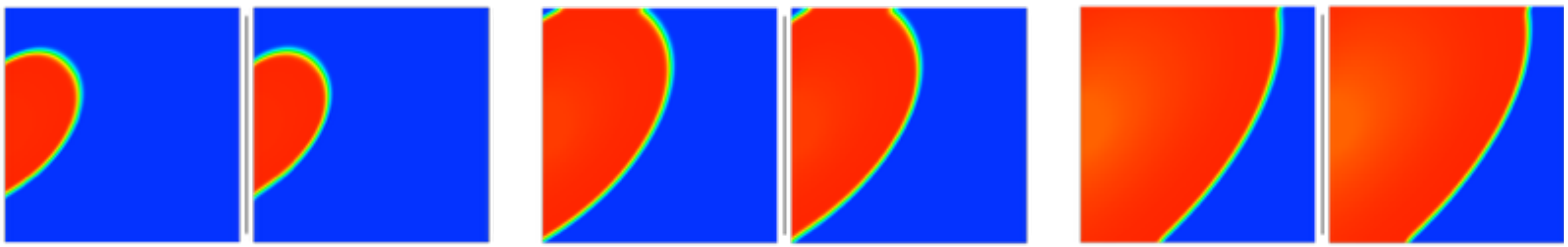
- Developed at Inria (M3DISIM and REO teams)
- Finite element library (started in 2010)
- Unified software environment which contains all the tools needed to perform simulations of complex cardiovascular models (electrophysiology, fluid and solid mechanics and coupling phenomena)
- C++, based on the PETSc library
- Opensource library

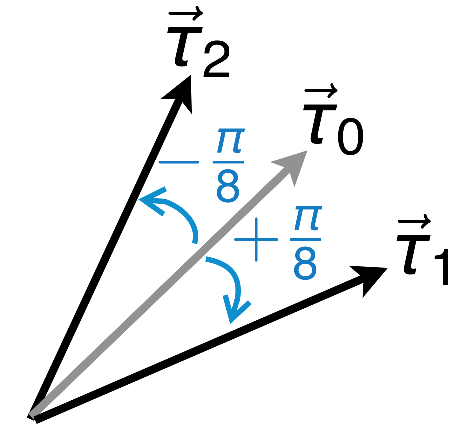
https://gforge.inria.fr/projects/felisce/

Bidomain surface model

- Numerical results
  - Results

Time = 30 ms | Time = 60 ms | Time = 100 ms





3D bidomain model
3D mesh
202,005 dofs

VS

Asymptotic bidomain model
Surface mesh
40,401 dofs (202,005/5)

  - Errors

$$\|u^{ref,3D} - u^{2D}\|_{l^2} = \frac{1}{V_{max} - V_{min}} \left(\frac{1}{\sharp Nodes} \sum_{nodes} |u^{ref,3D} - u^{2D}|^2 \right)^{\frac{1}{2}}$$

T(ms)	30	50	60	70	100	140	
$\lvert\cdot\rvert_{l^2}$	$6.764\,10^{-3}$	$1.179\,10^{-2}$	$1.442\,10^{-2}$	$1.624\,10^{-2}$	$1.933\,10^{-2}$	$1.934\,10^{-2}$	< 2%

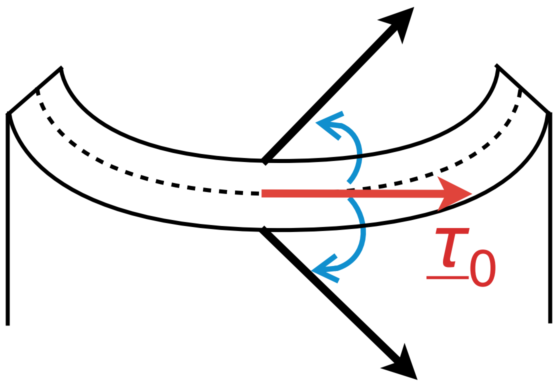
  - Computing time (15,000 iterations)

432 min (~7h30) $\xrightarrow{/\,17}$ 26 min

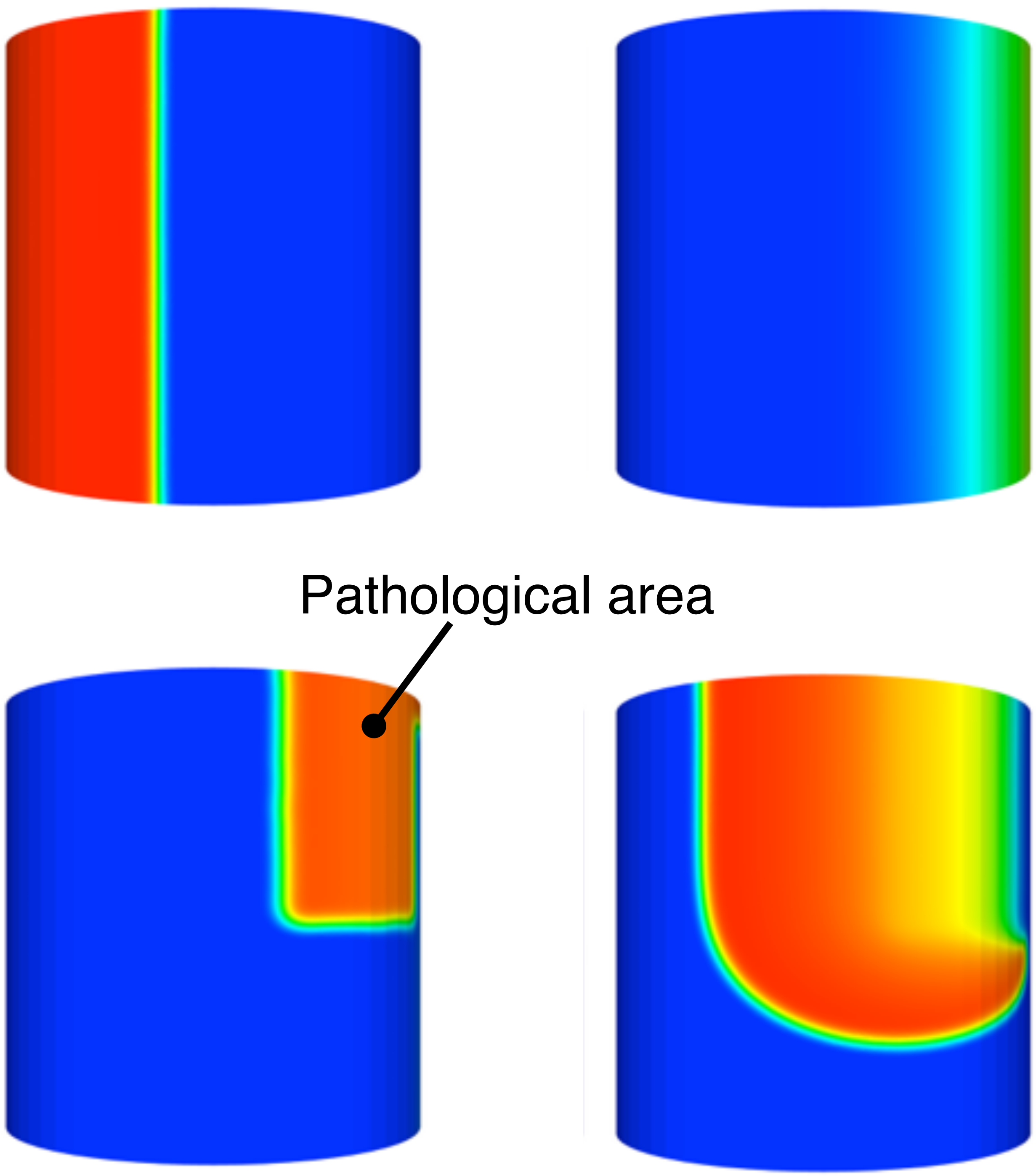
0 26 min 432 min

Bidomain surface model

- Spiral waves, an interesting benchmark in electrophysiology



More complex geometry
Less regular solution



Göektepe, Kuhl. Computational modeling of cardiac electrophysiology: a novel finite element approach. 2009.

Bidomain surface model

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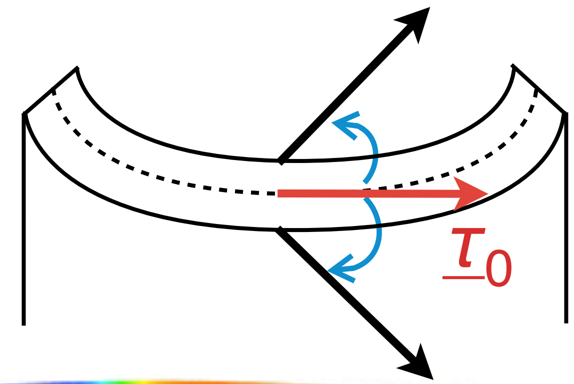

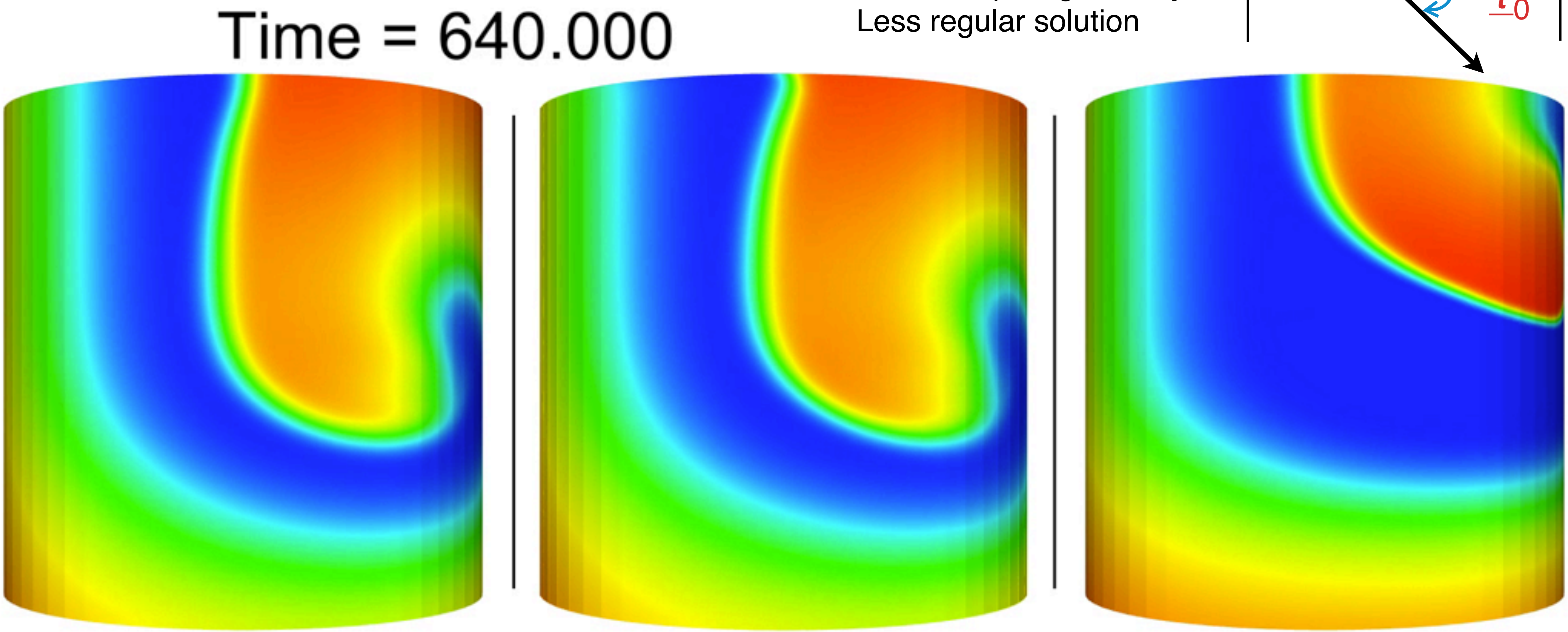

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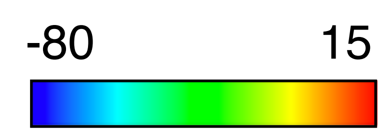
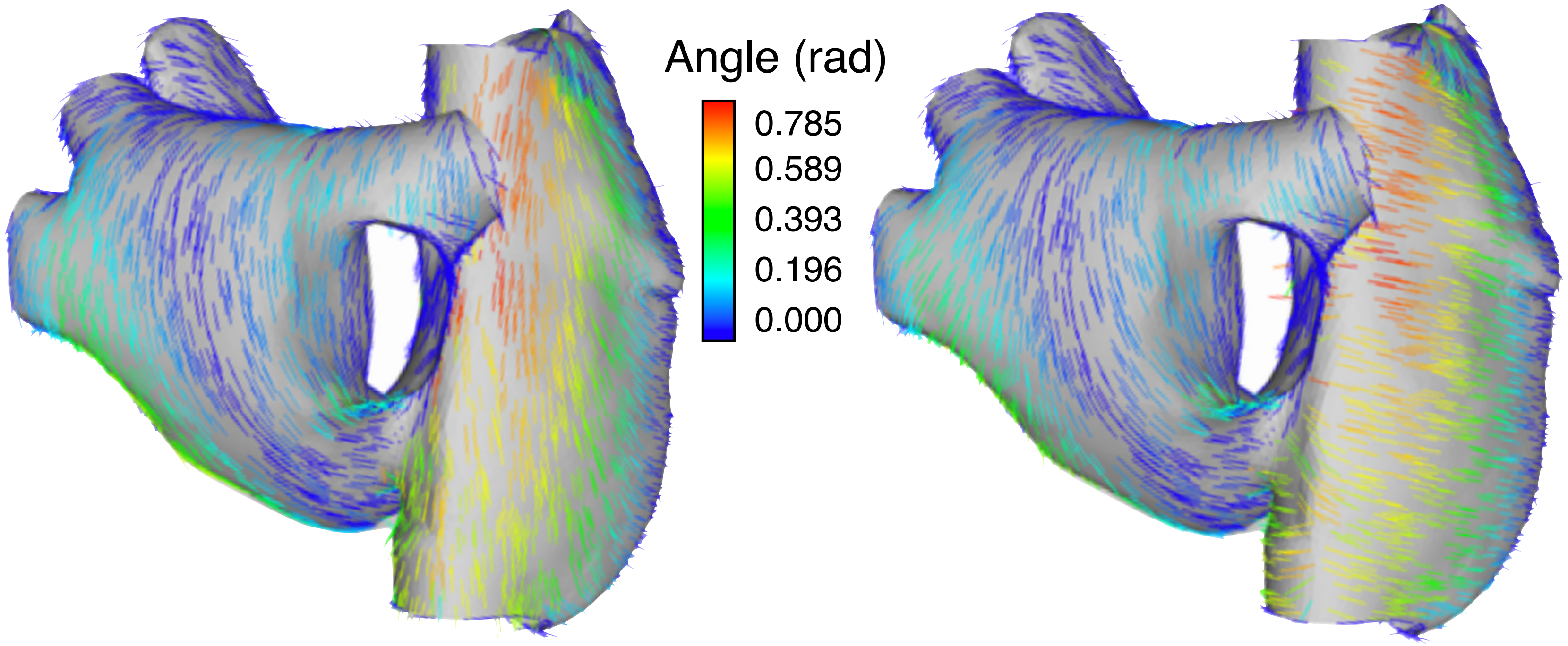


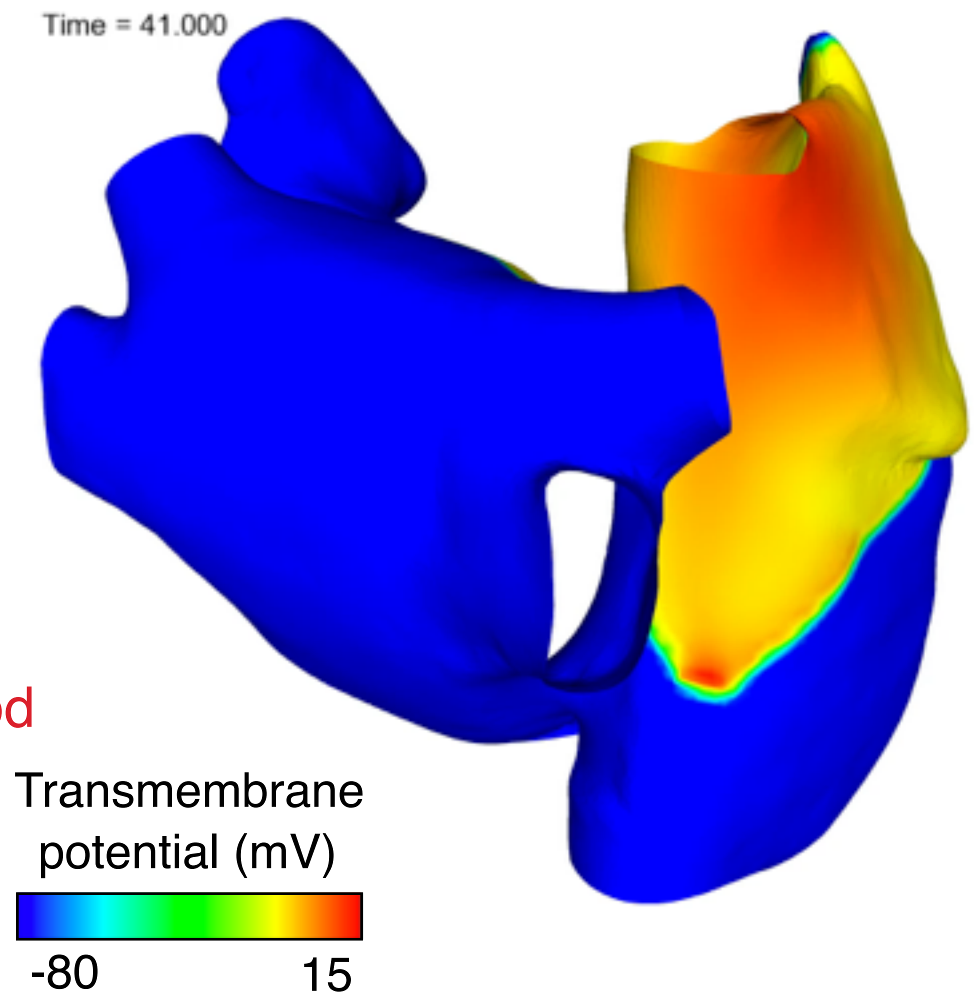
Bidomain surface model (left) versus **3D model** (center) and **2D naive model** (right)

