Anim3D Project

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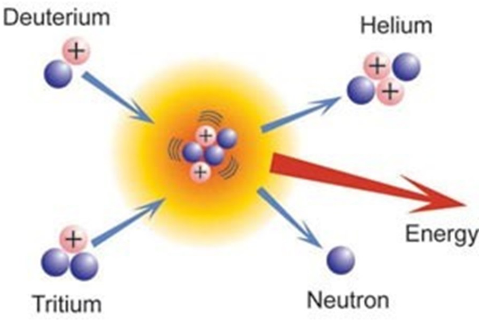
Outline

- Motivations
- IsoGeometric Analysis
- Numerical results
- Conclusion and perspectives

Plasma Physics

- **Plasma**: For very high temperatures, the gas are ionized and gives a plasma which can be controlled by magnetic and electric fields.
- **Thermonuclear fusion**: The MHD allows to describe some configuration where the collision are not so small or for long time behavior.
- **Astrophysics**: The MHD describe a lot of astrophysics configuration: supernovae explosion, solar wind and instabilities etc.
- **Context**: in the case we consider the application of the MHD to the simulation of Tokamak instabilities.



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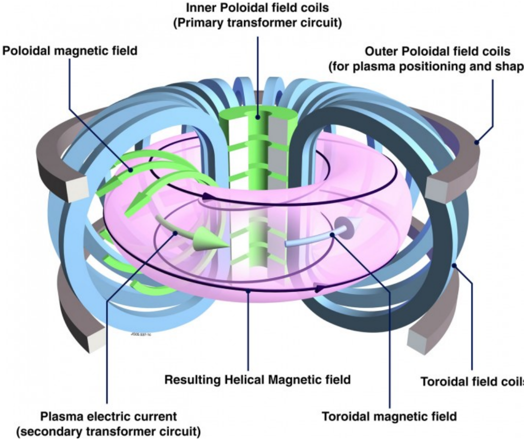


Figure: Tokamak

Example of Application : MHD and ELM

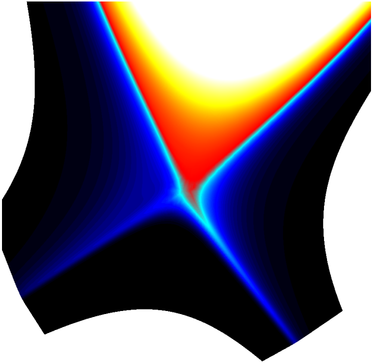
- In the tokamak some instabilities can appear in the plasma.
- The simulation of these instabilities is an important subject for ITER.

- Example of Instabilities in the tokamak :
  - **Disruptions**: Violent instabilities which can critically damage the Tokamak.
  - **Edge Localized Modes (ELM)**: Periodic edge instabilities which can damage the Tokamak.

- These instabilities are linked to the very large gradient of pressure and very large current at the edge.

- Many aspects of these instabilities are described by fluid models (MHD resistive and diamagnetic or extended)

- ELM simulation



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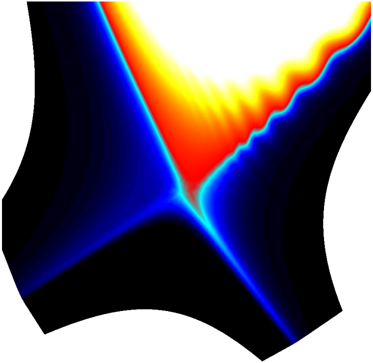

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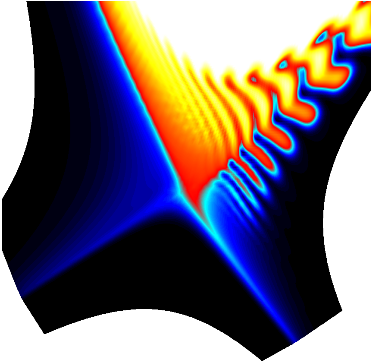

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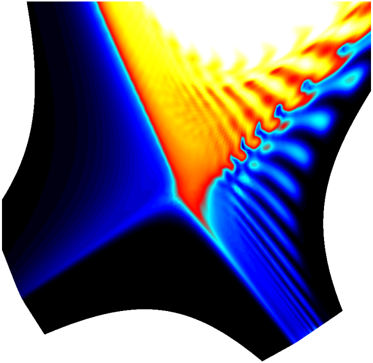

