CEMRACS 2016: project 'ANIM 3D '
ANisotropic Implicit MHD in 3D

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Abstract

This project has two main parts: enhancing the performance of our Isogeometric/Isoparametric finite elements framework by implementing the Jacobian-Free Krylov-Newton strategy and adding OpenMP4.0 multithreading and vectorization to our actual MPI parallelization. The second part of this work deals with stabilized finite elements and physics-based preconditioning techniques.

Introduction

The aim of magnetic confinement fusion is to develop a power plant that gains energy from the fusion of deuterium and tritium in a magnetically confined plasma. ITER, a tokamak type fusion experiment currently being built in the South of France, is the next step towards this goal.

One of the many challenges for the numerical simulation in a tokamak is the modeling and the simulation of magneto-hydrodynamics (MHD) instabilities as disruptions or ELMs. These instabilities which occur at the boundary of the plasma generate a loss of energy and can damage the wall of the tokamak. For this reason it is necessary to understand the behavior of these instabilities and find a way to control them using experiment and simulation. A physical model well suited to describe those large scale instabilities is the set of magneto-hydrodynamic equations (MHD) with resistivity and bi-fluid effects.

The Project

The numerical simulation of large-scale MHD instabilities in confined magnetic fusion devices (called a tokamak), relies on solving a Fluid model coupled with Maxwell equations

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \boldsymbol{u}) = 0, \\ \rho \partial_t \boldsymbol{u} + \rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \nabla p = \boldsymbol{J} \times \boldsymbol{B} + \nu \Delta \boldsymbol{u}, \\ \partial_t T + \boldsymbol{u} \cdot \nabla T + \gamma T \nabla \cdot \boldsymbol{u} - \nabla \cdot (K \nabla T) = (\gamma - 1)\eta |\boldsymbol{J}|^2 \\ \partial_t \boldsymbol{B} = \nabla \times \left(\boldsymbol{u} \times \boldsymbol{B} - \eta \boldsymbol{J} - \dfrac{m_i}{\rho e} (\boldsymbol{J} \times \boldsymbol{B}) \right), \\ \nabla \cdot \boldsymbol{B} = 0, \quad \nabla \times \boldsymbol{B} = \boldsymbol{J}. \end{cases}$$

with a magnetic field given by $\boldsymbol{B} = \boldsymbol{B}_{tor} + \boldsymbol{B}_{pol} = \frac{F_0}{R}\mathbf{e}_\phi + \frac{1}{R}\nabla\psi \times \mathbf{e}_\phi$ and $| \boldsymbol{B}_{tor} | >> | \boldsymbol{B}_{pol} |$.

Solving these equations in the complicated geometry of the tokamak is a highly demanding task due to the strong temporal and spatial multi-scale nature of the problem and the anisotropies induced by the magnetic field. Such simulations require large numerical codes like the European non-linear MHD code JOREK which solves the equations fully implicitly with cubic Hermite-Bézier finite elements, for the poloidal plane and a toroidal Fourier representation. Each time step requires to invert an ill-conditioned large sparse matrix. A new version of this code, JOREK-Django, has been developed and is based on Isoparametric /IsoGeometric approachs. The use of the k-refinement allows us to reduce the size of the sparse matrices, while the high order B-splines allows a local

refinement in the toroidal direction, compared to the Fourier discretization, which is suitable for Pellet-Injections. MHD models are divided into two families: reduced and full MHD models. In this work, we restrict our study to the case of the reduced MHD, where high order operators impose the use of at least C2 elements, or C1 elements in the case of a mixed formulation. For the time scheme we use a Cranck-Nicholson scheme coupled with a Preconditioned GMRES method for implicit scheme.The preconditioning is a physics based method which consists of approximating the solution of the full model by a relevant sequence of simple advection and diffusion problems. The preconditioning is obtained by solving accurately each sub-system using algebraic solvers.

Running a MHD simulation on a fine 3D grid is a highly demanding task. In order to reduce the CPU time for the assembly step we propose to add multithreading and vectorization using OpenMP. The memory consumption should also be improved thanks to the use of the Jacobian-Free Krylov-Newton procedure, which allows to avoid storing the different matrices arising from the Finite Elements discretization. The preconditioning step should be improved too with three strategies: using stabilized finite elements, in order to have well-posed operators for the sub-systems, using a new variant of physics-based preconditioning and finally, using specific multi-grids solver for high order finite element method.

Objectives/Expected Results

As a final application, we propose to study some realistic test cases (Kink instability, Tearing modes or Ballooning modes) using a reduced MHD model (without the parallel velocity). For this reason, we need first, to solve, for the poloidal flux ψ, the axisymetric equilibrium, a nonlinear elliptic partial differential equation, known as Grad-Shafranov-Schlüter equation

$$\Delta^* \psi = -R^2 \frac{dP}{d\psi} - F \frac{dF}{d\psi}$$

A perturbation is then applied, and the reduced MHD equations are solved using a Crank-Nicholson time scheme. The targeted grids are 64×64 and 128×128 in the poloidal plane, and $16 - 24$ elements in the toroidal direction, using high order B-Splines discretization or between 5 and 10 mods using Fourier expansion. An exhaustive comparison with the actual JOREK should be done, in terms of CPU time and memory consumption. An extension to more complex geometry and more complex models can be also prepared.

Expected participant and funding

Expected Participants :

- E. Franck (INRIA Nancy): 2 or 3 weeks
- A. Ratnani (IPP Garching): 1 week
- B. Nkonga (Nice) : 1 or 2 weeks
- H. Guillard (INRIA Nice) : 1 or 2 weeks
- J. Lakhili (Post doc IPP Garching) : 4 weeks
- M. Gaja (Phd IPP Garching) : 4 weeks
- J. Costa (Nice) : 2 weeks

- E. Estibals (Nice) : 2 weeks
- H. Heumann (INRIA Nice) : 2 weeks

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