Modelling - Bidomain surface model

- Numerical simulations of the Bidomain surface model

Endocardium (inner surface) *Epicardium (outer surface)*





Realistic geometry and fibers prescribed by a semi-automatic method

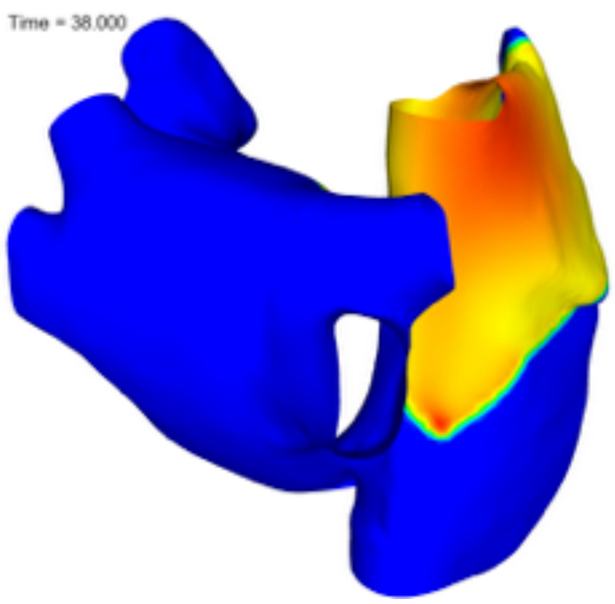
Collaboration with Michel Haïssaguerrre and Mélèze Hocini (CHU Bordeaux and IHU Liryc)

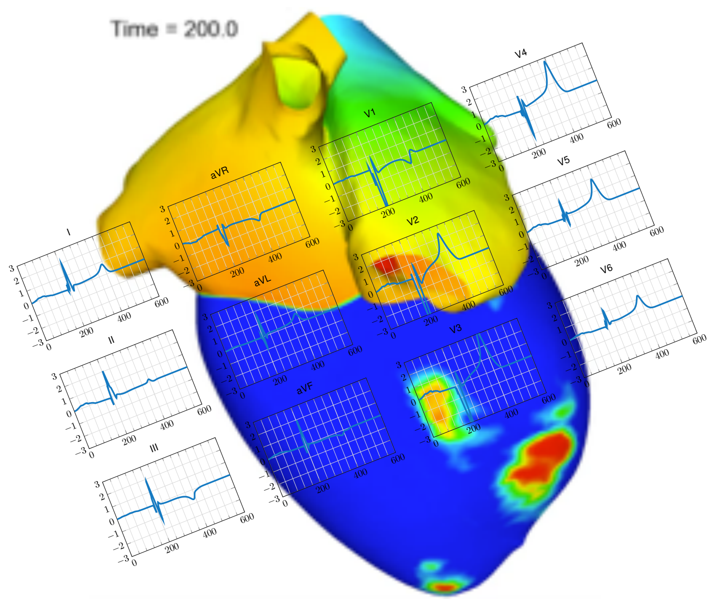
Good adequacy with 3D modelling studies

A.Collin, J.-F. Gerbeau, M. Hocini, M. Haïssaguerre, D. Chapelle. Surface-based electrophysiological modeling and assessment of physiological simulations in atria. Proc of FIMH 2013.

Conclusions / Perspectives (modelling part)

- Conclusions
  - Bidomain surface model adapted to thin cardiac structures (atria)
  - Full electrophysiological model (and full electrocardiograms)

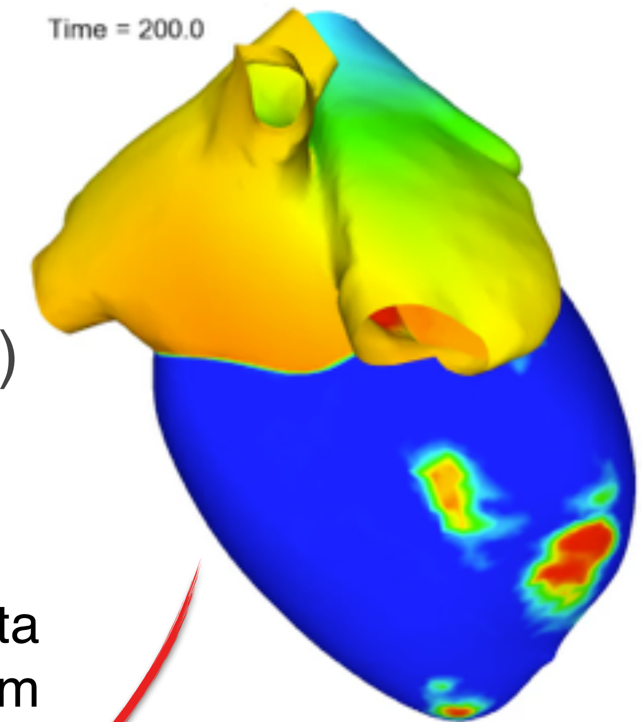


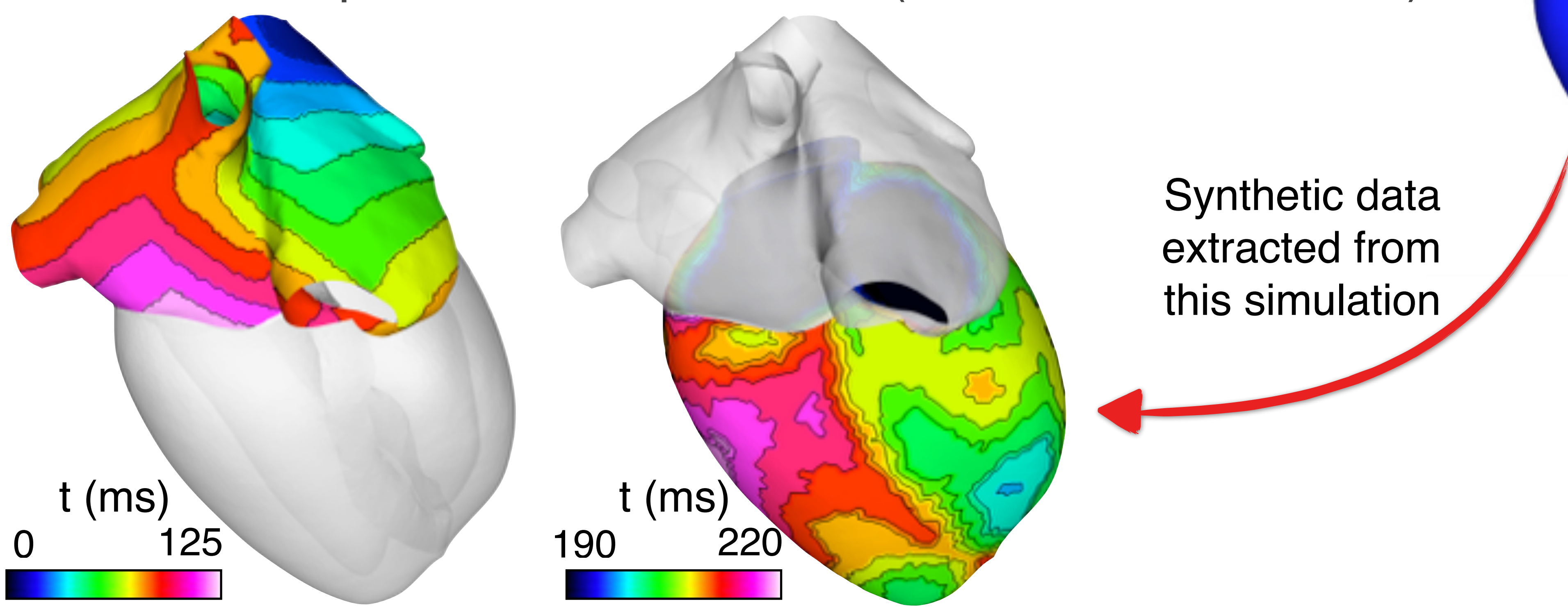


- Perspectives
  - Asymptotic electro-mechanical model: atria & right ventricle
  - Effects of mechanical deformations on the electrocardiograms
  - **And … patient specific model ?**

Data assimilation - Research focus

- Motivations
  - A personalised model typically for a patient
  - Can be used for predictive purposes: providing diagnosis and prognosis assistance
- Starting point
  - Very realistic simulations of the full heart (atria and ventricles)
- Which data ?
  - Depolarization maps obtained from ECGI (a multi-electrode vest)

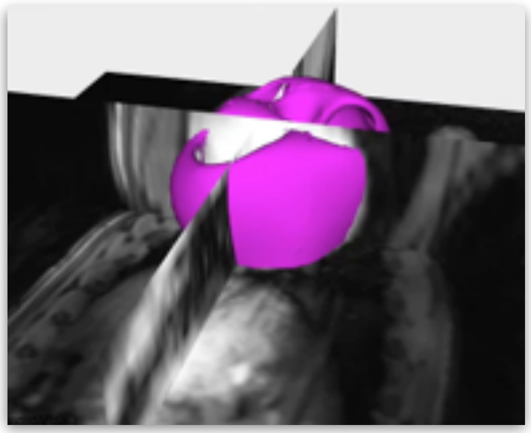




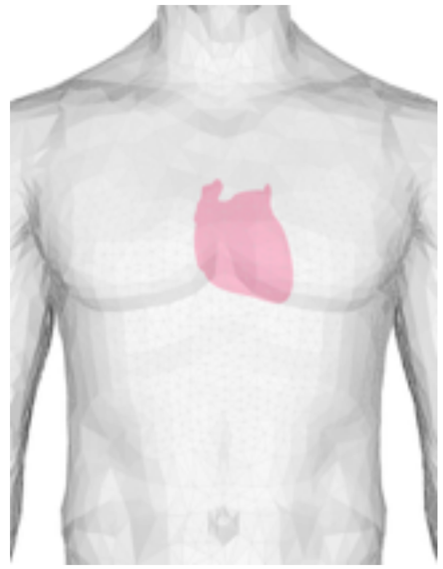
- Objective
  - An effective strategy for performing estimation in an electrophysiology model with data in the form of level sets of the electrical potential (isochrones)

Data assimilation - ECGI

- How are obtained the front level-set data ?

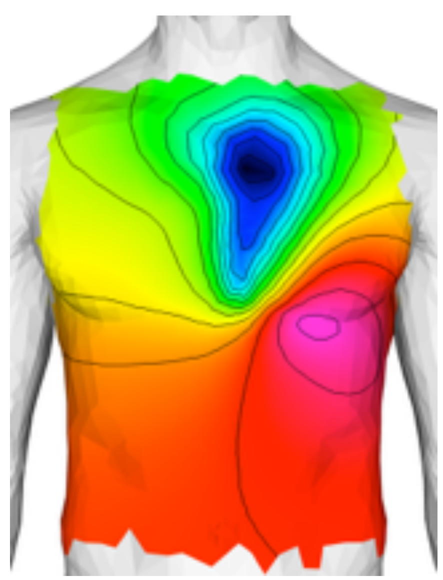


Images (CT, IRM)



Heart and torso geometries

Electrodes vest (multiple surface ECG recordings)

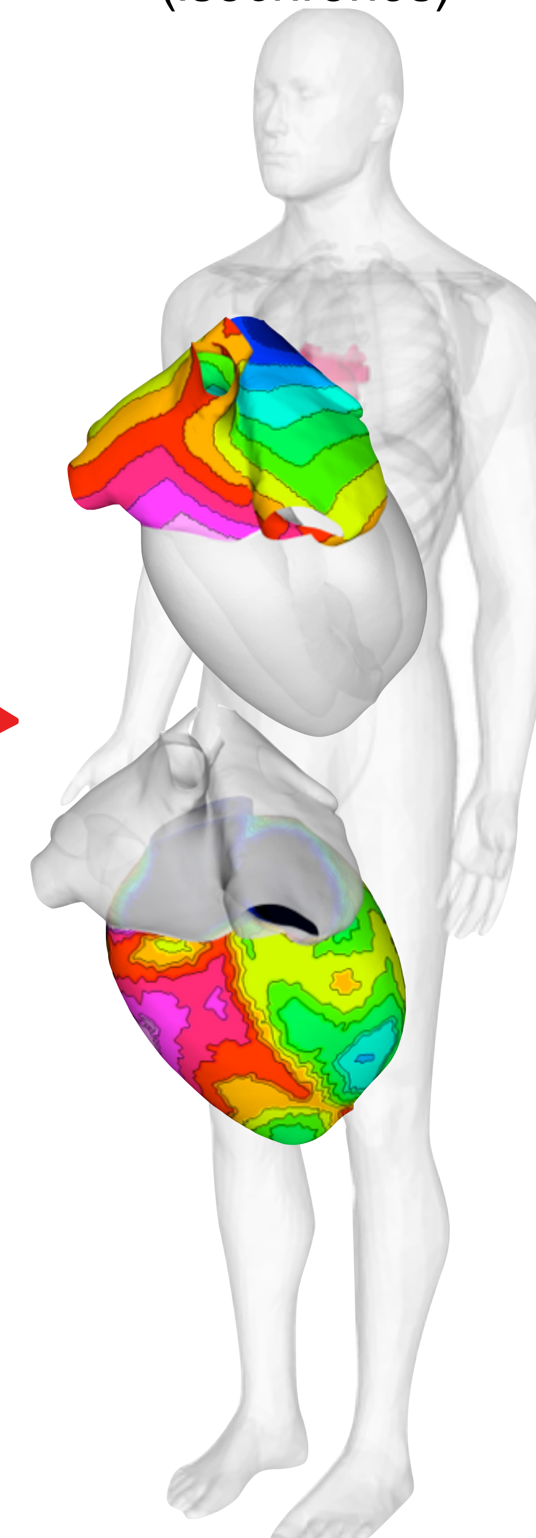


Body surface potentials

Computing inverse problem

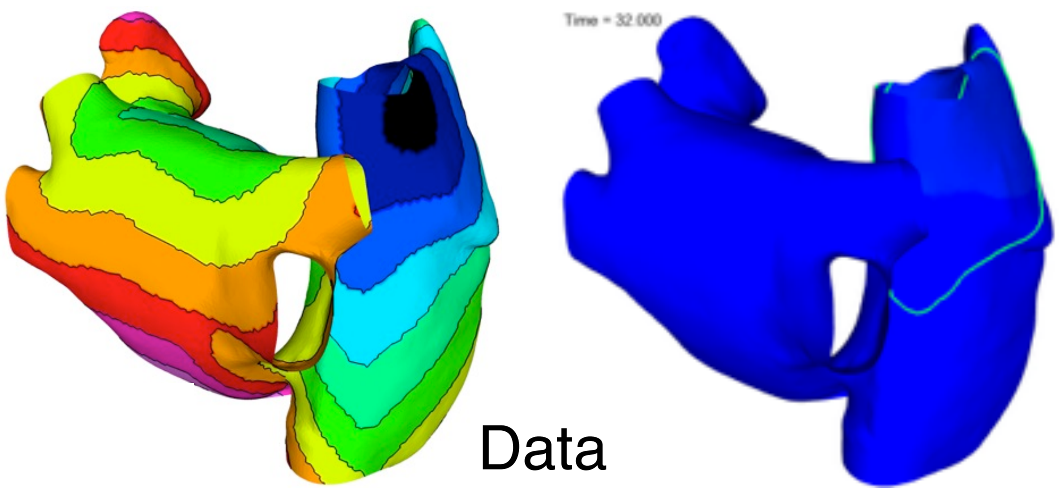
DATA

Maps of electrical activation (isochrones)



Data assimilation - Methodology

- Considering depolarisation-times data obtained from ECGI measurements and the corresponding reconstructed fronts, can we estimate the state and the parameters of the bidomain model for a patient specific adjustment ?



Data

$$
\begin{cases}
A_m\left(C_m \dfrac{\partial u}{\partial t} + I_{ion}(u, \cdots)\right) - \mathrm{div}(\vec{\vec{\sigma}}_i \cdot \vec{\nabla} u) \\
\qquad\qquad = \mathrm{div}(\vec{\vec{\sigma}}_i \cdot \vec{\nabla} u_e) + I_{app}, \\
\mathrm{div}\left((\vec{\vec{\sigma}}_i + \vec{\vec{\sigma}}_e) \cdot \vec{\nabla} u_e\right) = -\mathrm{div}(\vec{\vec{\sigma}}_i \cdot \vec{\nabla} u).
\end{cases}
$$

Model

Patient specific simulations ?

E. Konukoglu et al. Image guided personalization of reaction-diffusion type tumor growth models using modified anisotropic eikonal equations. 2010.

V. Moreau-Villegere et al. Building maps of local apparent conductivity of the epicardium with a 2D electrophysiological model of the heart. 2006.

- Methodological point of view

Sequential estimation on the complete bidomain model.

