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Feel++ for Red Blood Cells in 3D

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The simulation of suspensions of red blood cells (RBCs) in blood plasma is a cornerstone to understand the rheological properties of blood from a microscopic point of view. Indeed, red blood cells occupy the major volume fraction of formed constituents in the blood, and are therefore responsible for a large part of the complex macroscopic non-Newtonian behaviour of blood flow.

In 3D, RBC can be modelled as vesicles, i.e. non-porous inextensible membranes filled with a Newtonian fluid and added with two additional forces: a bending force accounting for the lipidic bilayered structure of the biological cell membrane, and a surface shear force.

The goal is then to numerically simulate such "enhanced" vesicles flowing in a Newtonian outer fluid with some given system geometry and inlet flow. The problem can be discretized within a finite-element framework, using the level-set method to track the position of the membranes. The fluids – both inner and outer – obey the Navier-Stokes or Stokes equations with position-dependent density and viscosity parameters, and take the forces exerted by the membranes into account. The level-set dynamics is governed by an advection equation involving the fluid velocity field. The fluid and level-set equations are coupled using an explicit non-monolithic approach. After natural time discretization (and after some linearization step in the Navier-Stokes case, using the Newton or Oseen iteration strategy), solving the problem amounts to inverting a large linear system a large number of times.

Results have already been obtained in 2D, where the surface shear force is irrelevant. In addition to the obvious difficulties pertaining to the resolution and preconditioning of the linear systems resulting from the fluid dynamics, one of the main challenges highlighted by these results is the treatment of the inextensibility constraint. Two methods have been considered: the elastic force and the Lagrange multiplier method. The first one consists in introducing a strong elastic force to "penalize" stretching of the membrane, while the second one exactly imposes local surface area conservation through the resolution of a saddle-point problem. If the latter is more accurate, it nonetheless involves the resolution of a much larger and ill-conditioned linear system for each iteration. The elastic force approach, formulated within an explicit coupling scheme between the fluid and the level-set is algebraically simpler, but it shows instabilities and requires carefull tuning of the simulation parameters in order to obtain a stable formulation and an accurate inextensibility while preserving reasonable computational times.

This project therefore aims at developing efficient numerical approaches to tackle the membrane inextensibility in 3D problems, and to incorporate surface shear forces in the computation. It will rely on the Feel++ finite-element C++ library for numerical aspects in order to design reliable methods and implement them following a high-performance computing approach.

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