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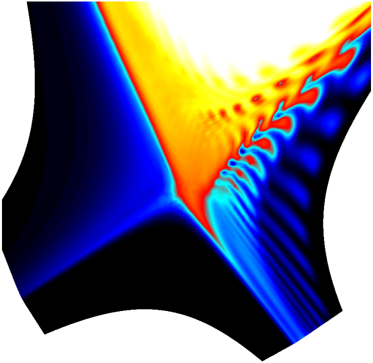

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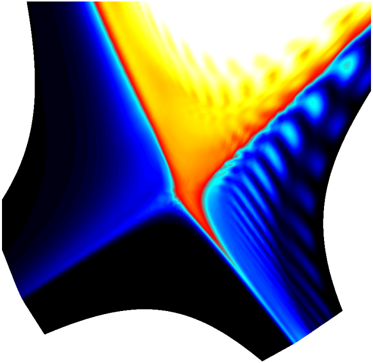


To create a family of *B-splines*, we need a non-decreasing sequence of knots $T = (t_i)_{1 \leqslant i \leqslant N+k}$, also called **knot vector**, with $k = p + 1$.
Each set of knots $T_j = \{t_j, \cdots, t_{j+p}\}$ will generate a *B-spline* N_j.

Definition (B-Spline serie)

The j-th B-Spline of order k is defined by the recurrence relation:

$$N_j^k = w_j^k N_j^{k-1} + (1 - w_{j+1}^k) N_{j+1}^{k-1}$$

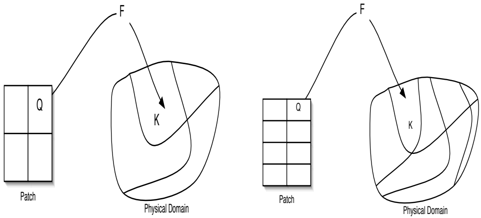
where,

$$w_j^k(x) = \frac{x - t_j}{t_{j+k-1} - t_j} \qquad N_j^1(x) = \chi_{[t_j, t_{j+1}[}(x)$$

for $k \geq 1$ and $1 \leq j \leq N$.

Discretization

The IsoGeometric Approach



Grid generation: the use of $h/p/k$-refinement keeps the mapping **F** unchanged.

- Compact support
- Partition of Unity
- Affine covariance
- IsoParametric concept
- Error estimates in Sobolev norms
- Exacte DeRham discrete sequence

Refinement strategies

Refining the grid can be done in 3 different ways. This is the most interesting aspects of B-splines basis.

- h-refinement: by inserting new knots. It is the equivalent of mesh refinement of the classical finite element method.
- p-refinement: by elevating the B-spline degree. It is the equivalent of using higher finite element order in the classical FEM.
- k-refinement: by increasing / decreasing the regularity of the basis functions (increasing / decreasing multiplicity of inserted knots).
- r-refinement: moving the control points to reduce a given error estimate

the use of **k-refinement** strategy is more efficient than the classical p-refinement, as it reduces the dimension of the basis.

Parallelism

Domain Decomposition

Available algorithms

- Tensor decomposition, when using Tensor Spaces
- Metis (ParMetis will be added later)

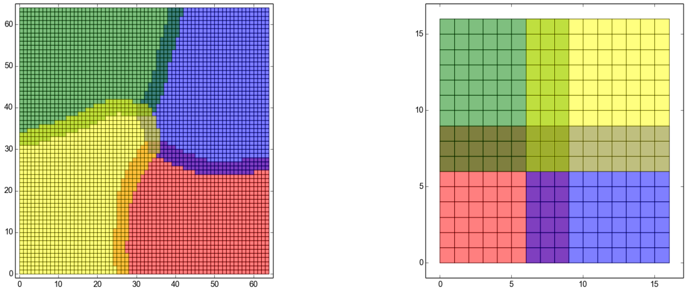
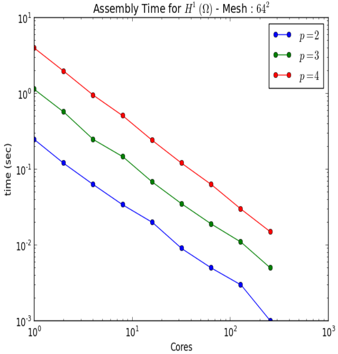
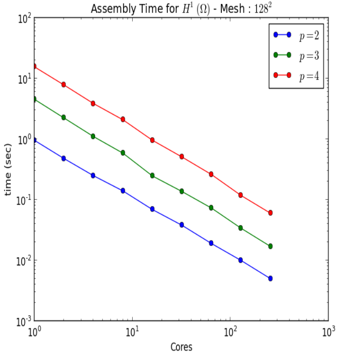


Figure: Metis (left) and tensor (right) partitioning.