Data assimilation - Sequential methods

- Target model

$$\begin{cases} \dot{u}(t) &= A(u, \theta, t) \\ u(0) &= u_\diamond + \zeta^u \\ \theta(0) &= \theta_\diamond + \zeta^\theta \end{cases}$$

$(u_\diamond, \theta_\diamond)$, *a priori*
(ζ^u, ζ^θ), unknown parts

- Data assimilation procedure (sequential methods) - Observer model

Data assimilation - Sequential methods

- Target model

$$\begin{cases} \dot{u}(t) &= A(u, \theta, t) \\ u(0) &= u_\diamond + \zeta^u \\ \theta(0) &= \theta_\diamond + \zeta^\theta \end{cases}$$

$(u_\diamond, \theta_\diamond)$, *a priori*
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- Data assimilation procedure (sequential methods) - Observer model

$$\begin{cases} \dot{\hat{u}}(t) &= A(\hat{u}, \theta, t) \\ \hat{u}(0) &= u_\diamond \\ \hat{\theta}(0) &= \theta_\diamond \end{cases}$$

Data assimilation - Sequential methods

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$$\begin{cases} \dot{u}(t) & = & A(u, \theta, t) \\ u(0) & = & u_\diamond + \zeta^u \\ \theta(0) & = & \theta_\diamond + \zeta^\theta \end{cases}$$

$(u_\diamond, \theta_\diamond)$, *a priori*
(ζ^u, ζ^θ), unknown parts

Observations z_u

- Data assimilation procedure (sequential methods) - Observer model

$$\begin{cases} \dot{\hat{u}}(t) & = & A(\hat{u}, \theta, t) \\ \hat{u}(0) & = & u_\diamond \\ \hat{\theta}(0) & = & \theta_\diamond \end{cases}$$

Data assimilation - Sequential methods

- Target model

$$\begin{cases} \dot{u}(t) &= A(u,\theta,t) \\ u(0) &= u_\diamond + \zeta^u \\ \theta(0) &= \theta_\diamond + \zeta^\theta \end{cases}$$

$(u_\diamond, \theta_\diamond)$, *a priori*
(ζ^u, ζ^θ), unknown parts

Observations z_u

- Data assimilation procedure (sequential methods) - Observer model

$$\begin{cases} \dot{\hat{u}}(t) &= A(\hat{u},\theta,t) + G_u\left(D(z_u,\hat{u})\right) \\ \hat{u}(0) &= u_\diamond \\ \hat{\theta}(0) &= \theta_\diamond \end{cases}$$

Data assimilation - Sequential methods

- Target model

$$\begin{cases} \dot{u}(t) & = & A(u, \theta, t) \\ u(0) & = & u_\diamond + \zeta^u \\ \theta(0) & = & \theta_\diamond + \zeta^\theta \end{cases}$$

$(u_\diamond, \theta_\diamond)$, *a priori*
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Observations z_u

- Data assimilation procedure (sequential methods) - Observer model

$$\begin{cases} \dot{\hat{u}}(t) & = & A(\hat{u}, \theta, t) + G_u\left(\boxed{D(z_u, \hat{u})}\right) \\ \hat{u}(0) & = & u_\diamond \\ \hat{\theta}(0) & = & \theta_\diamond \end{cases}$$

Discrepancy D (data fitting term)

Data assimilation - Sequential methods

- Target model

$$\begin{cases} \dot{u}(t) &= A(u, \theta, t) \\ u(0) &= u_\diamond + \zeta^u \\ \theta(0) &= \theta_\diamond + \zeta^\theta \end{cases}$$

$(u_\diamond, \theta_\diamond)$, *a priori*
(ζ^u, ζ^θ), unknown parts

Observations z_u

- Data assimilation procedure (sequential methods) - Observer model

$$\begin{cases} \dot{\hat{u}}(t) &= A(\hat{u}, \theta, t) + \boxed{G_u}\left(\boxed{D(z_u, \hat{u})}\right) \\ \hat{u}(0) &= u_\diamond \\ \hat{\theta}(0) &= \theta_\diamond \end{cases}$$

Discrepancy D (data fitting term)

Gain operator G_u

- Strategy
  - 1. Find a state observer for the Reaction - Diffusion model

Data assimilation - Sequential methods

- Target model

$$\begin{cases} \dot{u}(t) &= A(u, \theta, t) \\ u(0) &= u_\diamond + \zeta^u \\ \theta(0) &= \theta_\diamond + \zeta^\theta \end{cases}$$

$(u_\diamond, \theta_\diamond)$, *a priori*
(ζ^u, ζ^θ), unknown parts

Observations z_u

- Data assimilation procedure (sequential methods) - Observer model

$$\begin{cases} \dot{\hat{u}}(t) &= A(\hat{u}, \theta, t) + \boxed{G_u^\flat}\big(\boxed{D(z_u, \hat{u})}\big) \\ \dot{\hat{\theta}}(t) &= \boxed{G_\theta}\big(\boxed{D(z_u, \hat{u})}\big) \\ \hat{u}(0) &= u_\diamond \\ \hat{\theta}(0) &= \theta_\diamond \end{cases}$$

Discrepancy D (data fitting term)

Gain operators G_u and G_θ

$$\boxed{(\hat{u}, \hat{\theta}) \to (u, \theta)}$$

Moireau, **Chapelle**, and **LeTallec**. Joint state and parameter estimation for distributed mechanical systems. 2008.

[...] the methodology proposed to extend state estimation to joint state-parameter estimation is general and – indeed – applicable with any state filter.

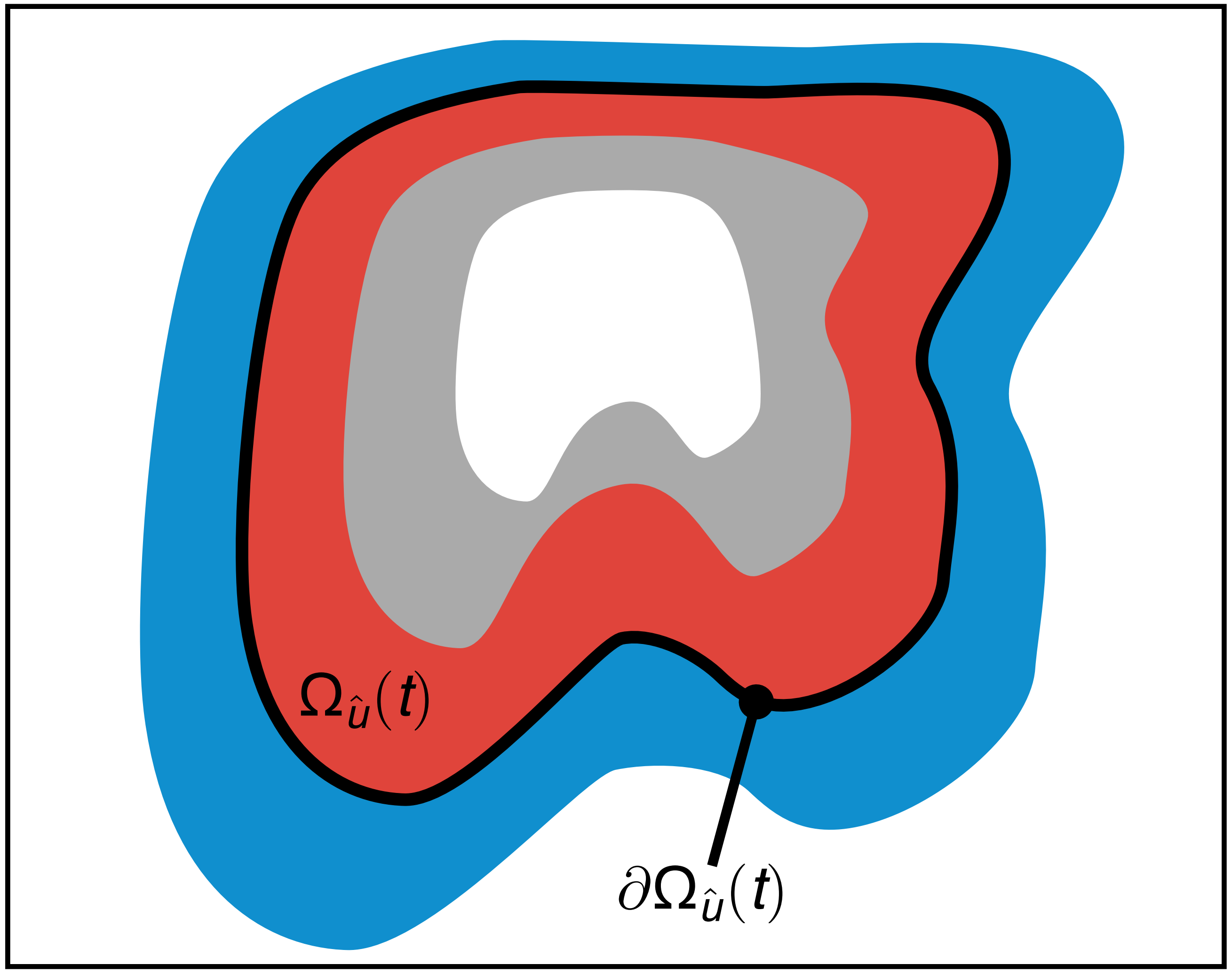
- Strategy
  - 1. Find a state observer for the Reaction - Diffusion model
  - 2. Extend to parameter observer

Data assimilation - Level set formulation

- Reaction-diffusion model

$$\begin{cases} \partial_t \hat{u} - \vec{\nabla} \cdot (D \vec{\nabla} \hat{u}) = k f(\hat{u}), & \Omega \times (0, T), \\ (D \vec{\nabla} \hat{u}) \cdot \underline{n} = 0, & \partial\Omega \times (0, T), \\ \hat{u}(\vec{x}, 0) = \hat{u}_0(\vec{x}), & \Omega. \end{cases}$$

- c_{th} is the depolarization constant
  i.e. $\Omega_{\hat{u}}(t) = \{\vec{x} \in \mathcal{B},\ \hat{u}(\vec{x}, t) > c_{th}\}$
  is the depolarized area at each time



- Level set formulation
  $\phi_{\hat{u}}$ a level set associated with $\Omega_{\hat{u}}(t)$
  i.e. $\Omega_{\hat{u}}(t) = \{\vec{x},\ \phi_{\hat{u}}(\vec{x},\ t) > 0\}$
  and $\partial\Omega_{\hat{u}}(t) = \Gamma_{\hat{u}}(t) = \{\vec{x},\ \phi_{\hat{u}}(\vec{x},\ t) = 0\}$

- Eikonal Equation model verified by $\phi_{\hat{u}}$

Data assimilation - Level set formulation

- Reaction-diffusion model

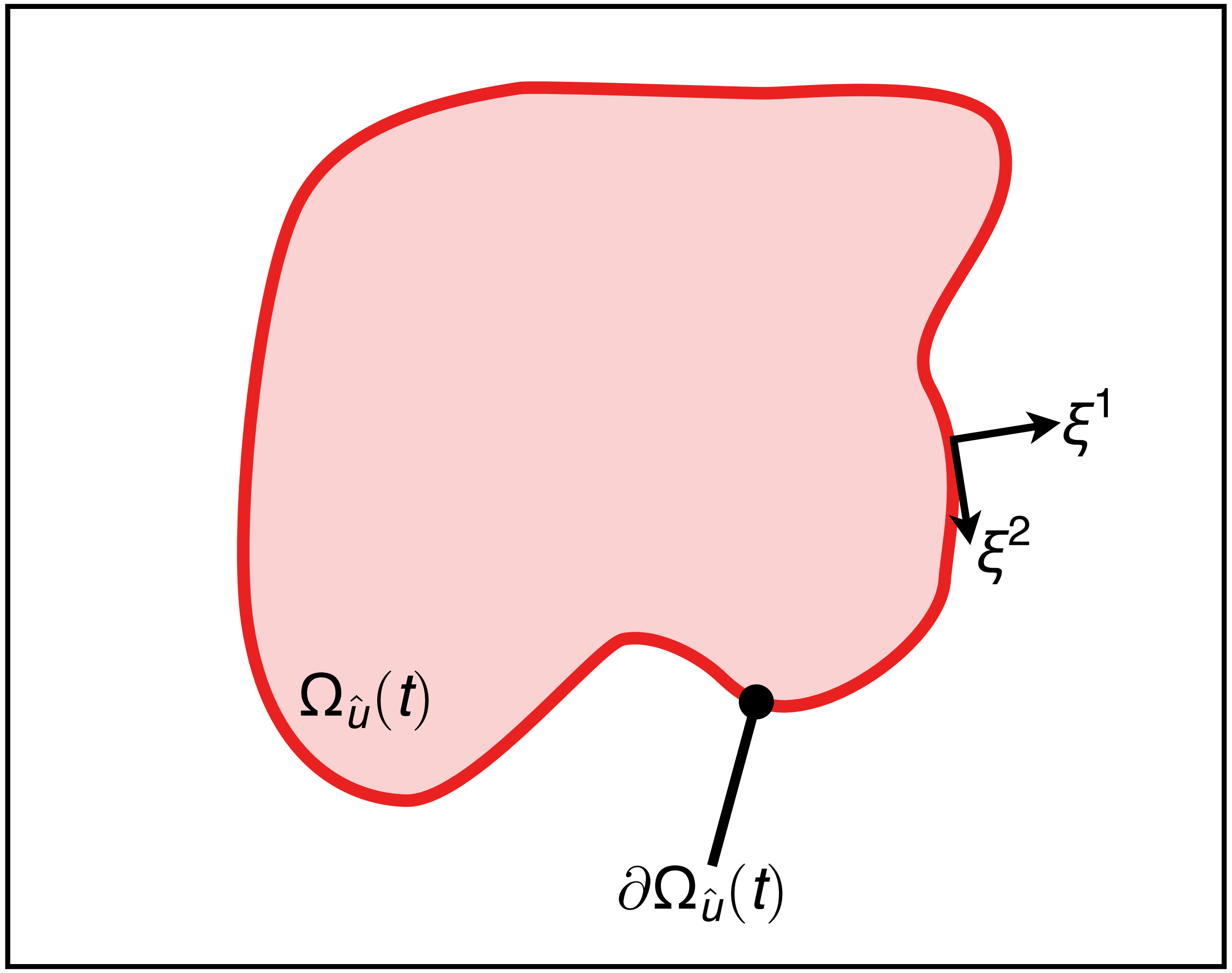
$$\left\{\begin{array}{rcll} \partial_t \hat{u} - \vec{\nabla}\cdot(D\vec{\nabla}\hat{u}) & = & k f(\hat{u}), & \Omega\times(0,T), \\ (D\vec{\nabla}\hat{u})\cdot \underline{n} & = & 0, & \partial\Omega\times(0,T), \\ \hat{u}(\vec{x},0) & = & \hat{u}_0(\vec{x}), & \Omega. \end{array}\right.$$

- Asymptotic analysis along the direction of the front i.e. along ξ_1

- Eikonal equation

$$\partial_t \phi_{\hat{u}} = |\vec{\nabla}\phi_{\hat{u}}|\Big(D\vec{\nabla}\cdot\Big(\frac{\vec{\nabla}\phi_{\hat{u}}}{|\vec{\nabla}\phi_{\hat{u}}|}\Big) + \sqrt{Dk}\,c_0\Big), \qquad \mathcal{B}\times(0,T).$$

Keener, An eikonal-curvature equation for action potential propagation in myocardium. 1991.



Data assimilation - Image processing

- Minimization of the following energy in object detection for image processing

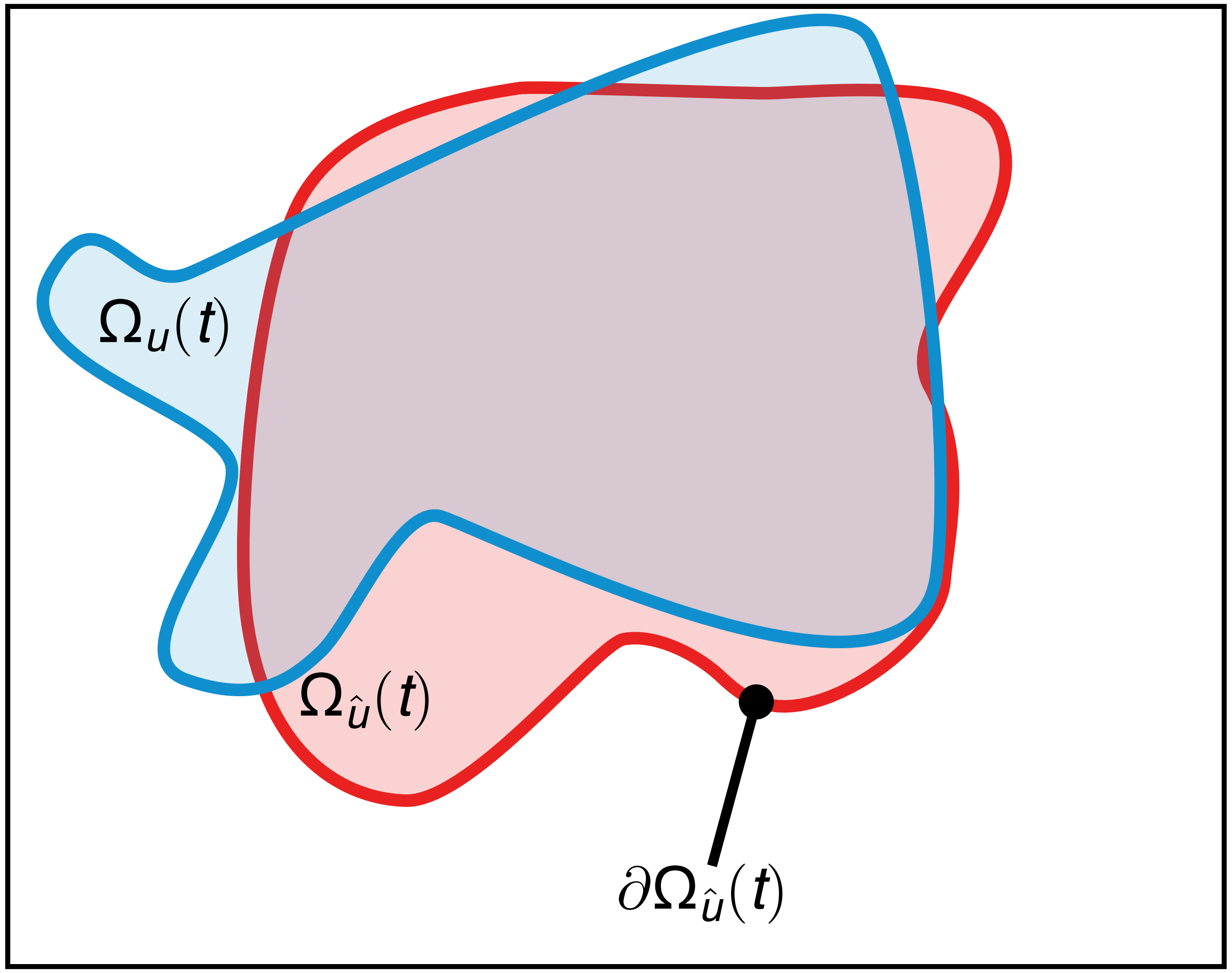
Courtesy Shawn Lankton

$$E(\phi_{\hat{u}}) = \text{regularization terms} + \boxed{\lambda_1 \int_{\Omega_{\hat{u}}} \left(z_u - C_1(\Omega_{\hat{u}})\right)^2 d\underline{x}} + \boxed{\lambda_2 \int_{\mathcal{B}\setminus\Omega_{\hat{u}}} \left(z_u - C_2(\Omega_{\hat{u}})\right)^2 d\underline{x}}.$$

Data terms

$$C_1(\Omega_{\hat{u}}) = \frac{1}{|\Omega_{\hat{u}}|} \int_{\Omega_{\hat{u}}} z_u \, d\underline{x}$$

$$C_2(\Omega_{\hat{u}}) = \frac{1}{|\mathcal{B} \setminus \Omega_{\hat{u}}|} \int_{\mathcal{B}\setminus\Omega_{\hat{u}}} z_u \, d\underline{x}$$



Osher, **Fedkiw**. Level Set Methods and Dynamic Implicit Surfaces. 2002.

