CEMRACS 2016 project: 'Schwarz for `TrioCFD`' Space-time domain decomposition for the Stokes system; application to the `TrioCFD` code

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Abstract

For time-dependent partial differential equations, the Optimized Schwarz Waveform Relaxation (OSWR) method has been introduced to provide parallel efficiency. It uses Robin type transmission conditions in which coefficients can be optimized to improve convergence rates. The objective is to extend the OSWR method for the Stokes problem describing viscous incompressible flows and combine OSWR and the algebraic domain decomposition developed in the `TrioCFD` code to take full advantage of the massively parallel computers available today. This work is part of the ANR project CINE-PARA.

1 Context and state of the art

The OSWR approach introduced in [12] is a space-time domain decomposition method derived at the continuous level so that the time-dependent problems are solved in each subdomain (resulting from a spatial decomposition) and the information is then exchanged over space-time interfaces between subdomains. As a result, different numerical schemes both in space and in time can be used in the subdomains [16] and less communication cost is needed (in terms of parallel computations) as the data is transferred only at the end of the time interval. Moreover, this iterative algorithm exchanges space-time boundary data through Robin or Ventcell transmission operators in which coefficients can be optimized to improve convergence rates.

The LAGA and CEA groups have expertise with space-time domain decomposition in the context of fluids and parabolic-type problems. The OSWR method has been developed and successfully used for advection-reaction-diffusion problems [18, 1] with applications to porous media flow and transport [15, 2], and hyperbolic problems [13] and the linear viscous Shallow Water system [19]. As far as the Stokes and Navier-Stokes systems are concerned we are aware of

- compressible Euler - Navier-Stokes coupling for aeroacoustics problems in [6], with promising numerical results but without any detailed description and analysis of the transmission conditions,
- the PhD Thesis [9] in which a (space) Schwarz method is proposed at each time step of a time marching integration for time dependent Stokes and Navier-Stokes equations. One chapter deals also with preliminary theoretical aspects of Schwarz waveform relaxation for the Oseen equations, without any numerical experiment nor convergence analysis,
- the PhD Thesis [20] which studies the coupling of the 2D shallow water equations and the 3D hydrostatic Navier-Stokes system, without any incompressibility constraint,
- Euler and compressible Navier-Stokes equations [10], with overlapping subdomains and Robin parameters chosen heuristically, and with a study of different methods to combine the nonlinear solver and the domain decomposition method (Newton outside - DD inside or DD outside - Newton inside),

but there has not been, up to now, any use and analysis of this type of method for the time dependent, *incompressible* Stokes and Navier-Stokes equations.

The OSWR method can be formulated as an interface problem on the space-time interfaces between subdomains (see e.g. [15]) which is then solved by an iterative method (Jacobi, Krylov,...), in which each iteration needs to solve independent local problems and leads to a natural parallel implementation.

2 Objectives of the project and working program

The final objective within the project CINE-PARA is the extension to the Navier-Stokes problem for the OSWR method. Such approaches will be very useful for the numerical studies in the nuclear industry, in particular the interaction of turbulent flow with the solid structures of nuclear reactors. Such applications are currently investigated by a CFD code, called `TrioCFD` (formerly named `Trio_U`, see [7, 5]), which has been developed for about 20 years in the Nuclear Energy Division of the CEA. The code is designed to treat efficiently various physical problems, such as turbulent flows, fluid/solid coupling, multiphase flows or flows in porous media (see for example [8, 3]). It was developed for local and small-scale calculations using especially Reynolds-Averaged Navier-Stokes (RANS) and Large Eddy Simulation (LES) models [4]. For such simulations, more than 150 million cells and one billion degrees of freedom may be needed over hundreds of thousands of time steps. This necessitates the use of parallel computations. The code `TrioCFD` currently uses massive parallelism (10,000 processors) to handle such simulations. However the parallelism is used only to speed up matrix-vector products that occur within the iterative methods used to solve large linear systems, without a real domain decomposition strategy and thus with a cost which may increase rapidly without preconditioning strategies. An advantage of OSWR space-time domain decomposition is that it both allows partitioning the computation and serves as a preconditioning tool based on the continuous problem. Moreover the parallel approaches currently developed in `TrioCFD` could be used in each subdomain of the DD method.