Numerical results: Parallel runs

The $2D$ case





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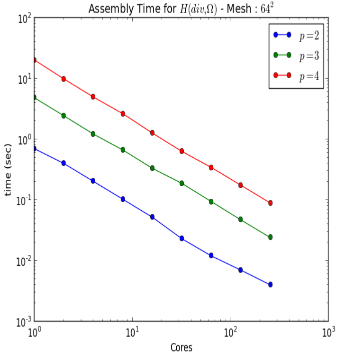

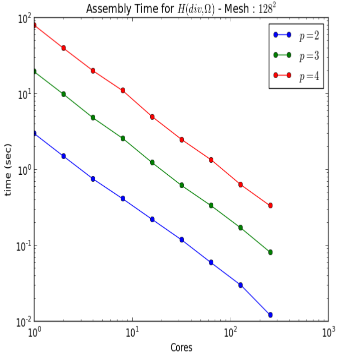

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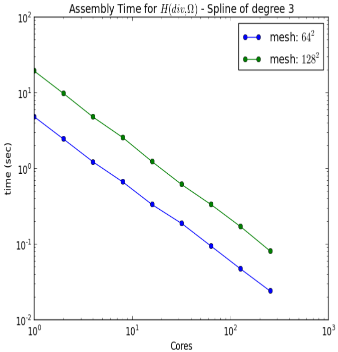
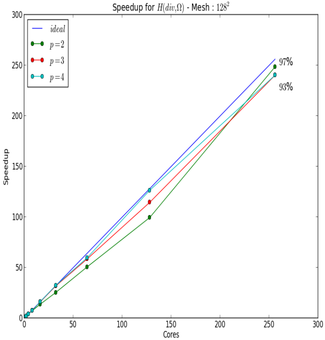
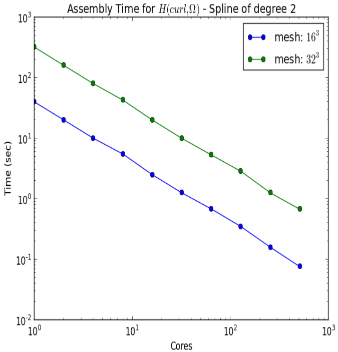
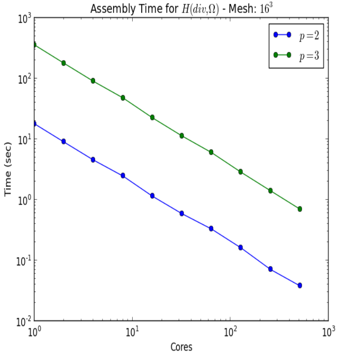




Numerical results: Parallel runs

The $3D$ case





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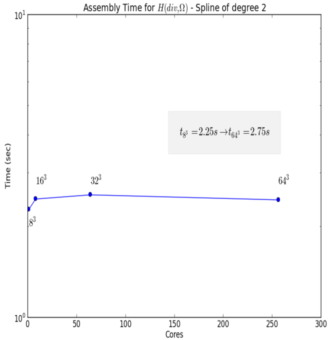

Statistics: Quadratic Splines on a grid 32^3:

- $23'101'440$ non zeros for $H(curl)$
- $98'304$ dofs for $H(curl)$
- $13'860'864$ non zeros for $H(div)$
- $98'304$ dofs for $H(div)$

Equilibrium

- We consider the resistive MHD with $\mathbf{v} = 0$. We obtain the following equilibrium

$$\begin{cases} \mathbf{v} = 0, \quad \mathbf{J} \times \mathbf{B} = \nabla p \\ \partial_t \mathbf{B} = \frac{\eta}{\nu_0} \Delta \mathbf{B} \end{cases}$$

- $\tau << \tau_{diff}$ with τ and τ_{diff} the characteristic time of transport and the diffusion.