Chan, Vese. Active contour without edges. 2001.

Hintermüller, **Laurin**. Multiphase image segmentation and modulation recovery based on shape and topological sensitivity. 2002.

Data assimilation - Image processing

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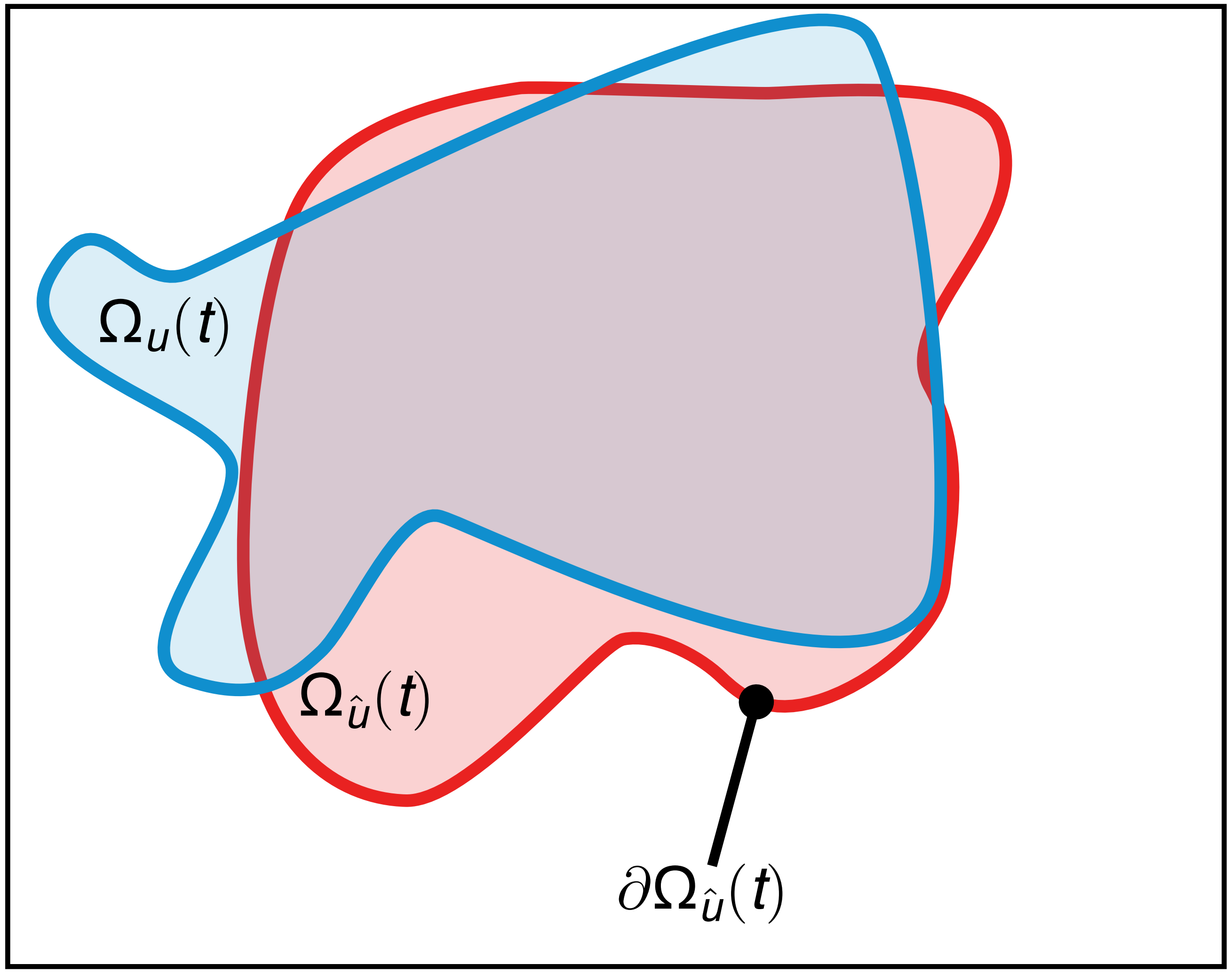

Osher, **Fedkiw**. Level Set Methods and Dynamic Implicit Surfaces. 2002.

Chan, Vese. Active contour without edges. 2001.

Hintermüller, **Laurin**. Multiphase image segmentation and modulation recovery based on shape and topological sensitivity. 2002.

Data assimilation - State observer

- Minimization of the functional by gradient projection method

$$\partial_t \phi_{\hat{u}} = -\partial_{\phi_{\hat{u}}} E$$

$$\partial_t \phi_{\hat{u}} = \cdots + \delta(\phi_{\hat{u}})\Big(-\lambda_1 \big(z_u - C_1(\Omega_{\hat{u}})\big)^2 + \lambda_2 \big(z_u - C_2(\Omega_{\hat{u}})\big)^2\Big)$$

- State observer for eikonal equation

$$\partial_t \phi_{\hat{u}} = |\vec{\nabla} \phi_{\hat{u}}| \Big(D\, \vec{\nabla} \cdot \Big(\frac{\vec{\nabla} \phi_{\hat{u}}}{|\vec{\nabla} \phi_{\hat{u}}|}\Big) + \sqrt{Dk} c_0\Big)$$
$$+ \lambda \delta(\phi_{\hat{u}}) \Big(-\Big(z_u - C_1(\Omega_{\hat{u}})\Big)^2 + \Big(z_u - C_2(\Omega_{\hat{u}})\Big)^2\Big)$$

Asymptotic analysis

- State observer for Reaction-Diffusion model

$$\partial_t \hat{u} = \vec{\nabla} \cdot (D \vec{\nabla} \hat{u}) + k f(\hat{u})$$
$$+ \lambda \delta(\phi_{\hat{u}}) \frac{1}{|\vec{\nabla} \phi_{\hat{u}}|} \frac{\vec{\nabla} \phi_{\hat{u}}}{|\vec{\nabla} \phi_{\hat{u}}|} \cdot \vec{\nabla} \hat{u} \Big[-\Big(z_u - C_1(\Omega_{\hat{u}})\Big)^2 + \Big(z_u - C_2(\Omega_{\hat{u}})\Big)^2\Big]$$

Data assimilation - State observer

- Minimization of the functional by gradient projection method

$$\partial_t \phi_{\hat{u}} = -\partial_{\phi_{\hat{u}}} E$$

$$\partial_t \phi_{\hat{u}} = \dots + \delta(\phi_{\hat{u}})\Big(-\lambda_1 \big(z_u - C_1(\Omega_{\hat{u}})\big)^2 + \lambda_2 \big(z_u - C_2(\Omega_{\hat{u}})\big)^2\Big)$$

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$$\partial_t \phi_{\hat{u}} = |\vec{\nabla}\phi_{\hat{u}}|\Big(D\,\vec{\nabla}\cdot\Big(\frac{\vec{\nabla}\phi_{\hat{u}}}{|\vec{\nabla}\phi_{\hat{u}}|}\Big) + \sqrt{Dk}c_0\Big)$$
$$+ \lambda\delta(\phi_{\hat{u}})\Big(-\Big(z_u - C_1(\Omega_{\hat{u}})\Big)^2 + \Big(z_u - C_2(\Omega_{\hat{u}})\Big)^2\Big)$$

Asymptotic analysis

- State observer for Reaction-Diffusion model

$$\partial_t \hat{u} = \vec{\nabla}\cdot(D\vec{\nabla}\hat{u}) + kf(\hat{u})$$
$$+ \lambda\delta(\hat{u} - c_{th})\Big[-\Big(z_u - C_1(\Omega_{\hat{u}})\Big)^2 + \Big(z_u - C_2(\Omega_{\hat{u}})\Big)^2\Big]$$

Remark : $\phi_{\hat{u}} = \hat{u} - c_{th}$ is a level set associated with $\partial\Omega_{\hat{u}}$

Data assimilation - State observer

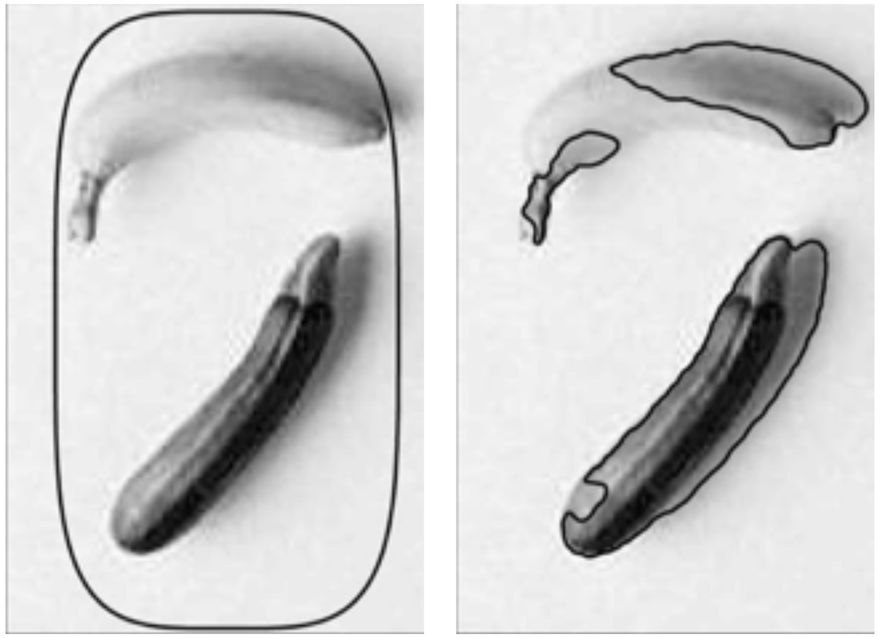
- Mathematical study

Theorem

If the data front is sufficiently contrasted (i.e. sharpness of the front) then the observer term is a stabilizing term.

- Error model $\tilde{u} = u - \hat{u}$
- Linearisation using shape derivatives
- Constrast condition

Courtesy, Hintermüller and Ring

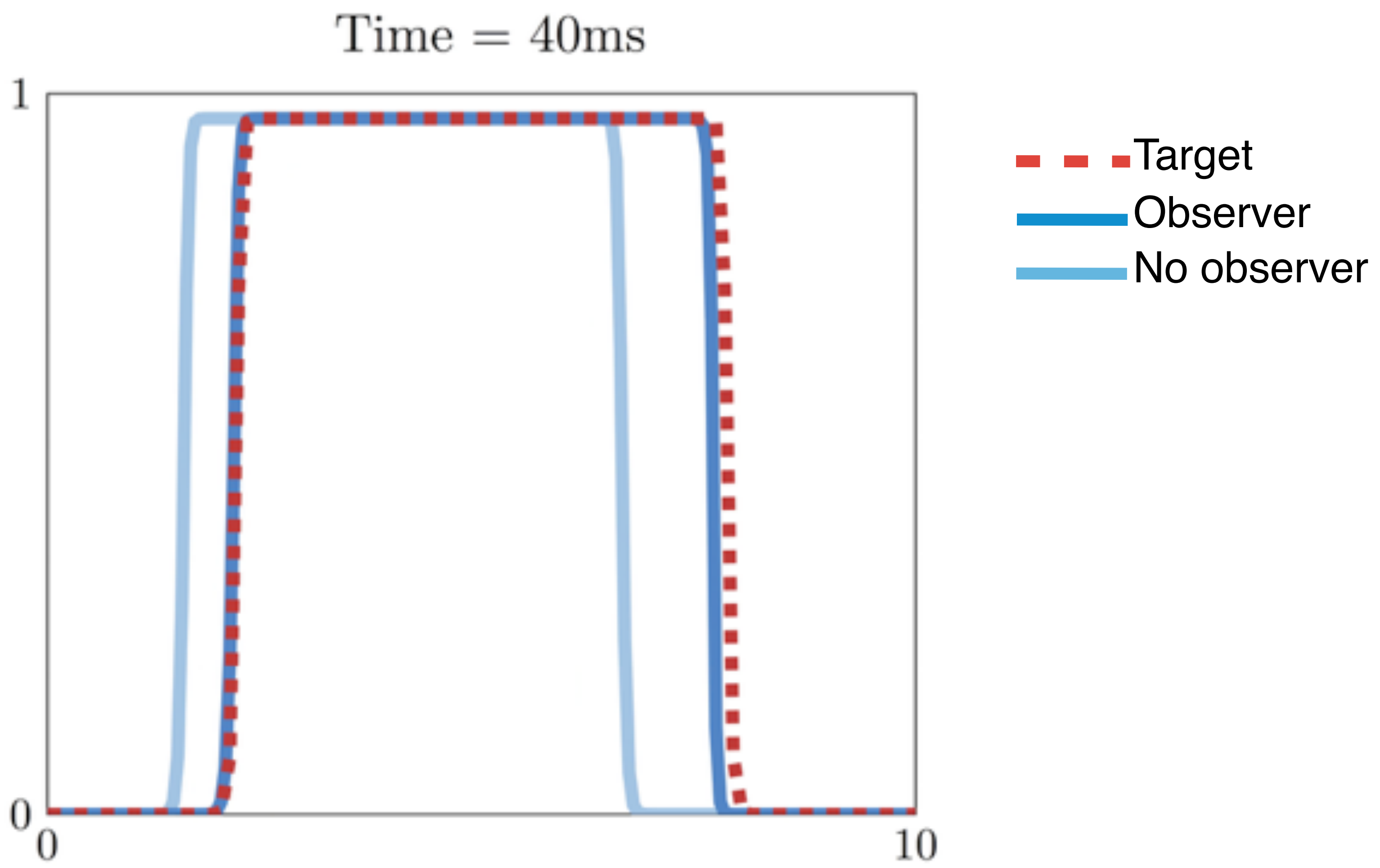


A.Collin, D. Chapelle and P. Moireau. A Luenberger observer for reaction-diffusion models with front position data. JCP 2015.

Data assimilation - 1D simulations

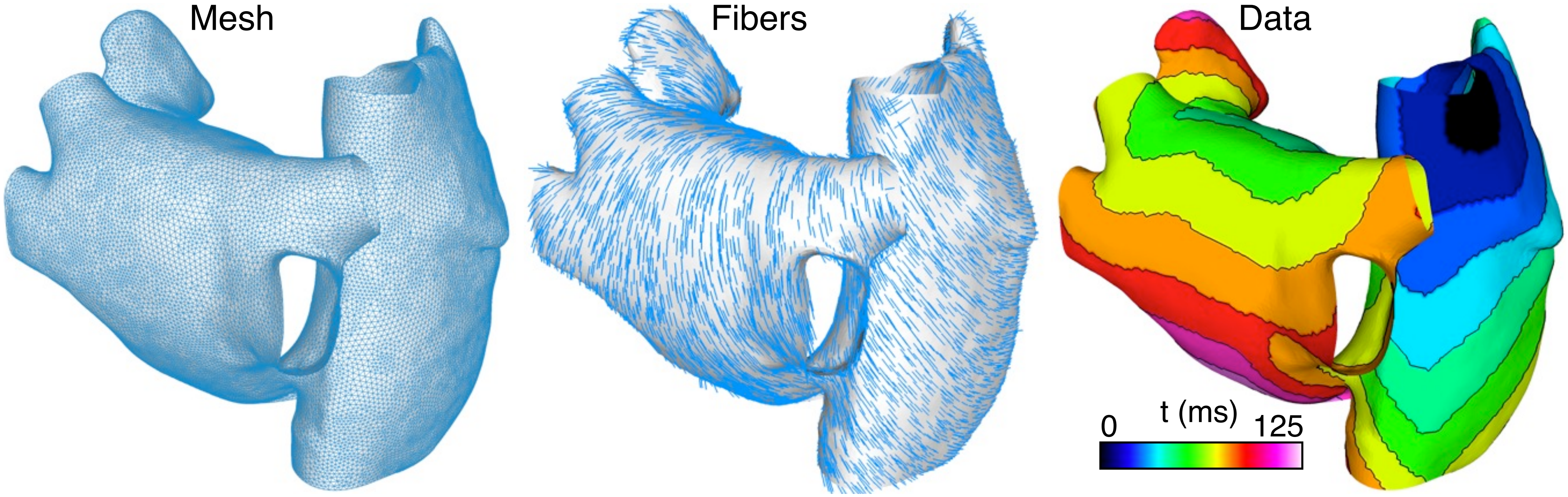
- An illustrative example in 1D
- With Mitchell-Schaeffer ionic model

