CEMRACS 2016 project: 'RB-component' Component Mapping Automation for Parametric Component Reduced Basis Techniques

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Objectives of the project, working program and funding

The idea is to develop some techniques for automation of the mappings (to reference domain) required by reduced basis methods : the development of geometry mappings is indeed often a substantial impediment in the implementation of reduced basis techniques, especially in the context of reduced element strategies. In the Akselos context, the geometry mappings are applied at the level of components.

The proposed procedure (and hence related research program) is as follows:

1. Describe the boundary surfaces of a component in terms of parametrized level sets, or intersections of level sets. Level sets can be an intuitive and relatively easy way to describe parametrized boundaries.

2. Develop a good mesh — a mesh which anticipates parametric transformation and hence deformation — for the reference domain.

3. Apply methods from optimal transport (related to "registration" in the image-processing business) to develop mappings — velocities and then displacements — for the boundary surfaces, and in particular the boundary nodes of the reference domain mesh, as a function of parameter.

   We would solve a surface $L^2 + H^1$ minimization problem for node velocities subject to a convection equation which constrains the points to reside on the parametrized level-set which defines the boundary; we would then integrate this velocity to obtain the associated displacements. (Note the H^1 part of the objective function would permit imposition of end conditions on the velocity for level sets which are not closed; this is mathematically related to variational approaches for surface tension boundary conditions in fluid mechanics.)

4. Solve the equations of linear elasticity (for Poisson's ratio = 0) for the displacement boundary conditions developed in 3) to obtain the parametric mapping for the entire domain — boundary surfaces and interior; zero normal displacement can also be a convenient boundary condition in certain cases.

   In actual practice of course we would not consider full FE as this would be too expensive (online). Instead, we have two options:

(a) RB : In this case we would, in the online stage, first solve a single RB (elasticity) problem for the mapping functions, and then the actual RB problem(s) for the PDE of interest. In the component context, there are actually many RB problems for the PDE — an RB bubble problem for each port mode — such that the cost of the mapping RB is arguably negligible.

    Note in the offline stage we would pre-store the various inner products between mapping RB functions and bubble RB functions, much as we now store inner products between mapping EIM functions and bubble RB functions.

(b) Restricted-Domain EIM. Here we apply a variant of the EIM scheme in which the magic points are required to reside on a restricted subdomain of the full domain. In this case, the restricted domain would be the boundary surfaces (for which the mapping function is known prior to solution of the linear elasticity lifting — such that we can indeed apply EIM).

5. Test the approach on various cases to apprehend the limits of the approach and try to foresee and overcome the possible failures.

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