For the OSWR algorithm, a critical ingredient to the formulation and the success of the method is the proper choice of transmission conditions, so that the correct physical quantities are exchanged across the subdomain interface. The choice of interface conditions can have a dramatic influence on the success of Schwarz domain decomposition methods (see for instance [17, 18, 1].

In a first step, we will neglect the nonlinear advection term and consider the application to the incompressible Stokes equations,

$$\partial_t \boldsymbol{u} + \nabla p - \nu \Delta \boldsymbol{u} = \boldsymbol{f} \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0. \tag{2}$$

It will involve Robin transmission operator in the form

$$\alpha(\boldsymbol{u})(-\nu \nabla \boldsymbol{u} + pI) \cdot \boldsymbol{n} + \boldsymbol{u} \tag{3}$$

which induces the presence of both flux and pressure in the transmission conditions.

This kind of mixed problem with mixed boundary condition has been studied in the simpler context of a mixed formulation of the advection-diffusion equation in [14], which could be a good starting point for our own study. However, we expect that one major additional difficulty will come from the incompressibility constraint.

As far as the space discretization is concerned, we will consider finite elements or hybrid finite-volume discretization, like the finite-volume elements (V.E.F.) scheme used in `TrioCFD` for tetrahedra [11]. The velocity variables are located at the center of the cell faces, while pressure unknowns are located both at the nodes and cell centers. An important issue here will be the correct discretization of the optimized transmission conditions prior to an implementation of the method in `TrioCFD`.

Precisely, the goal of the work during the CEMRACS is:

- to study Robin transmission conditions for the Stokes problem at the continuous PDE level (well-posedness, equivalence between the multi- and mono- domain formulations),
- to study the numerical scheme implemented in `TrioCFD`,
- to study how to adapt `TrioCFD` to handle Robin conditions, and to propose a multidomain numerical scheme that will be equivalent to the mono-domain one,
- to analyze a possible way to adapt this to `TrioCFD` time discretization of the time-dependent Stokes system, and propose a strategy accordingly.
- to implement the space-time domain decomposition method in `TrioCFD` and show numerical results for a standard benchmarks such as lid-driven cavity flows (Re = 400 and 1000).
- to study efficient parallelization of the space-time domain decomposition for the Stokes equation, implement it and evaluate the speed-up obtained on parallel platforms.

3 Partners, participants and funding

Project partners:

• **CEA** has expertise in the nuclear industry, in particular the interaction of turbulent flow with the solid structures of nuclear reactors, currently investigated by `TrioCFD` code. It aims to develop numerical methods (domain decomposition, parallelism in time) for solving the Navier-Stokes equations and for Large eddy simulation (LES) models for turbulence.

• **LAGA** (University Paris 13) has an expertise in numerical methods for PDEs and scientific computing, especially in domain decomposition methods (optimized Schwarz methods) for fluid dynamics.

• **LIPN** (University Paris 13) has an expertise in High Performance Computing and large-scale parallel applications.

This project is part of the **ANR CINE-PARA** (*Parallelisation methods for complex kinetics*, ANR-15-CE23-0019) led by Yvon Maday, which brings together LJLL, CEA, LAGA, INRIA, CEREMADE and ENPC/NAVIER teams, and aims to bring domain decomposition and parallel in time methods to larger classes of parallel architectures, increase the theoretical understanding and expand the domains of application including industrial size problem.

Project participants:

- Katia Ait Ameur (M2 UPMC, Master internship at CEA, 6 weeks)
- Camille Coti (LIPN, University Paris 13, 2 weeks)
- Caroline Japhet (LAGA, University Paris 13, 4 weeks)
- Pascal Omnes (CEA and University Paris 13, 2 weeks)
- Mathieu Peybernes (CEA, 1 week)
- Thomas Rubiano (PhD, LCR, LIPN, University Paris 13, 6 weeks)

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