Shifted and delayed initial condition

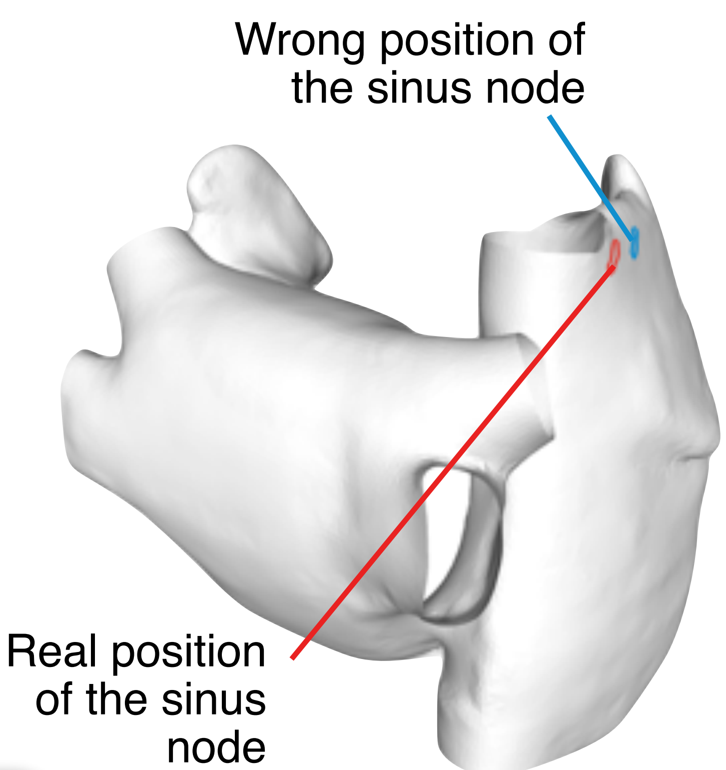


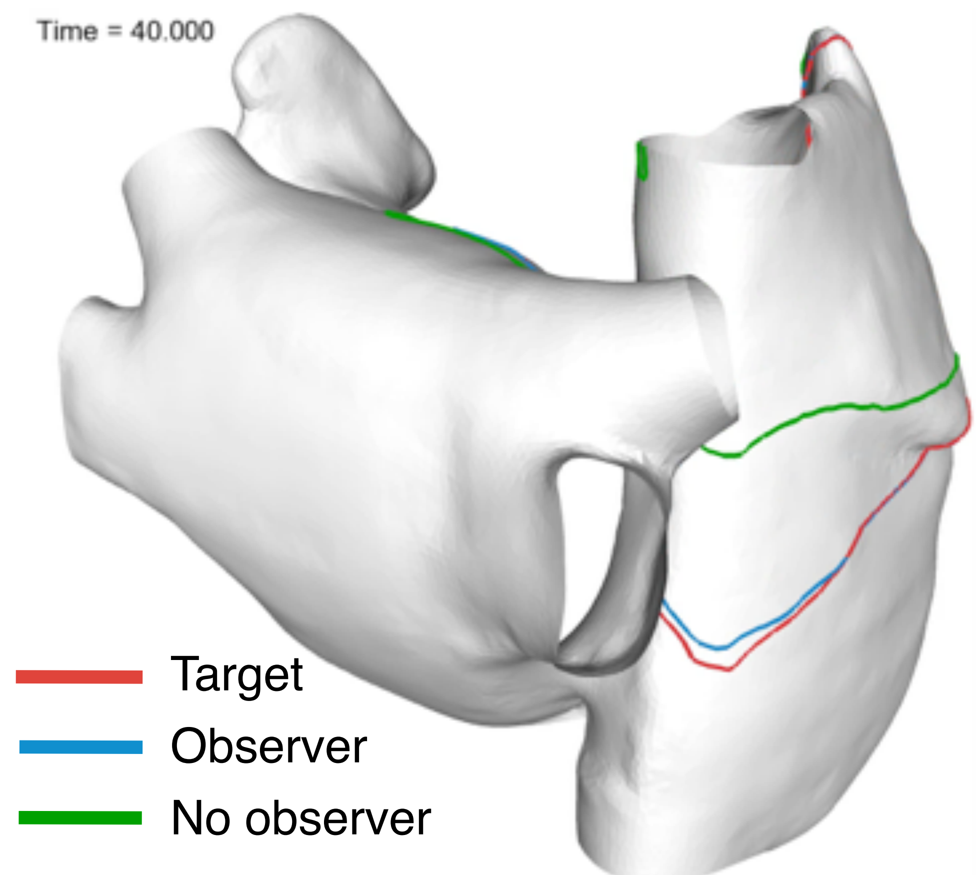
Data assimilation - Atrial simulations

- Specific patient



- Data assimilation
  - State observer





Data assimilation - Parameters estimation

- Joint state and parameter strategy
  - Automatic strategy

Moireau, **Chapelle**, and **LeTallec**. Joint state and parameter estimation for distributed mechanical systems. 2008.

[...] the methodology proposed to extend state estimation to joint state-parameter estimation is general and – indeed – applicable with any state filter.

  - Procedure (recall)

$$\left\{\begin{array}{rcl} \dot{\hat{u}}(t) & = & A(\hat{u}, \theta, t) + \boxed{G_u}\left(\boxed{D(z_u, \hat{u})}\right) \\ \hat{u}(0) & = & u_\diamond \\ \hat{\theta}(0) & = & \theta_\diamond \end{array}\right.$$

Discrepancy D

Gain operator G_u

Data assimilation - Parameters estimation

- Joint state and parameter strategy
  - Automatic strategy

Moireau, **Chapelle**, and **LeTallec**. Joint state and parameter estimation for distributed mechanical systems. 2008.

[...] the methodology proposed to extend state estimation to joint state-parameter estimation is general and – indeed – applicable with any state filter.

  - Procedure (recall)

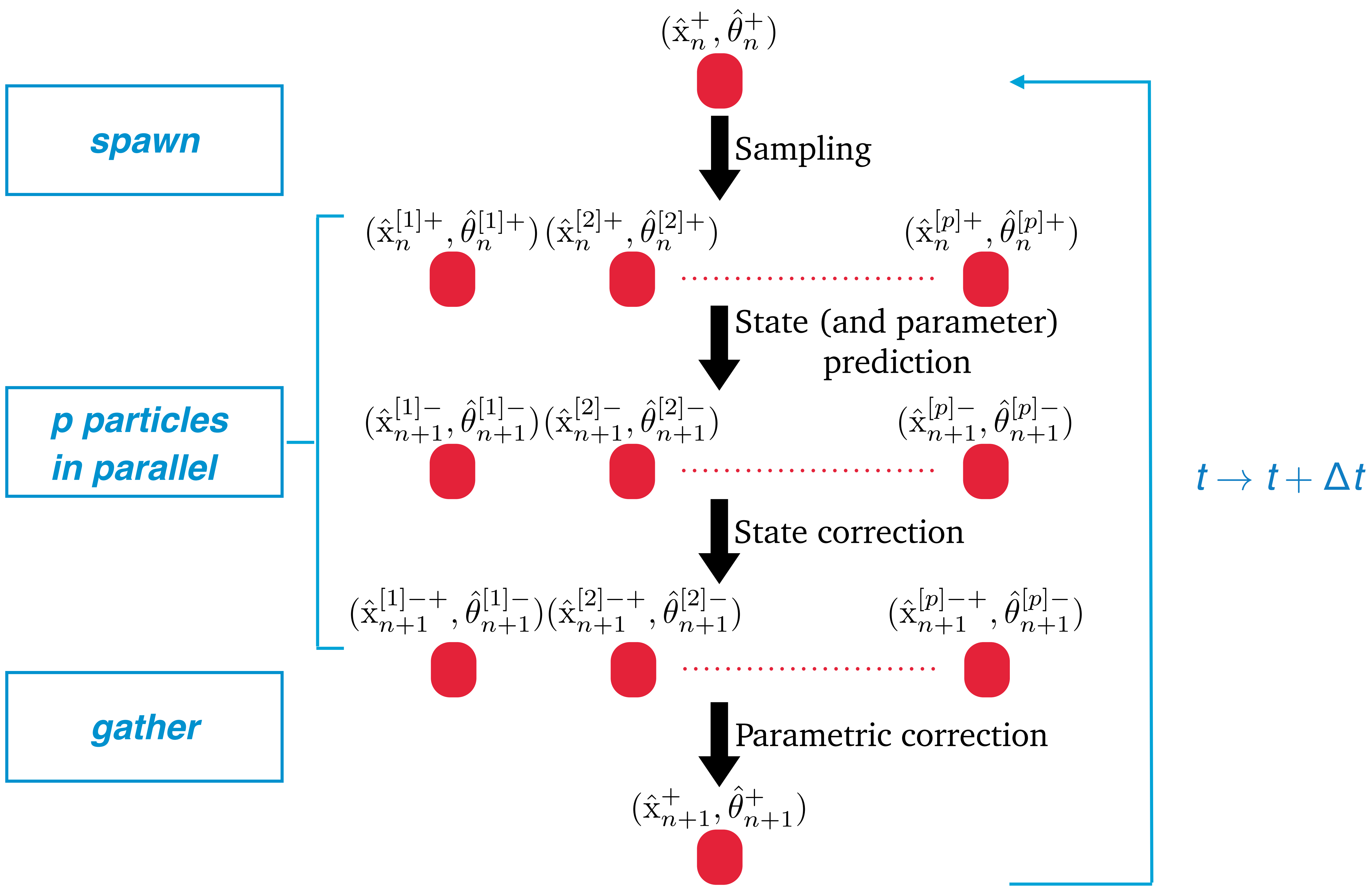
$$\begin{cases} \dot{\hat{u}}(t) &= A(\hat{u}, \theta, t) + \boxed{G_u^\flat}\left(\boxed{D(z_u, \hat{u})}\right) \\ \dot{\hat{\theta}}(t) &= \boxed{G_\theta}\left(\boxed{D(z_u, \hat{u})}\right) \\ \hat{u}(0) &= u_\diamond \\ \hat{\theta}(0) &= \theta_\diamond \end{cases}$$

Discrepancy D

Gain operators G_u and G_θ

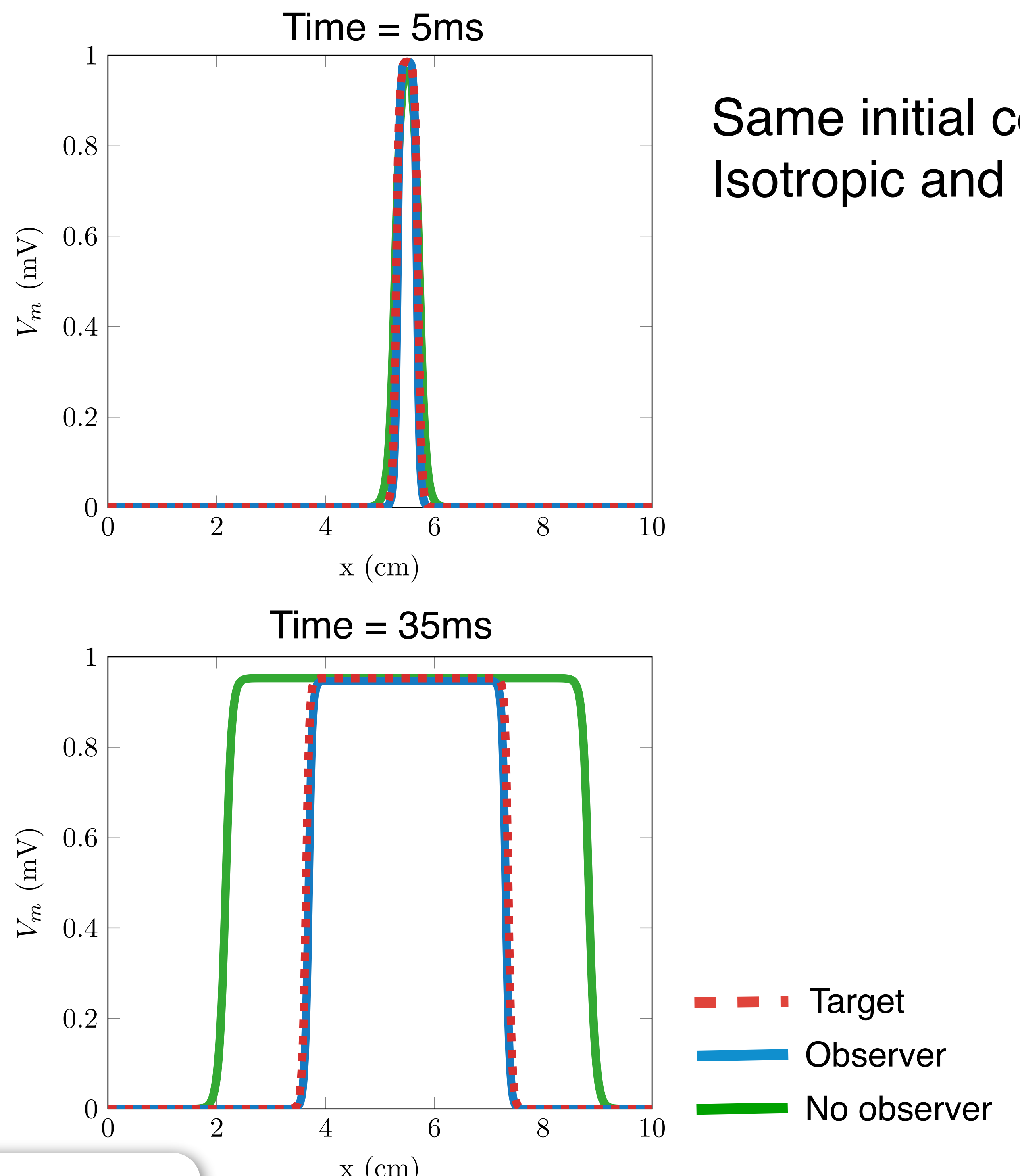
$$\boxed{(\hat{u}, \hat{\theta}) \to (u, \theta)}$$

Data assimilation - Parallel algorithm

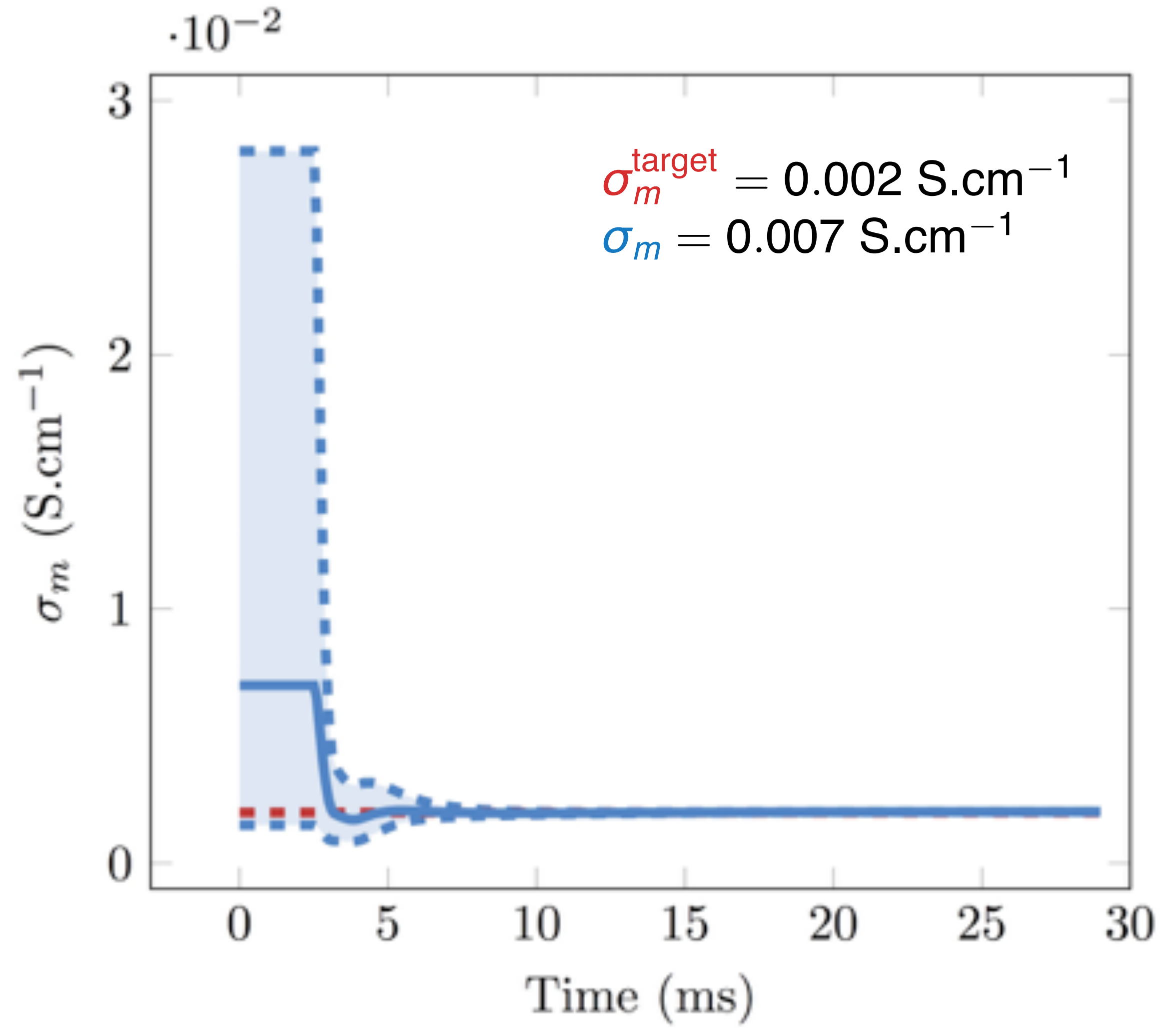


Data assimilation - Parameters estimation

- Illustrative examples in 1D
- Observations generated with the solution of the target model (in red)



Same initial condition - **wrong parameter**
Isotropic and spacewise-constant diffusion tensor = σ_m