MHD equilibrium

- The equilibrium is mainly defined by the force balanced

$$\mathbf{J} \times \mathbf{B} = \nabla p$$

- The equilibrium induces that $\mathbf{B} \cdot \nabla p = 0$, $\nabla \cdot \mathbf{J} = 0$ and we assume that $\nabla p \cdot \mathbf{e}_\phi = 0$.
- In a Tokamak we assume that

$$\mathbf{B} = \mu_0 \frac{F(\psi, Z)}{R} \mathbf{e}_\phi + \frac{1}{R} (\nabla \psi \times \mathbf{e}_\phi)$$

- After some computation we obtain the following equation

Grad-Shafranov equation

$$\Delta^* \psi = -\mu_0 R^2 \frac{dp(\psi)}{d\psi} - \mu_0^2 F(\psi) \frac{dF(\psi)}{d\psi}$$

Grad-Shafranov Shift and β plasma

Shift

- **Property of GS operator**: induce a shift of the magnetic surface
- **Shift estimation**: $\frac{\Delta}{r} \approx \beta_p \frac{r}{R_0}$
- with r and R_0 the minor and major radius.
- $\beta_p = \frac{2\mu_0 |p|}{|\mathbf{B}_p|}$ the ratio of the pressure and poloidal magnetic pressure.

Test case

- **Discretization**: 32*32*8 cells with third order B-Splines
- **Solvers** Picard nonlinear solver + GMRES
- **Physical problems**: $\beta_p \approx 10^{-2}$, $R_0 = 3$ and $\frac{\beta}{\beta_p} \approx 10$

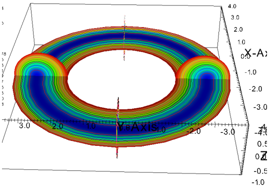

Figure: 3D equilibrium

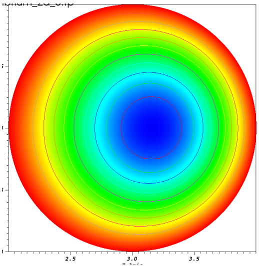

Figure: poloidal cut of equilibrium

Anisotropic diffusion

- Model :

$$\partial_t T - \nabla \cdot \Big((k_{\parallel} - k_{\perp})(\mathbf{b} \otimes \mathbf{b}) \nabla T + k_{\perp} \nabla T \Big) = 0$$

  with $k_{\parallel} >> k_{\perp}$.
- The magnetic field is construct solving the equilibrium.
- In this case $k_{\parallel} = 200$ and $k_{\perp} = 0$ .The diffusion is along the magnetic lines.

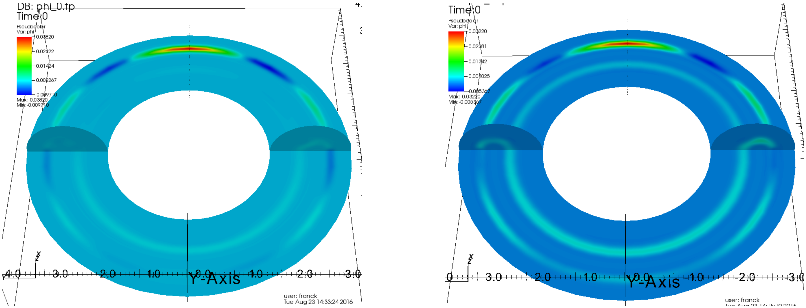

Figure: Left: solution after time $T = 0.5$. Right: solution after time $T = 7$

Reduce MHD model

Single fluid resistive MHD

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0, \\ \rho \partial_t \mathbf{v} + \rho \mathbf{v} \cdot \nabla \mathbf{v} + \nabla p = \mathbf{J} \times \mathbf{B} - \nabla \cdot \overline{\overline{\mathbf{\Pi}}}, \\ \partial_t p + \mathbf{v} \cdot \nabla p + p \nabla \cdot \mathbf{v} + \nabla \cdot \mathbf{q} = 0 \\ \partial_t \mathbf{B} = -\nabla \times (-\mathbf{v} \times \mathbf{B} + \eta \mathbf{J}), \\ \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \mathbf{J}. \end{cases}$$

- **Reduced MHD model**: Reduce the number of variables and eliminate the fast waves in the reduced MHD model.
- We consider the cylindrical coordinate $(R, Z, \phi) \in \Omega \times [0, 2\pi]$.

Reduced MHD: Assumption

$$\mathbf{B} = \frac{F_0}{R}\mathbf{e}_\phi + \frac{1}{R}\nabla \psi \times \mathbf{e}_\phi, \quad \mathbf{v} = -R\nabla u \times \mathbf{e}_\phi + v_{||}\mathbf{B}$$

with u the electrical potential, ψ the magnetic poloidal flux, $v_{||}$ the parallel velocity.

- **Initialization**: we use ψ and pressure equilibrium, a zero velocity ($u = v_{||} = 0$).
- **Wave structure**: low Mach and low β regime $\rightarrow$ a large ratio between wave speeds.
- This problem coupled with hyperbolic structure generate ill-conditioned problem.