Data assimilation - Parameters estimation

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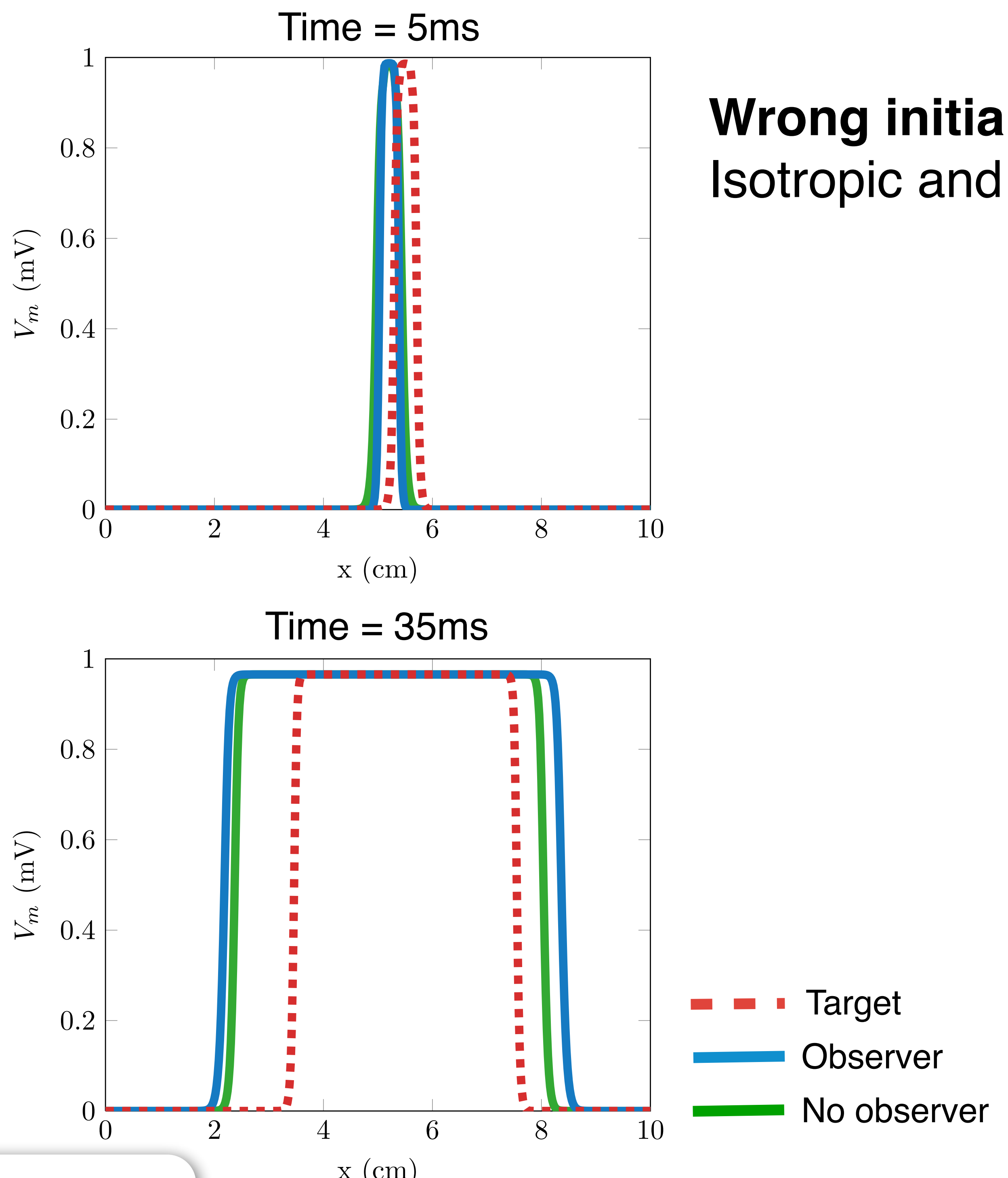
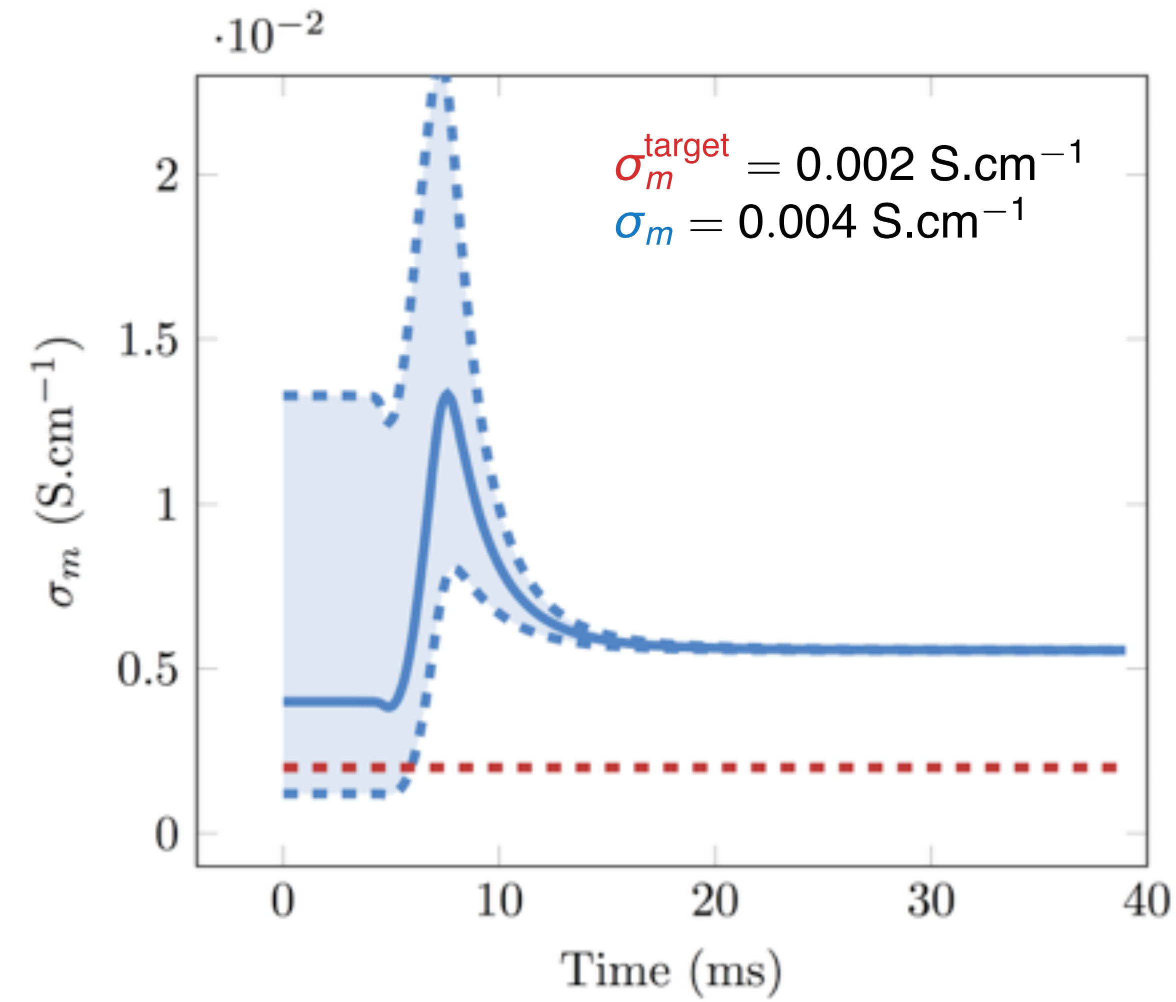


Wrong initial condition - wrong parameter
Isotropic and spacewise-constant diffusion tensor = σ_m



Data assimilation - Parameters estimation

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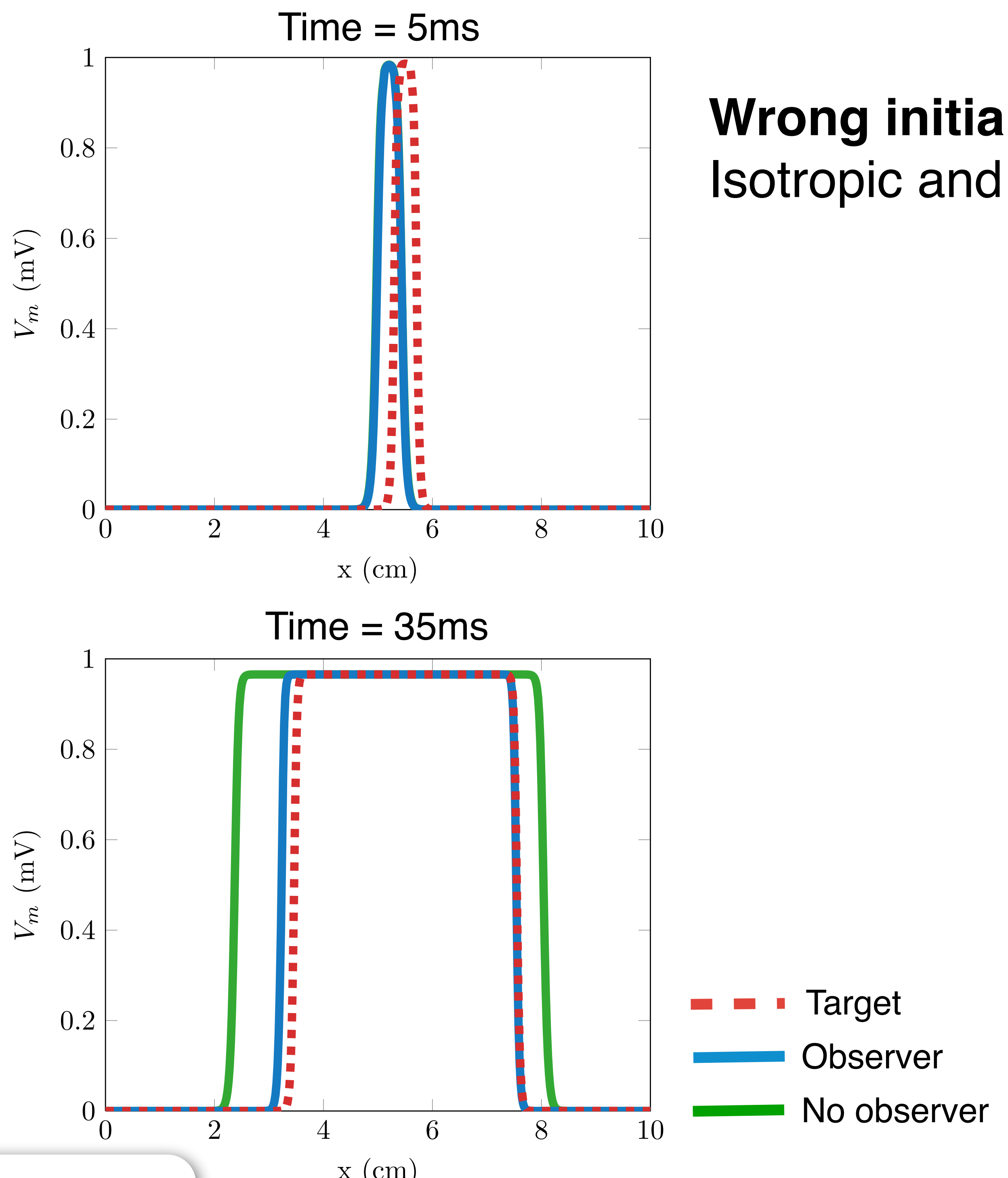

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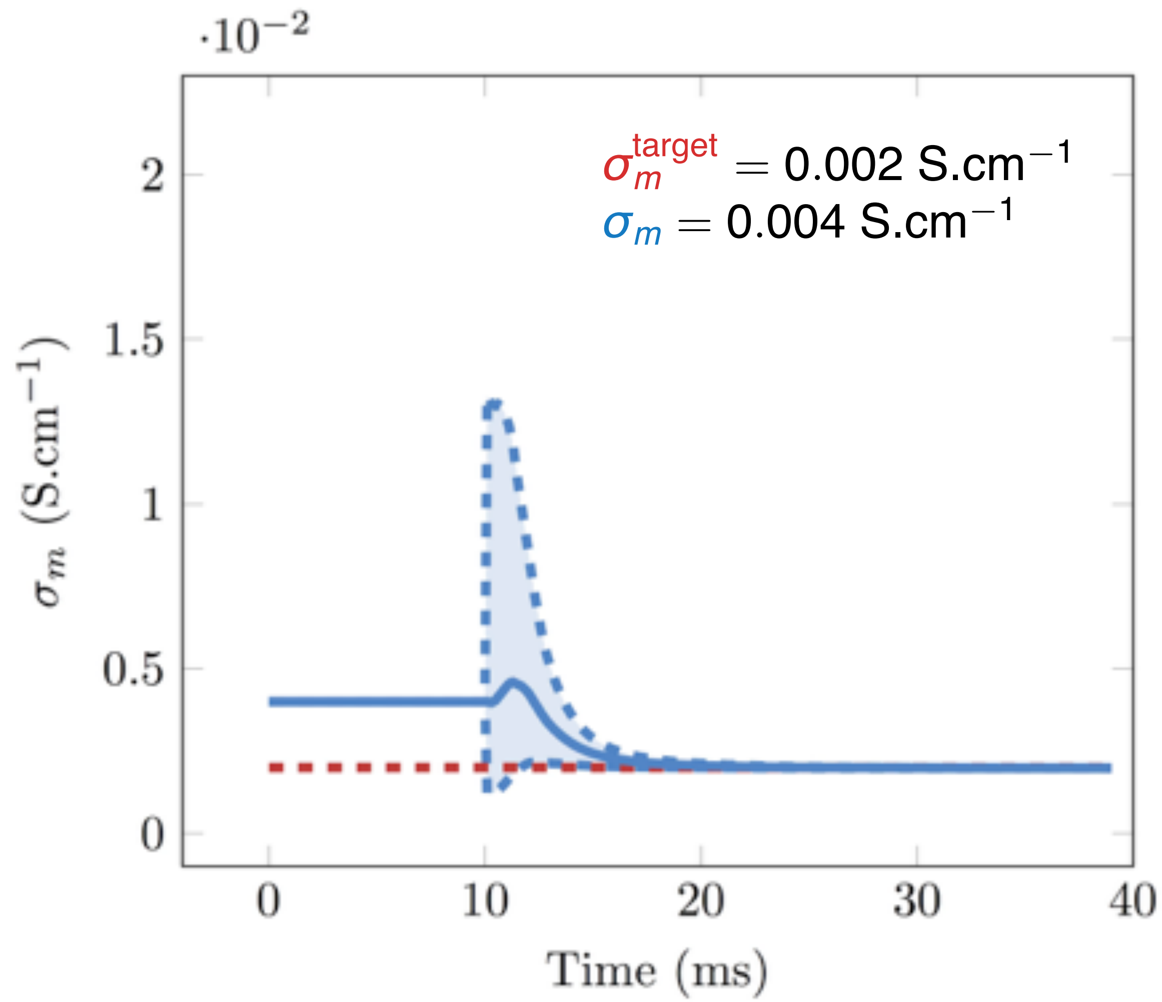
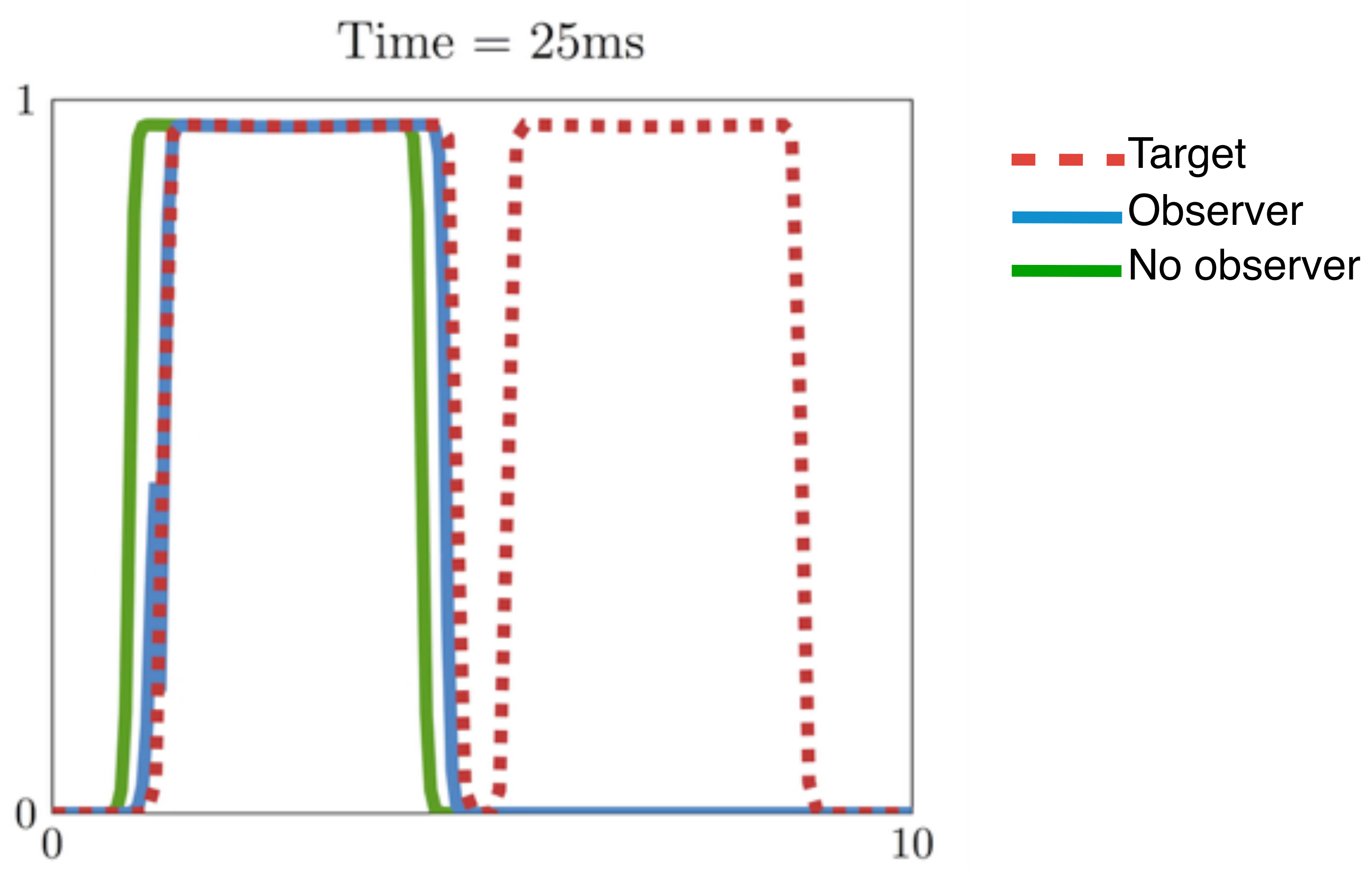


State and delay parameter observers

Data assimilation - Limitations

- An illustrative example in 1D
- With Mitchell-Schaeffer ionic model

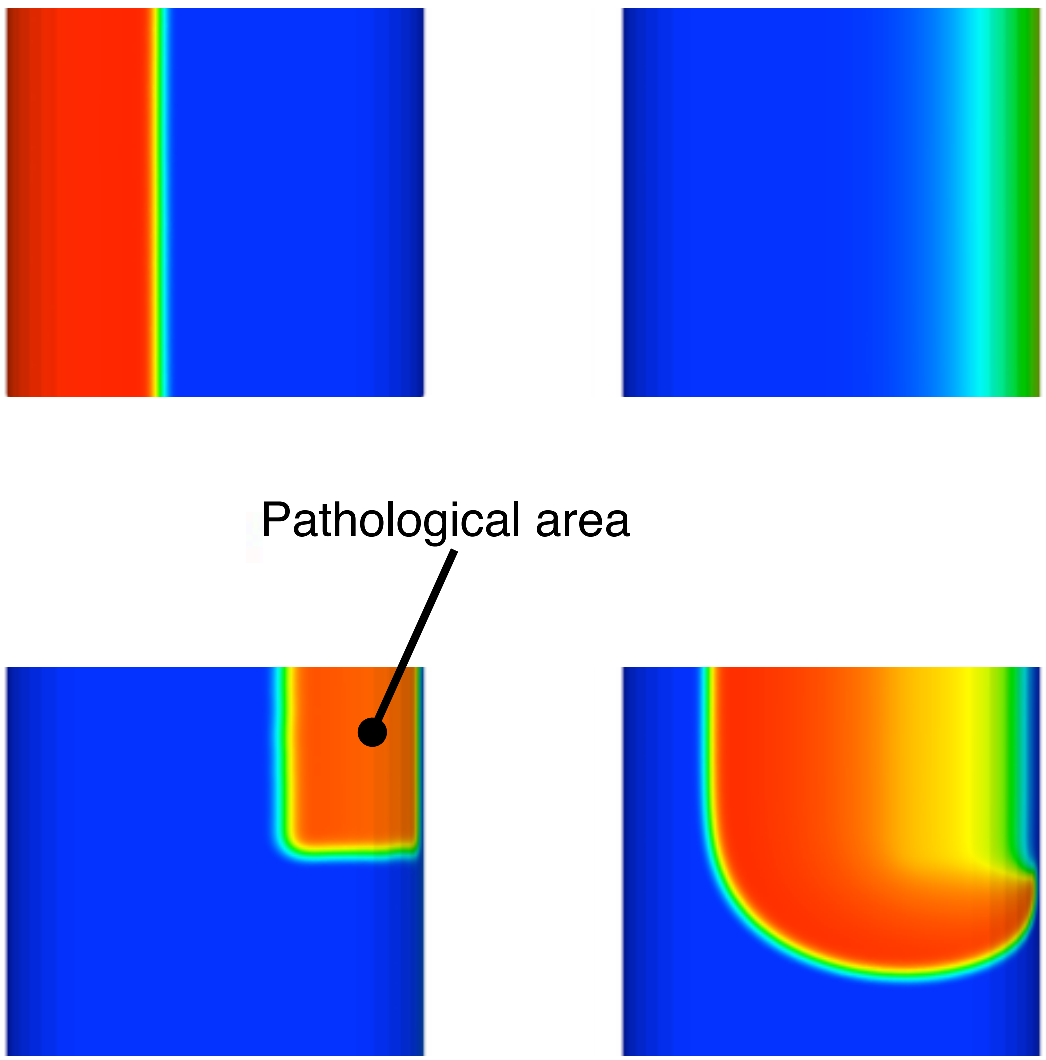
Two fronts for the target
But only one front for the observer



Data assimilation - Limitations

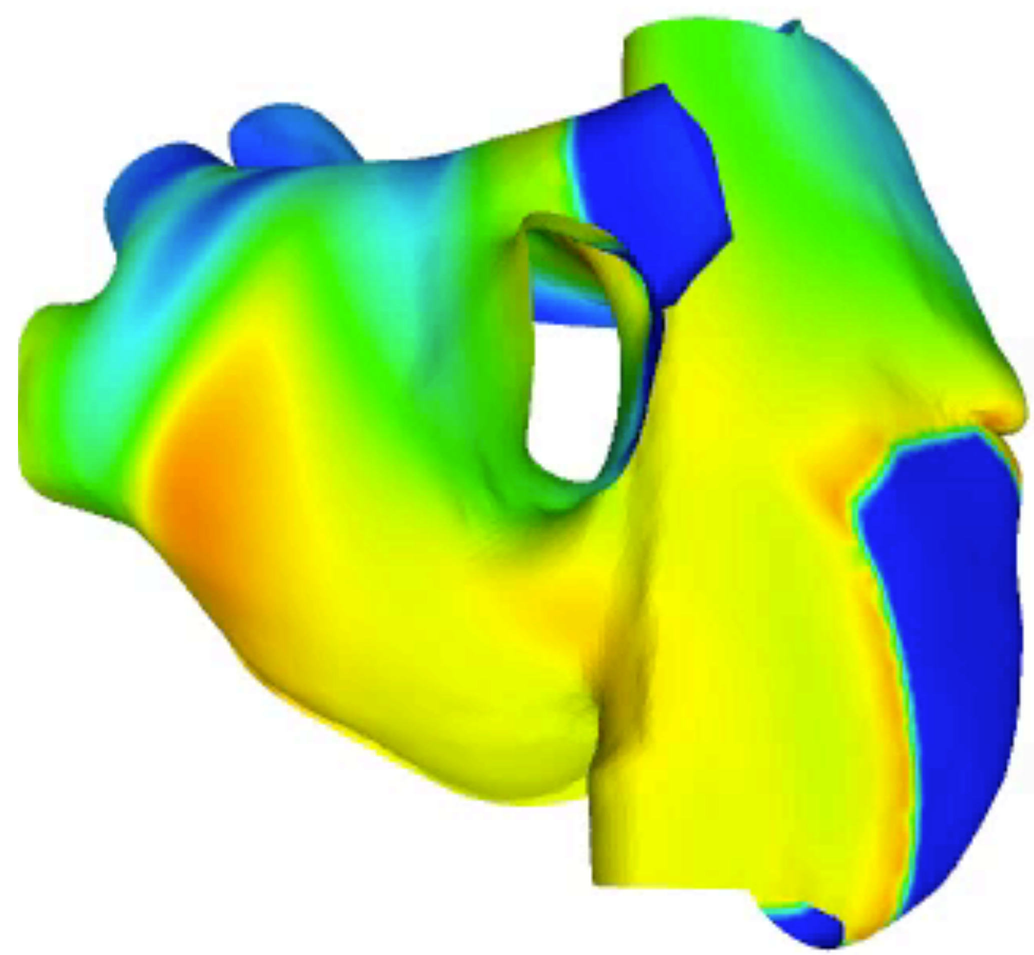
- Atrial fibrillation is an abnormal heart rhythm characterized by rapid and irregular beating

Spiral waves
(no information about the initial condition)



standard S1 - S2 protocol

Atrial fibrillation simulation



- Atrial fibrillation: limitation of the shape space observer

Data assimilation - A shape-based observer

- In image processing, we consider the following energy

$$E_{DA} = \int_{\Omega_{\hat{u}}} \left(z_u - C_1(\Omega_{\hat{u}})\right)^2 d\underline{x} + \int_{\mathcal{B}\setminus\Omega_{\hat{u}}} \left(z_u - C_2(\Omega_{\hat{u}})\right)^2 d\underline{x}$$

- We define a *shape* derivative

$$\vec{\nabla}_{\text{sh}} E_{DA} = \frac{\hat{\delta}(\Gamma_{\hat{u}}, \vec{x})}{|\vec{\nabla} V_m|} \Big(\left(z_u - C_1(\Omega_{V_m})\right)^2 - \left(z_u - C_2(\Omega_{V_m})\right)^2 \Big)$$

- And the previous observer can be written

$$\begin{cases} A_m C_m \int_\omega \Big(\frac{\partial V_m}{\partial t} + I_{ion} \Big) \phi \, dS + C_m \int_\omega \Big(\underline{\underline{\sigma}}_i \cdot (\underline{\nabla} V_m + \underline{\nabla} u_e) \Big) \cdot \underline{\nabla} \phi \, dS \\ \qquad - \int_\omega \lambda \vec{\nabla}_{\text{sh}} E_{DA} \, \phi \, dS = \int_\omega I_{app} \phi \, dS, \\ \int_\omega \Big((\underline{\underline{\sigma}}_i + \underline{\underline{\sigma}}_e) \cdot \underline{\nabla} u_e \Big) \cdot \underline{\nabla} \psi \, dS + \int_\omega \Big(\underline{\underline{\sigma}}_i \cdot \underline{\nabla} V_m \Big) \cdot \underline{\nabla} \psi \, dS = 0 \end{cases}$$

- How to circumvent the limitation of this *shape-based* observer ?

Data assimilation - A shape-topological observer

- Idea : complement the shape derivative by a topological derivative that represents the sensitivity of E_{DA}

$$E_{DA} = \int_{\Omega_{\hat{u}}} \left(z_u - C_1(\Omega_{\hat{u}})\right)^2 d\underline{x} + \int_{\mathcal{B}\backslash\Omega_{\hat{u}}} \left(z_u - C_2(\Omega_{\hat{u}})\right)^2 d\underline{x}$$

- We define a *topological* derivative

$$\vec{\nabla}_{\text{top}} E_{DA} = \Big(1 + \text{sign}\big(dE_{DA} \times (\hat{u} - c_{th})\big)\Big) dE_{DA}$$

with

$$dE_{DA}(\Omega_{\hat{u}})(\vec{x}) = (z_u(\vec{x}) - C_1(\Omega_{\hat{u}}))^2 - (z_u(\vec{x}) - C_2(\Omega_{\hat{u}}))^2$$

- We propose the shape / topological observer

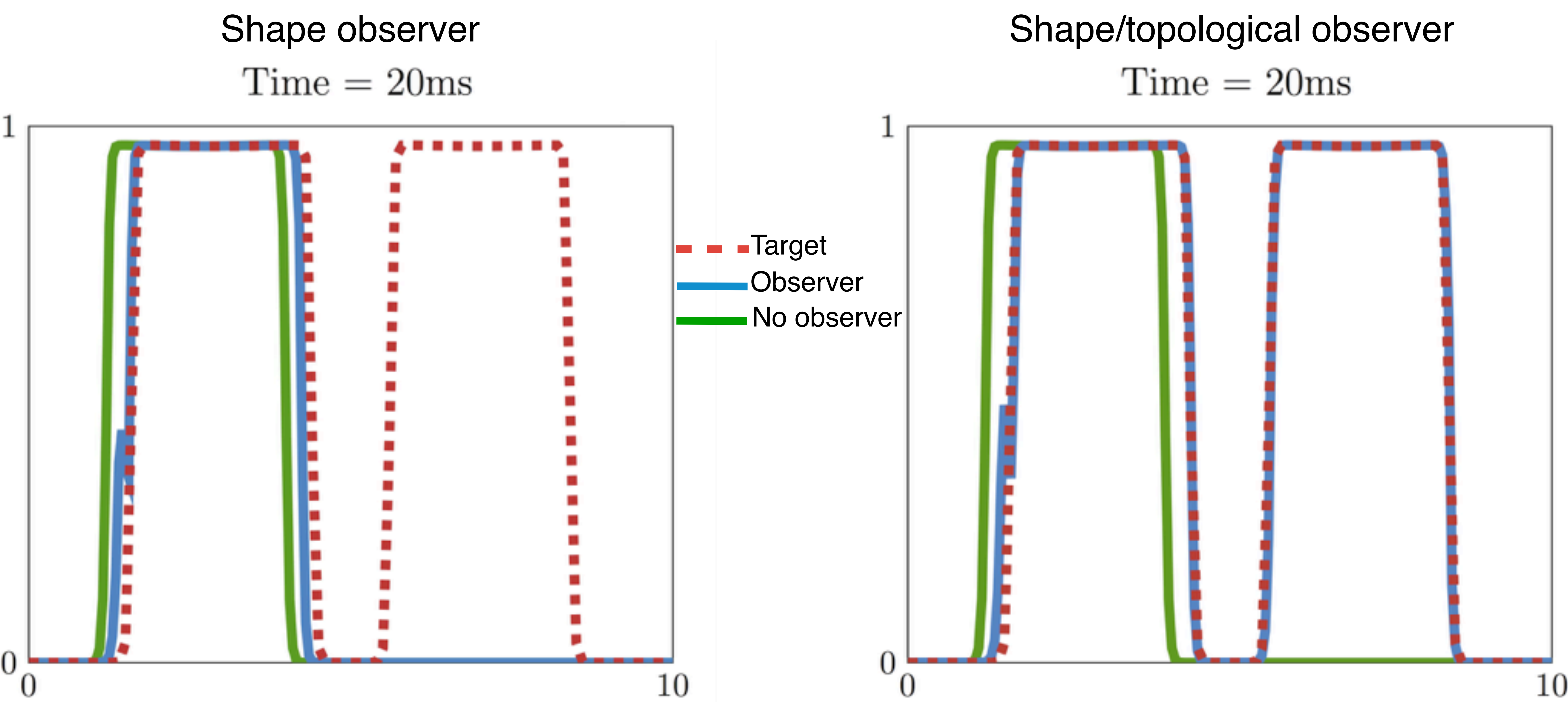
$$\begin{cases} A_m C_m \displaystyle\int_\omega \Big(\frac{\partial V_m}{\partial t} + I_{ion}\Big) \phi \, dS + C_m \int_\omega \Big(\underline{\underline{\sigma}}_i \cdot (\underline{\nabla} V_m + \underline{\nabla} u_e)\Big) \cdot \underline{\nabla} \phi \, dS \\ \qquad - \displaystyle\int_\omega (\lambda \vec{\nabla}_{\text{sh}} E_{DA} + \mu \vec{\nabla}_{\text{top}} E_{DA}) \, \phi \, dS = \int_\omega I_{app} \phi \, dS, \\ \displaystyle\int_\omega \Big((\underline{\underline{\sigma}}_i + \underline{\underline{\sigma}}_e) \cdot \underline{\nabla} u_e\Big) \cdot \underline{\nabla} \psi \, dS + \int_\omega \Big(\underline{\underline{\sigma}}_i \cdot \underline{\nabla} V_m\Big) \cdot \underline{\nabla} \psi \, dS = 0 \end{cases}$$

A.Collin, D. Chapelle and P. Moireau. Sequential state estimation for electrophysiology models with front level-set data using topological gradient derivations. FIMH 2015.

Data assimilation - Shape/topological observer

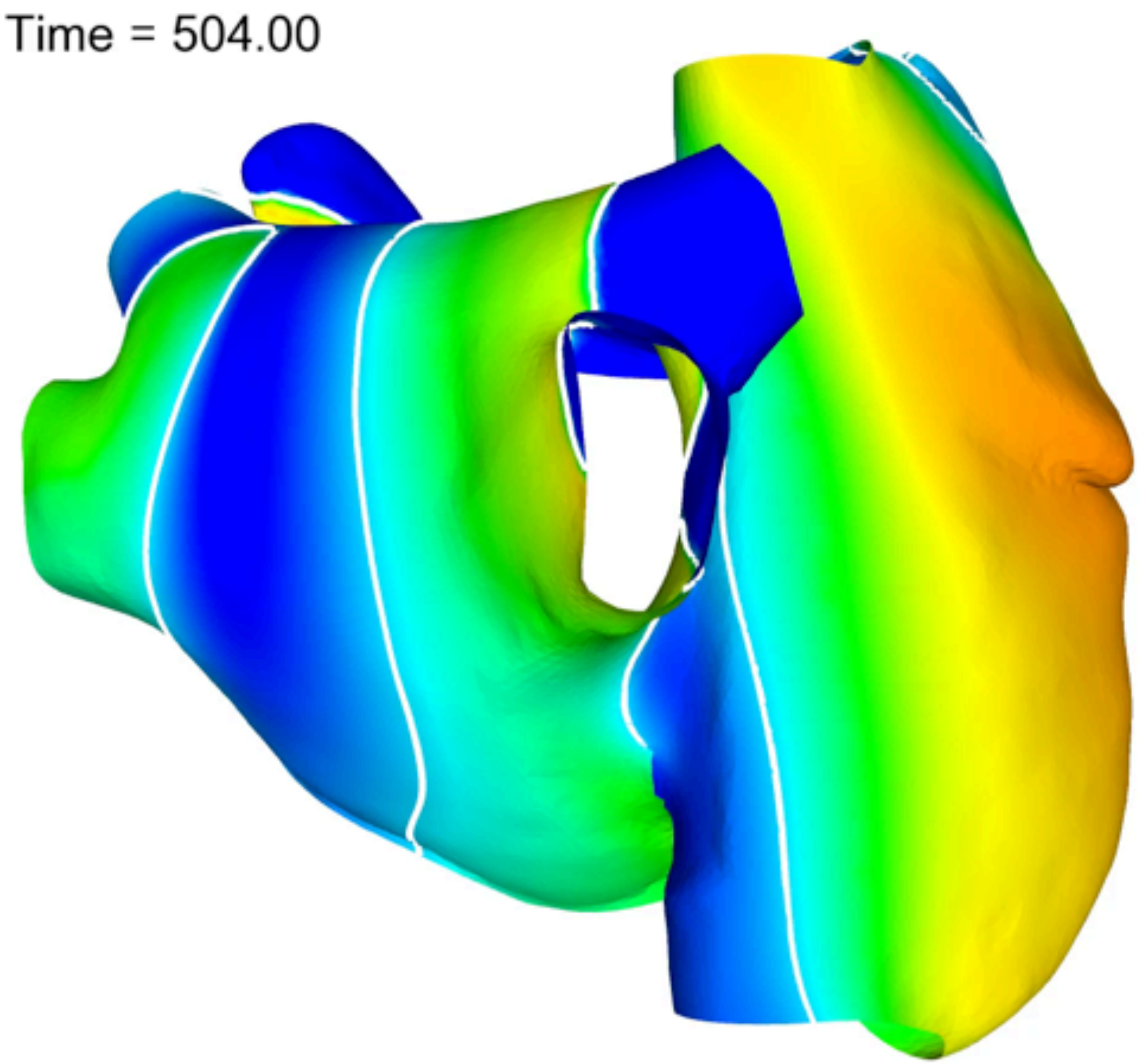
- An illustrative example in 1D
- With Mitchell-Schaeffer ionic model