Preconditioning

- The implicit system after linearization is given by

$$\begin{pmatrix} \mathbf{B}^{n+1} \\ p^{n+1} \\ \mathbf{u}^{\mathbf{n+1}} \end{pmatrix} = \begin{pmatrix} A_{\mathbf{B},p} & C_{\mathbf{B},p,\mathbf{u}} \\ C_{\mathbf{u},\mathbf{B},p} & A_{\mathbf{u}} \end{pmatrix}^{-1} \begin{pmatrix} R_{\mathbf{B}} \\ R_p \\ R_{\mathbf{u}} \end{pmatrix}$$

- with $A_{\mathbf{B},p}$ and $A_{\mathbf{u}}$ the advection terms linked to $\mathbf{B}$ and p (resp $\mathbf{u}$), $C_{\mathbf{B},p,\mathbf{u}}$ and $C_{\mathbf{u},\mathbf{B},p}$ the coupling terms which gives the Alfven and acoustic waves.
- The solution of the system is given by

$$\begin{pmatrix} \mathbf{B}^{n+1} \\ p^{n+1} \\ \mathbf{u}^{\mathbf{n+1}} \end{pmatrix} = \begin{pmatrix} I_d & A_{\mathbf{B},p}^{-1} C_{\mathbf{B},p,\mathbf{u}} \\ 0 & I_d \end{pmatrix} \begin{pmatrix} A_{\mathbf{B},p}^{-1} & 0 \\ 0 & P_{schur}^{-1} \end{pmatrix} \begin{pmatrix} I_d & 0 \\ -C_{\mathbf{u},\mathbf{B},p} A_{\mathbf{B},p}^{-1} & I_d \end{pmatrix} \begin{pmatrix} R_{\mathbf{B}} \\ R_p \\ R_{\mathbf{u}} \end{pmatrix}$$

- Using the previous Schur decomposition, we obtain the following algorithm:

$$\begin{cases} \text{Predictor}: & A_{\mathbf{B},p} \begin{pmatrix} \mathbf{B}^* \\ p^* \end{pmatrix} = \begin{pmatrix} R_{\mathbf{B}} \\ R_p \end{pmatrix} \\ \text{Velocity evolution}: & P_{schur} \mathbf{u}^{n+1} = \left(-C_{\mathbf{u},\mathbf{B},p} \begin{pmatrix} \mathbf{B}^{n+1} \\ p^{n+1} \end{pmatrix} + R_{\mathbf{u}} \right) \\ \text{Corrector}: & A_{\mathbf{B},p} \begin{pmatrix} \mathbf{B}^{n+1} \\ p^{n+1} \end{pmatrix} = A_{\mathbf{B},p} \begin{pmatrix} \mathbf{B}^* \\ p^* \end{pmatrix} - C_{\mathbf{B},p,\mathbf{u}} \mathbf{u}_{n+1} \end{cases}$$

- **Preconditioning**: we approximate the Schur complement by a multi-scale elliptic operator.
- Using classical Multi-grids and auxiliary-space theory we can perform the invert of the Schur approximation.

JorekDjango Framework

These developments were done in the **JorekDjango** framework. It is written in *Fortran2008*, using MPI.

- **CLAPPIO** Input/Output Library
- **PLAF** Parallel Linear Algebra Library
- **SPL** Library for NURBS/B-Splines
- **DISCO** Abstract Discretization Context Library
- **FEMA** Library of Finite Elements Assemblers

JorekDjango Framework

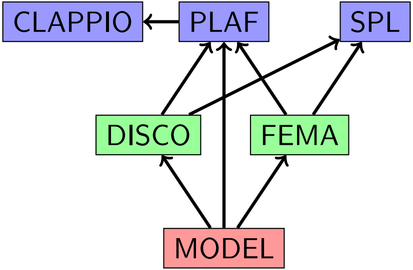

Figure: Strucutre of the JorekDjango Framework

Conclusion and persecptives

Conclusions

- Weak scalability. In $2D$, we get a speedup of 104% on H^1 and $94 - 97\%$ for $H(div)$
- Different models have been implemented during this summer-school (Equilibrium, Anistropic Diffusion, Reduced-MHD)

Ongoing work and Perspectives

- Validation of OpenMP
- Physics-Based Preconditioner for the Reduced-MHD (model199)
- Add more physics (model303)

Thanks!