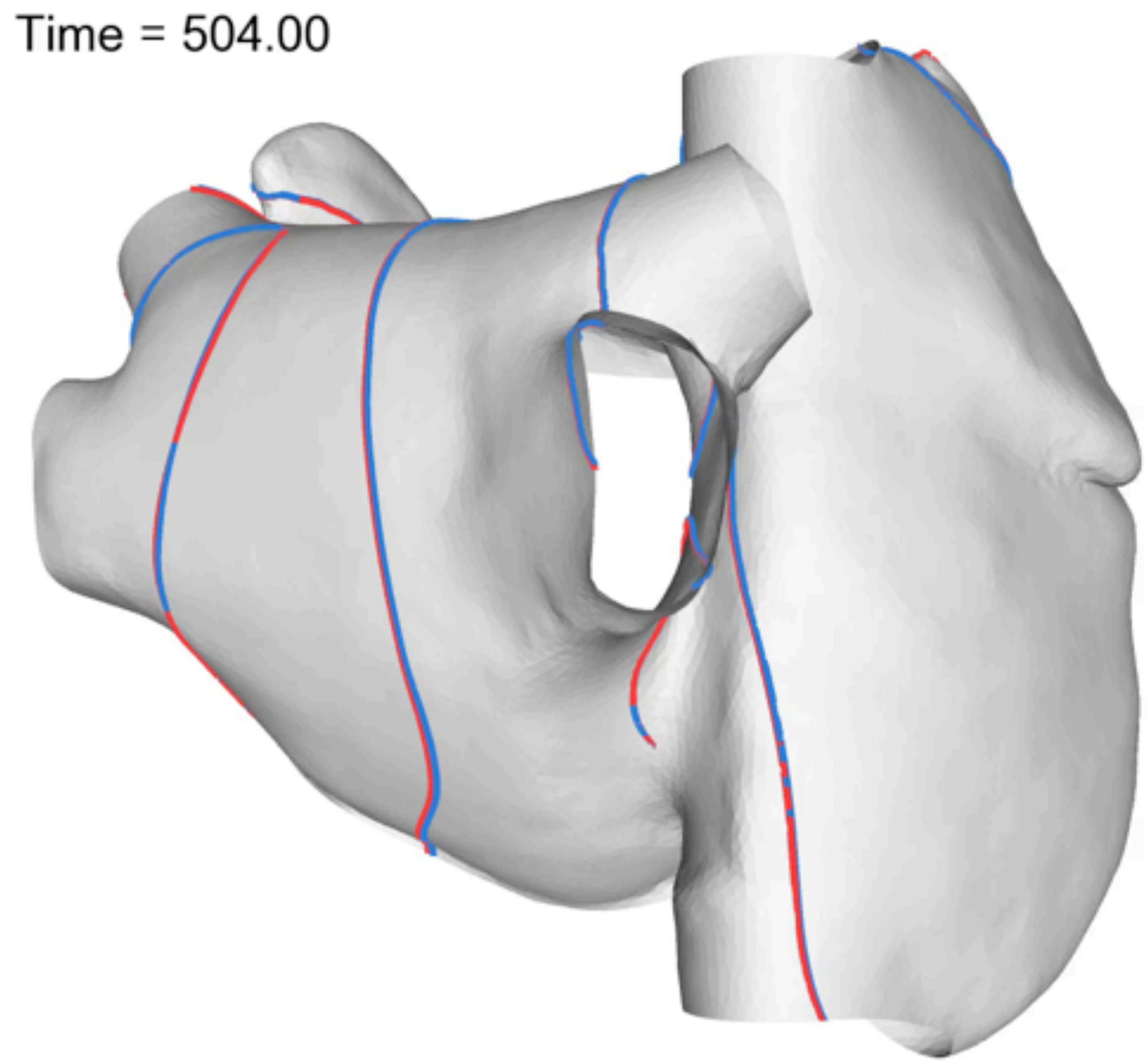
Two fronts for the target
But only one front for the observer



Data assimilation - Shape/topological observer

- Observer solution (with a small time delay before applying the observer)





Able to track complex patterns

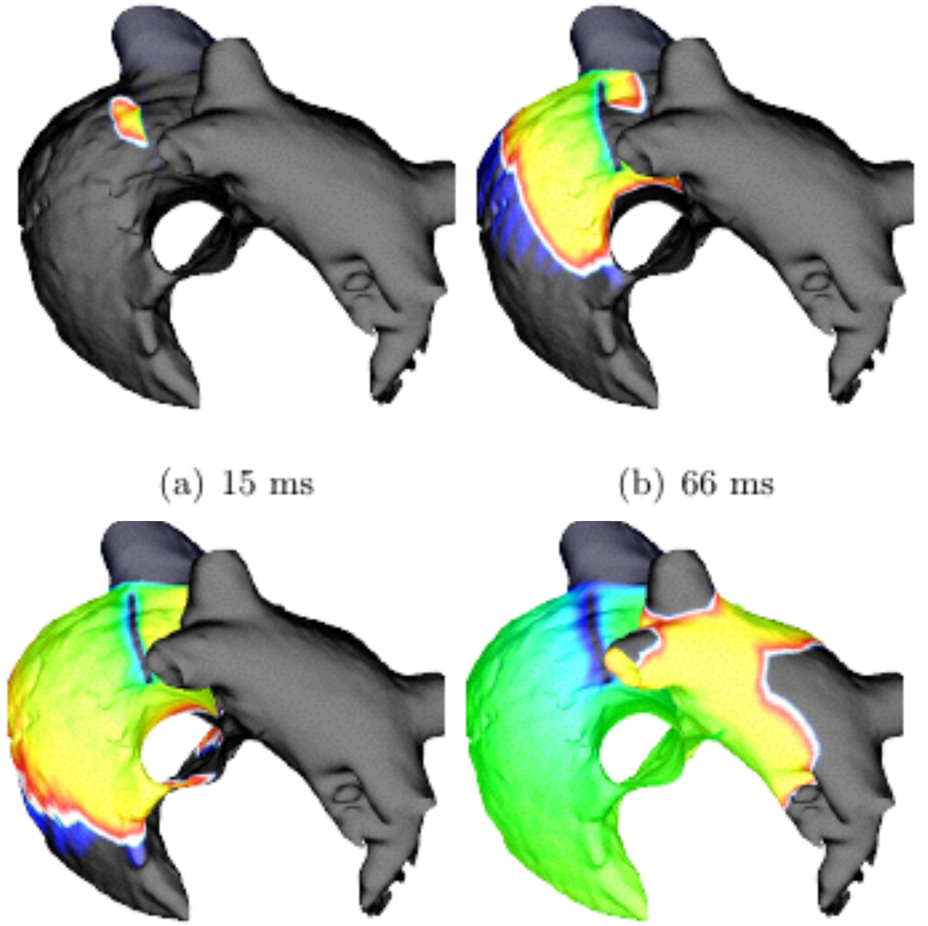
Perspectives (data assimilation part) - Real data

- A complementary model
  - Liryc institute and Carmen Inria team
  - Bilayer model

Y.Coudière, J.Henry, S.Labarthe. A two layers monodomain model of cardiac electrophysiology of the atria. Math. Biol. 2015.

$$\begin{cases} A_m\Big(C_m\,\partial_t V_m^{epi} + I_{ion}\Big) + \underline{\nabla}\cdot\Big(\underline{\underline{\sigma}}^{\,epi}\cdot\underline{\nabla} V_m^{epi}\Big) + \gamma_\epsilon\big(V_m^{end} - V_m^{epi}\big) = I_{app}, & \mathcal{S}^{epi} \\ A_m\Big(C_m\,\partial_t V_m^{end} + I_{ion}\Big) + \underline{\nabla}\cdot\Big(\underline{\underline{\sigma}}^{\,end}\cdot\underline{\nabla} V_m^{end}\Big) + \gamma_\epsilon\big(V_m^{epi} - V_m^{end}\big) = I_{app}, & \mathcal{S}^{end} \end{cases}$$

  - Real data



(a) 15 ms (b) 66 ms

(c) 90 ms (d) 180 ms

Monodomain model
Bilayer meshes
Bilayer fibers

Bidomain model
Mean surface mesh
Linear variation Fibers

Jason Bayer, Yves Coudière, Antoine Gérard, Philippe Moireau

CEMRACS 2016

Φsio's project

Perspectives (data assimilation part) - Fire propagation

- Monodomain model (simplified version)

$$A_m\Big(C_m\frac{\partial V_m}{\partial t} + I_{ion}(V_m,\cdots)\Big) - \mathrm{div}(\vec{\vec{\sigma}}\cdot\vec{\nabla} V_m) = I_{app}, \quad \text{in } \mathcal{B}\times(0,T)$$

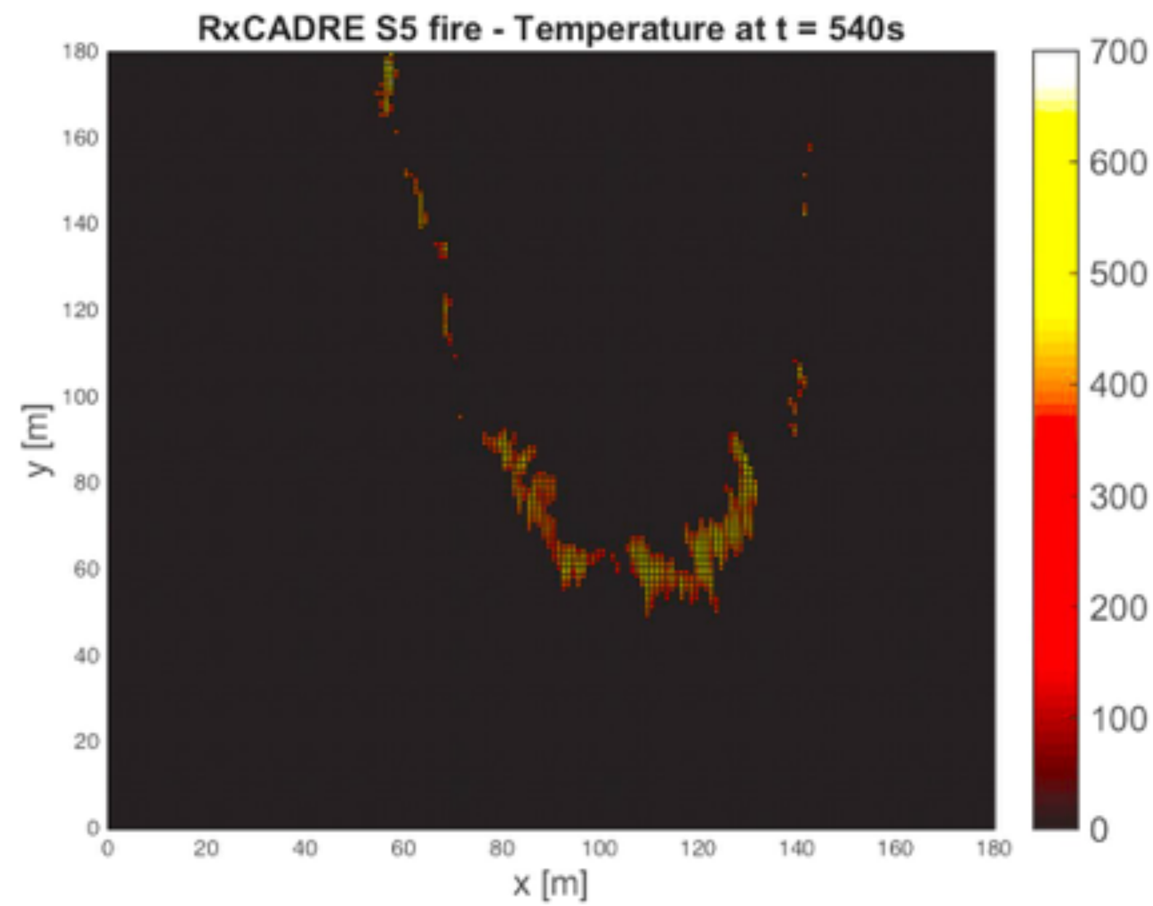
- Works also for reaction diffusion model and front equations

$$\begin{cases} \partial_t u - \mathrm{div}(D\vec{\nabla}u) &= kf(u), & \mathcal{B}\times(0,T),\\ (D\vec{\nabla}u)\cdot \underline{n} &= 0, & \partial\mathcal{B}\times(0,T),\\ u(\vec{x},0) &= u_0(\vec{x}), & \mathcal{B}. \end{cases}$$

$$\partial_t\phi = ROS\,|\nabla\phi|$$

- Data - position of the front

M Rochoux, B Delmotte, B Cuenot, S Ricci, A Trouvé. Regional-scale simulations of wildland fire spread informed by real-time flame front observations.2013.

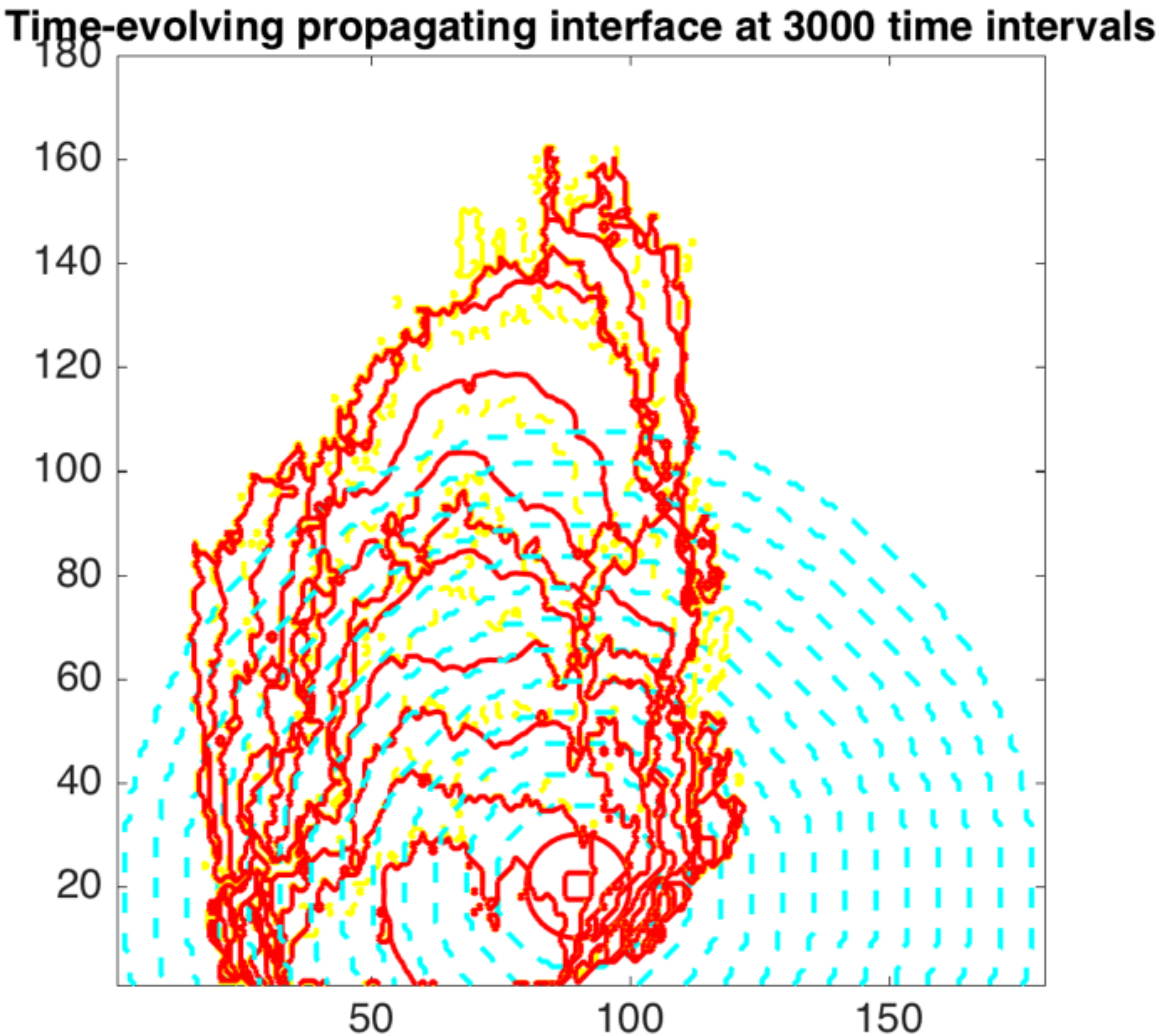


CEMRACS 2016

Flam's project

Didier Lucor
Philippe Moireau
Sophie Ricci
Mélanie Rochoux
Cong Zhang

Perspectives (data assimilation part) - Fire state estimation



MODELING AND DATA ASSIMILATION IN CARDIAC ELECTROPHYSIOLOGY

Collaborators: Dominique Chapelle, Jean-Frédéric Gerbeau,
Philippe Moireau, Sébastien Impériale, Elisa Schenone